

8.2 Answers

GCSE and Preparatory A-Level Mathematics Conic Sections (Simultaneous Equations IV)

Answer 1

$$2x^2 + 10x + 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with $a = 2$, $b = 10$ and $c = 5$ gives

$$x = \frac{-10 \pm \sqrt{10^2 - 4 \times 2 \times 5}}{2 \times 2}$$

$$= \frac{-10 \pm \sqrt{100 - 40}}{4}$$

$$= \frac{-10 \pm \sqrt{60}}{4}$$

$$= \frac{-10}{4} \pm \frac{\sqrt{4 \times 15}}{4}$$

$$= -\frac{10}{4} \pm \frac{2\sqrt{15}}{4}$$

$$= -\frac{5}{2} \pm \frac{\sqrt{15}}{2} \quad \square$$

Answer 2

$$2x^2 + 3y^2 = 5$$

$$y = 2x + 1$$

$$2x^2 + 3[(2x + 1)^2] = 5$$

$$2x^2 + 3[4x^2 + 4x + 1] = 5$$

$$2x^2 + 12x^2 + 12x + 3 - 5 = 0$$

$$14x^2 + 12x - 2 = 0$$

Divide through by 2

$$7x^2 + 6x - 1 = 0$$

$$(7x - 1)(x + 1) = 0$$

Either $7x - 1 = 0$ or $x + 1 = 0$

$$x = \frac{1}{7} \text{ or } x = -1$$

Use $y = 2x + 1$ to get the y-part of the coordinates

Answer : Points of intersection are $\left(\frac{1}{7}, \frac{9}{7}\right)$ and $(-1, -1)$

Answer 3

$$x^2 - 5x - 36 = (x - 9)(x + 4)$$

Answer 4

$$x + 2y = -0.5$$

$$3x - y = 16$$

$$\begin{array}{rcl} x + 2y & = & -0.5 \\ 6x - 2y & = & 32 \quad \text{ADD} \\ \hline 7x & = & 31.5 \\ x & = & 4.5 \quad \therefore y = -2.5 \end{array}$$

Answer 5

$$2x^2 - 7x + 6 = (2x - 3)(x - 2)$$

Answer 6

Square both sides of the equation of the hyperbola,

$$y = \frac{12}{x} \Rightarrow y^2 = \frac{144}{x^2}$$

Then substitute this in place of the y^2 in the equation of the circle,

$$x^2 + \left(\frac{144}{x^2} \right) = 24$$

Multiply through by x^2

$$x^2 \times x^2 + \frac{x^2}{1} \times \left(\frac{144}{x^2} \right) = x^2 \times 24$$

$$x^4 + 144 = 24x^2$$

$$x^4 - 24x^2 + 144 = 0$$

$$\text{Let } z = x^2 \Rightarrow z^2 = x^4$$

$$z^2 - 24z + 144 = 0$$

$$(z - 12)(z - 12) = 0$$

$$\therefore z - 12 = 0 \Rightarrow z = 12$$

$$\therefore x^2 = 12 \Rightarrow x = \pm\sqrt{12}$$

Use the equation for the hyperbola to get the y part of the points,

$$y = \frac{12}{x}$$

The two points of intersection are,

$$(-\sqrt{12}, -\sqrt{12}), (\sqrt{12}, \sqrt{12})$$

Answer 7

$$4x^2 + 2x - 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with $a = 4$, $b = 2$ and $c = -7$ gives

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{2^2 - 4 \times 4 \times (-7)}}{2 \times 4} \\ &= \frac{-2 \pm \sqrt{4 + 112}}{8} \\ &= \frac{-2 \pm \sqrt{116}}{8} \\ &= \frac{-2}{8} \pm \frac{\sqrt{4 \times 29}}{8} \\ &= -\frac{1}{4} \pm \frac{\sqrt{29}}{4} \end{aligned}$$

To 2 significant figures the solutions are 1.1 or -1.6

Answer 8**(a)**

Area of a trapezium is half the sum of the parallel sides time the distance in between

$$\therefore 133 = \frac{1}{2} ((3x - 2) + (x + 5)) \times (2x - 3)$$

$$2 \times 133 = (4x + 3) \times (2x - 3)$$

$$266 = 8x^2 - 12x + 6x - 9$$

$$8x^2 - 6x - 9 - 266 = 0$$

$$8x^2 - 6x - 275 = 0 \quad \square$$

(b)

Guessing the brackets is not impossible but also not easy !

Possibly it's quicker to use the Q formula, being careful with minus signs...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with $a = 8$, $b = -6$ and $c = -275$ gives

$$x = \frac{6 \pm \sqrt{6^2 - 4 \times 8 \times (-275)}}{2 \times 8}$$

$$= \frac{6 \pm \sqrt{36 + 8800}}{16}$$

$$= \frac{6 \pm \sqrt{8836}}{16}$$

$$= \frac{6 \pm 94}{16}$$

$$= -\frac{11}{2}, \frac{25}{4}$$

Only the positive value of x makes sense in the context of the question

Note : The factorisation into brackets would be $(2x + 11)(4x - 25)$

Answer 9

$$(10x - 3)(x + 1) = 6x$$

$$10x^2 + 10x - 3x - 3 - 6x = 0$$

$$10x^2 + x - 3 = 0$$

$$(5x + 3)(2x - 1) = 0$$

$$\text{Either } 5x + 3 = 0 \text{ or } 2x - 1 = 0$$

$$x = -\frac{3}{5} \text{ or } x = \frac{1}{2}$$

Use $y = 6x$ to get the y -part of the coordinates

Points of intersection are $\left(-\frac{3}{5}, -\frac{18}{5}\right)$ or $\left(\frac{1}{2}, 3\right)$

or, as decimals, $(-0.6, -3.6)$ or $(0.5, 3)$

$$\text{Midpoint} \left(\frac{-0.6 + 0.5}{2}, \frac{-3.6 + 3}{2} \right) = (-0.05, -0.3)$$