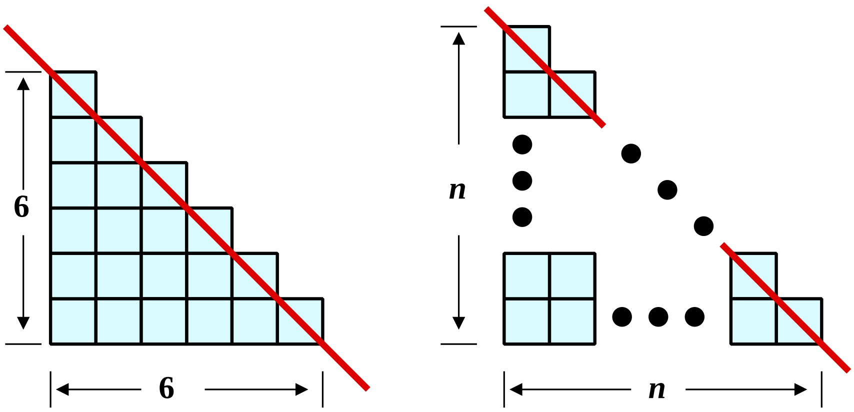
1.3 Solutions

1.3.1 Answer to Teaching Video Example



The diagrams do most of the work !

For the specific case of the sixth triangular number, the idea is to obtain the answer in a way that can then be generalised,

$$\sum_{1}^{6} r = 1 + 2 + 3 + 4 + 5 + 6 \quad \text{from which the diagram is drawn}$$

$$= \frac{1}{2} \times 6 \times 6 + 6 \times \frac{1}{2} \quad \text{areas from the diagram}$$

$$= 18 + 3$$

$$= 21$$

From which the general case is deduced,

$$\sum_{1}^{n} r = 1 + 2 + \dots + (n - 1) + n \quad \text{from which the diagram is drawn}$$

$$= \frac{1}{2} \times n \times n + n \times \frac{1}{2} \quad \text{areas from the diagram}$$

$$= \frac{1}{2} n (n + 1) \quad \square$$

[6 marks]

1.3.2 Answers to 1.2 Exercise

Answer 1

$$\sum_{1}^{100} r = \frac{1}{2} \times 100 \times 101$$

$$= 5050$$

[2 marks]

Answer 2

$$\sum_{1}^{200} r = \frac{1}{2} \times 200 \times 201$$

$$= 20100$$

[2 marks]

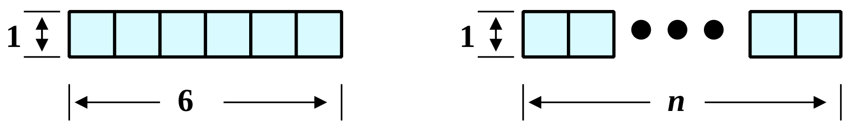
Answer 3

$$\sum_{1}^{500} r = \frac{1}{2} \times 500 \times 501$$

$$= 125250$$

[2 marks]

Answer 4



For the specific case,

$$\sum_{1}^{6} 1 = 1 + 1 + 1 + 1 + 1 + 1 \quad \text{from which the diagram is drawn}$$

$$= 1 \times 6 \quad \text{area from the diagram}$$

$$= 6$$

From which the general case is deduced,

$$\sum_{1}^{n} 1 = 1 + 1 + \dots + 1 + 1 \quad \text{from which the diagram is drawn}$$

$$= 1 \times n \quad \text{area from the diagram}$$

$$= n \qquad \square$$

[4 marks]

Answer 5

(i) 50

(ii) 115

[2 marks]

Answer 6

(i)

$$\sum_{1}^{6} (3r - 2) = 1 + 4 + 7 + 10 + 13 + 16$$

$$= 51$$

(ii)

$$3 \sum_{1}^{6} r - 2 \sum_{1}^{6} 1 = 3 \times \frac{1}{2} \times 6 \times 7 - 2 \times 6$$

$$= 63 - 12$$

$$= 51$$

[4 marks]

Answer 7

$$\sum_{1}^{14} (4r + 5) = 4 \sum_{1}^{14} r + 5 \sum_{1}^{14} 1$$

$$= 4 \times \frac{1}{2} \times 14 \times 15 + 5 \times 14$$

$$= 420 + 70$$

$$= 490$$

[4 marks]

Answer 8

$$\sum_{1}^{30} (2r + 7) = 2 \sum_{1}^{30} r + 7 \sum_{1}^{30} 1$$

$$= 2 \times \frac{1}{2} \times 30 \times 31 + 7 \times 30$$

$$= 930 + 210$$

$$= 1140$$

[4 marks]

Answer 9

$$\sum_{4}^{7} r = \sum_{1}^{7} r - \sum_{1}^{3} r$$

$$= \frac{1}{2} \times 7 \times 8 - \frac{1}{2} \times 3 \times 4$$

$$= 28 - 6$$

$$= 22$$

[4 marks]

Answer 10

$$\frac{1}{100}\sum_{80}^{120} r = \frac{1}{100}\left(\sum_{1}^{120} r - \sum_{1}^{79} r\right)$$

$$= \frac{1}{100}\left(\frac{1}{2}\times 120 \times 121 - \frac{1}{2}\times 79 \times 80\right)$$

$$= \frac{1}{100}\,(\,7260 - 3160\,)$$

$$= \frac{1}{100} \times 4100$$

$$= 41$$

[4 marks]

Answer 11

(i)

$$\text{LHS} = \sum_{r=1}^{n} (\,7r - 4\,)$$

$$= 7\sum_{1}^{n} r - 4\sum_{1}^{n} 1$$

$$= 7 \times \frac{1}{2}\, n\,(\,n + 1\,) - 4n$$

$$= \frac{7}{2}n^2 + \frac{7}{2}n - \frac{8}{2}n$$

$$= \frac{7}{2}n^2 - \frac{1}{2}n$$

$$= \frac{1}{2}\, n\,(\,7n - 1\,)$$

$$= \text{RHS} \qquad \square$$

[4 marks]

(ii)

$$\sum_{r=20}^{50} (\,7r - 4\,) = \sum_{1}^{50} (\,7r - 4\,) - \sum_{1}^{19} (\,7r - 4\,)$$

$$= \frac{1}{2}\times 50 \times 349 - \frac{1}{2} \times 19 \times 132$$

$$= 8725 - 1254$$

$$= 7471$$

[4 marks]

Answer 12

$$\sum_{r=1}^{n} r = 528$$

$$\frac{1}{2}n(n+1) = 528$$

$$n^2 + n = 1056$$

$$n^2 + n - 1056 = 0$$

$$\left[\begin{array}{c} FACT\{1056\} = 2^5 \times 3 \times 11 \\ \sqrt{1056} = 32.5 \end{array}\right]$$

$$(n-32)(n+33) = 0$$

$$\text{Either } n = 32 \text{ or } n = -33 \text{ (rejected)}$$

$$\therefore n = 32$$

[4 marks]

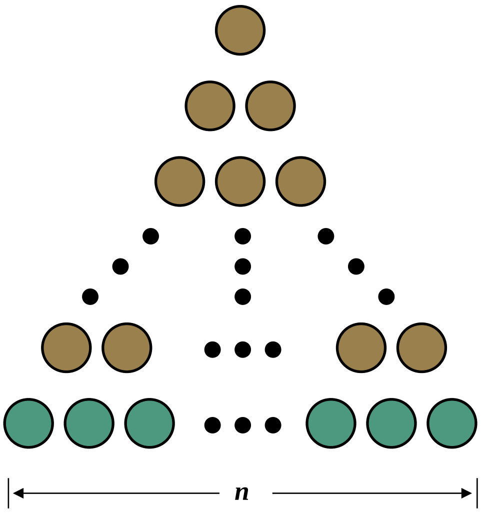
Answer 13

(i)

$$\begin{aligned}\text{LHS} &= T_n - T_{n-1} \\ &= \sum_{1}^{n} r - \sum_{1}^{n-1} r \\ &= \frac{1}{2} n(n+1) - \frac{1}{2}(n-1)n \\ &= \frac{1}{2} n((n+1) - (n-1)) \\ &= \frac{1}{2} n(2) \\ &= n \\ &= \text{RHS} \qquad \square\end{aligned}$$

[4 marks]

(ii)



Any triangular number T_n (brown and green) take away the triangular number immediately before T_{n-1} (brown) leaves only the base row (green) of n spots.

Thus,

$$T_n - T_{n-1} = n \qquad \square$$

[6 marks]