

## 7.5 Answers to 7.3 Exercise

### GCSE and Preparatory A-Level Mathematics Conic Sections (Simultaneous Equations IV)

#### Answer 1

$$x^2 + y^2 - 2y = 24$$

$$y = x + 2$$

#### METHOD 1

$$x^2 + [(x + 2)^2] - 2(x + 2) = 24$$

$$x^2 + [x^2 + 4x + 4] - 2x - 4 = 24$$

$$x^2 + x^2 + 4x + 4 - 2x - 4 = 24$$

$$2x^2 + 2x - 24 = 0$$

Divide through by 2

$$x^2 + x - 12 = 0$$

$$(x + 4)(x - 3) = 0$$

$$\text{Either } x + 4 = 0 \text{ or } x - 3 = 0$$

$$x = -4 \text{ or } x = 3$$

Use  $y = x + 2$  to get the y-part of the coordinates

Answer : Points of intersection are  $(-4, -2)$  or  $(3, 5)$

#### METHOD 2

$$[(y - 2)^2] + y^2 - 2y = 24$$

$$y^2 - 4y + 4 + y^2 - 2y = 24$$

$$2y^2 - 6y - 20 = 0$$

Divide through by 2

$$y^2 - 3y - 10 = 0$$

$$(y - 5)(y + 2) = 0$$

$$\text{Either } y - 5 = 0 \text{ or } y + 2 = 0$$

$$y = 5 \text{ or } y = -2$$

Use  $x = y - 2$  to get the y-part of the coordinates

Answer : Points of intersection are  $(-4, -2)$  or  $(3, 5)$

**Answer 2**

$$2x^2 + 3y^2 = 14$$

$$x = 2y - 3$$

$$2 \left[ (2y - 3)^2 \right] + 3y^2 = 14$$

$$2 \left[ 4y^2 - 12y + 9 \right] + 3y^2 = 14$$

$$8y^2 - 24y + 18 + 3y^2 - 14 = 0$$

$$11y^2 - 24y + 4 = 0$$

$$(11y - 2)(y - 2) = 0$$

$$\text{Either } 11y - 2 = 0 \text{ or } y - 2 = 0$$

$$y = \frac{2}{11} \text{ or } y = 2$$

Use  $x = 2y - 3$  to get the  $y$ -part of the coordinates

Answer : Points of intersection are  $\left(-\frac{29}{11}, \frac{2}{11}\right)$  or  $(1, 2)$

**Answer 3**

$$3x^2 + 6x - 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with  $a = 3$ ,  $b = 6$  and  $c = -5$  gives

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 3 \times (-5)}}{2 \times 3}$$

$$= \frac{-6 \pm \sqrt{36 + 60}}{6}$$

$$= \frac{-6 \pm \sqrt{96}}{6}$$

$$= \frac{-6}{6} \pm \frac{\sqrt{16 \times 6}}{6}$$

$$= -1 \pm \frac{4\sqrt{6}}{6}$$

$$= -1 \pm \frac{2\sqrt{6}}{3}$$

To 3 significant figures the solutions are  $-2.63$  or  $0.633$

**Answer 4**

$$x^2 - 3xy + 2y^2 = 0$$

$$y = 9 - x$$

$$x^2 - 3x(9 - x) + 2(9 - x)^2 = 0$$

$$x^2 - 27x + 3x^2 + 2[81 - 18x + x^2] = 0$$

$$x^2 - 27x + 3x^2 + 162 - 36x + 2x^2 = 0$$

$$6x^2 - 63x + 162 = 0$$

$$2x^2 - 21x + 54 = 0$$

$$(x - 6)(2x - 9) = 0$$

$$\text{Either } x - 6 = 0 \text{ or } 2x - 9 = 0$$

$$x = 6 \text{ or } x = \frac{9}{2}$$

Use  $y = 9 - x$  to get the y-part of the coordinates

Answer : Points of intersection are ( 6, 3 ) or ( 4.5, 4.5 )

**Answer 5**

$$2x + y = -3 \Leftrightarrow y = -2x - 3$$

$$y = (-1)(2x + 3)$$

$$y^2 = (2x + 3)^2$$

$$2y^2 + x^2 = -6x + 42$$

$$2(2x + 3)^2 + x^2 = -6x + 42$$

$$2(4x^2 + 12x + 9) + x^2 = -6x + 42$$

$$8x^2 + 24x + 18 + x^2 + 6x - 42 = 0$$

$$9x^2 + 30x - 24 = 0$$

$$3x^2 + 10x - 8 = 0$$

$$(3x - 2)(x + 4) = 0 \text{ (Or use the Q formula)}$$

$$\text{Either } x = \frac{2}{3} \text{ or } x = -4$$

Use  $y = (-1)(2x + 3)$  to get the y-part of the coordinates

Points of intersection are  $\left(\frac{2}{3}, -\frac{13}{3}\right)$  or  $(-4, 5)$