## 2.4 Answers



### 2.4.1 Solutions to 2.2 Teaching Video Example

$$\sum_{r=n}^{2n} (15 - 2r) = 0$$

$$\sum_{1}^{2n} (15 - 2r) - \sum_{1}^{n} (15 - 2r) = 0$$

$$15 \sum_{1}^{2n} 1 - 2 \sum_{1}^{2n} r - 15 \sum_{1}^{n-1} 1 + 2 \sum_{1}^{n-1} r = 0$$

$$15 \times 2n - 2 \times \frac{1}{2} (2n)(2n + 1) - 15(n - 1) + 2 \times \frac{1}{2} (n - 1)(n) = 0$$

$$30n - 4n^2 - 2n - 15n + 15 + n^2 - n = 0$$

$$-3n^2 + 12n + 15 = 0$$

divide through by $(-3)$

$$n^2 - 4n - 5 = 0$$

$$(n - 5)(n + 1) = 0$$

Either $n = 5$ or $n = -1$ (reject)

$$\therefore\ n = 5$$

Writing out the sequence helps to make sense of what has been achieved;

| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Term | 13 | 11 | 9 | 7 | 5 | 3 | 1 | $-1$ | $-3$ | $-5$ | $-7$ |

**[ 6 marks ]**

### 2.4.2 Solutions to 2.3 Exercise

**Answer 1**

**( i )** $7 + 13 + 19 + 25 + 31 = 5 \times 19$

**[ 1 mark ]**

**( ii )**

$$\begin{aligned} \text{LHS} &= \sum_{r=1}^{n} ( 6r + 1 ) \\ &= 6 \sum_{1}^{n} r + \sum_{1}^{n} 1 \\ &= 6 \times \frac{1}{2} \times n \times ( n + 1 ) + n \\ &= 3 n^2 + 3n + n \\ &= n ( 3n + 4 ) \\ &= \text{RHS} \qquad \square \end{aligned}$$

**[ 4 marks ]**

**Answer 2**

**( i )** $1 + 5 + 9 + 13 + 17 = 5 \times 9$

**[ 1 mark ]**

**( ii )**

$$\sum_{1}^{n} ( 4r - 3 ) = n ( 2n - 1 )$$

**[ 2 marks ]**

**( iii )**

$$\begin{aligned} \text{LHS} &= \sum_{1}^{n} ( 4r - 3 ) \\ &= 4 \sum_{1}^{n} r - 3 \sum_{1}^{n} 1 \\ &= 4 \times \frac{1}{2} \times n ( n + 1 ) - 3n \\ &= 2 n^2 + 2n - 3n \\ &= 2 n^2 - n \\ &= n ( 2n - 1 ) \\ &= \text{RHS} \qquad \square \end{aligned}$$

**[ 4 marks ]**

**Answer 3**

$$\text{LHS} = \sum_{r=1}^{2n} (5r - 4)$$

$$= 5\sum_{r=1}^{2n} r - 4\sum_{r=1}^{2n} 1$$

$$= 5 \times \frac{1}{2} \times 2n \times (2n + 1) - 4 \times 2n$$

$$= 10n^2 + 5n - 8n$$

$$= 10n^2 - 3n$$

$$= n(10n - 3)$$

$$= \text{RHS} \qquad \square$$

**[ 4 marks ]**

**Answer 4**

**( i )**

$$\sum_{r=1}^{2n-1} r = \frac{1}{2} \times (2n - 1)\, 2n$$

$$= n(2n - 1)$$

**[ 3 marks ]**

**( ii )**

$$\text{LHS} = \sum_{r=n+1}^{2n-1} r$$

$$= \sum_{1}^{2n-1} r - \sum_{1}^{n} r$$

$$= n(2n - 1) - \frac{1}{2} \times n \times (n + 1)$$

$$= \left(\frac{1}{2}n\right)(4n - 2 - n - 1)$$

$$= \frac{1}{2}n(3n - 3)$$

$$= \frac{3}{2}n(n - 1)$$

$$= \text{RHS} \qquad \square$$

**[ 3 marks ]**

**Answer 5**

$$
\begin{aligned}
\text{RHS} &= n^2\, T_{n-1} + n\, T_n \\
&= n^2 \sum_{1}^{n-1} r + n \sum_{1}^{n} r \\
&= n^2 \times \frac{1}{2} \times (n-1)\, n + n \times \frac{1}{2} \times n\,(n+1) \\
&= \left(\frac{1}{2} n^2\right) \left( (n-1)\, n + (n+1) \right) \\
&= \left(\frac{1}{2} n^2\right) \left( n^2 - n + n + 1 \right) \\
&= \left(\frac{1}{2} n^2\right)\left( n^2 + 1 \right) \\
&= \frac{1}{2} n^2 \left( n^2 + 1 \right) \\
&= T_{n^2} \\
&= \text{LHS} \qquad \square
\end{aligned}
$$

**[ 5 marks ]**

**Answer 6**

**( i )** $a = 1$

**[ 1 mark ]**

**( ii )** $d = 1$

**[ 1 mark ]**

**( iii )**

$$
\begin{aligned}
S_n &= \frac{n}{2} \{ 2a + (n-1)\, d \} \\
\therefore \sum_{r=1}^{n} r &= \frac{n}{2} \{ 2 \times 1 + (n-1) \times 1 \} \\
&= \frac{n}{2} \{ 2 + n - 1 \} \\
&= \frac{n}{2} \{ n + 1 \} \\
&= \frac{1}{2} n\,(n+1)
\end{aligned}
$$

**[ 3 marks ]**

**Answer 7**

$$\sum_{r=n}^{3n} (80 - 3r) = 54$$

$$\sum_{1}^{3n} (80 - 3r) - \sum_{1}^{n-1} (80 - 3r) = 54$$

$$80 \sum_{1}^{3n} 1 - 3 \sum_{1}^{3n} r - 80 \sum_{1}^{n-1} 1 + 3 \sum_{1}^{n-1} r = 54$$

$$80 \times 3n - \frac{3}{2} 3n (3n + 1) - 80 (n - 1) + \frac{3}{2} (n - 1) n = 54$$

Multiply through by 2

$$160 \times 3n - 9n (3n + 1) - 160 (n - 1) + 3 (n - 1) n = 108$$

$$480n - 27n^2 - 9n - 160n + 160 + 3n^2 - 3n - 108 = 0$$

$$-24n^2 + 308n + 52 = 0$$

Divide through by $(-4)$

$$6n^2 - 77n - 13 = 0$$

$$(6n + 1)(n - 13) = 0$$

Either $n = -\frac{1}{6}$ (reject) or $n = 13$

| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Term | 77 | 74 | 71 | 68 | 65 | 62 | 59 | 56 | 53 | 50 |

| Position | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| Term | 47 | 44 | 41 | 38 | 35 | 32 | 29 | 26 | 23 | 20 |

| Position | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| Term | 17 | 14 | 11 | 8 | 5 | 2 | − 1 | − 4 | − 7 | − 10 |

| Position | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
|---|---|---|---|---|---|---|---|---|---|---|
| Term | − 13 | − 16 | − 19 | − 22 | − 25 | − 28 | − 31 | − 34 | − 37 | − 40 |

Between $n = 13$ and $n = 39$ ( positives − negatives ) = 301 − 247 = 54

**[ 8 marks ]**