

2.4 Answers

Further A-Level Pure Mathematics : Core 1 Series and Visual Proof

2.4.1 Solutions to 2.2 Teaching Video Example

$$\begin{aligned}\sum_{r=n}^{2n} (15 - 2r) &= 0 \\ \sum_1^{2n} (15 - 2r) - \sum_1^n (15 - 2r) &= 0 \\ 15 \sum_1^{2n} 1 - 2 \sum_1^{2n} r - 15 \sum_1^{n-1} 1 + 2 \sum_1^{n-1} r &= 0 \\ 15 \times 2n - 2 \times \frac{1}{2} (2n)(2n+1) - 15(n-1) + 2 \times \frac{1}{2} (n-1)(n) &= 0 \\ 30n - 4n^2 - 2n - 15n + 15 + n^2 - n &= 0 \\ -3n^2 + 12n + 15 &= 0 \\ \text{divide through by } (-3) & \\ n^2 - 4n - 5 &= 0 \\ (n-5)(n+1) &= 0 \\ \text{Either } n = 5 \text{ or } n = -1 \text{ (reject)} & \\ \therefore n = 5 &\end{aligned}$$

Writing out the sequence helps to make sense of what has been achieved;

Position	1	2	3	4	5	6	7	8	9	10	11
Term	13	11	9	7	5	3	1	-1	-3	-5	-7

[6 marks]

2.4.2 Solutions to 2.3 Exercise

Answer 1

(i) $7 + 13 + 19 + 25 + 31 = 5 \times 19$

[1 mark]

(ii)

$$\begin{aligned}\text{LHS} &= \sum_{r=1}^n (6r + 1) \\&= 6 \sum_{r=1}^n r + \sum_{r=1}^n 1 \\&= 6 \times \frac{1}{2} \times n \times (n + 1) + n \\&= 3n^2 + 3n + n \\&= n(3n + 4) \\&= \text{RHS} \quad \square\end{aligned}$$

[4 marks]

Answer 2

(i) $1 + 5 + 9 + 13 + 17 = 5 \times 9$

[1 mark]

(ii)

$$\sum_{r=1}^n (4r - 3) = n(2n - 1)$$

[2 marks]

(iii)

$$\begin{aligned}\text{LHS} &= \sum_{r=1}^n (4r - 3) \\&= 4 \sum_{r=1}^n r - 3 \sum_{r=1}^n 1 \\&= 4 \times \frac{1}{2} \times n(n + 1) - 3n \\&= 2n^2 + 2n - 3n \\&= 2n^2 - n \\&= n(2n - 1) \\&= \text{RHS} \quad \square\end{aligned}$$

[4 marks]

Answer 3

$$\begin{aligned}
\text{LHS} &= \sum_{r=1}^{2n} (5r - 4) \\
&= 5 \sum_{r=1}^{2n} r - 4 \sum_{r=1}^{2n} 1 \\
&= 5 \times \frac{1}{2} \times 2n \times (2n + 1) - 4 \times 2n \\
&= 10n^2 + 5n - 8n \\
&= 10n^2 - 3n \\
&= n(10n - 3) \\
&= \text{RHS} \quad \square
\end{aligned}$$

[4 marks]**Answer 4****(i)**

$$\begin{aligned}
\sum_{r=1}^{2n-1} r &= \frac{1}{2} \times (2n - 1) 2n \\
&= n(2n - 1)
\end{aligned}$$

[3 marks]**(ii)**

$$\begin{aligned}
\text{LHS} &= \sum_{r=n+1}^{2n-1} r \\
&= \sum_1^{2n-1} r - \sum_1^n r \\
&= n(2n - 1) - \frac{1}{2} \times n \times (n + 1) \\
&= \left(\frac{1}{2} n \right) (4n - 2 - n - 1) \\
&= \frac{1}{2} n (3n - 3) \\
&= \frac{3}{2} n (n - 1) \\
&= \text{RHS} \quad \square
\end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned}
\text{RHS} &= n^2 T_{n-1} + n T_n \\
&= n^2 \sum_1^{n-1} r + n \sum_1^n r \\
&= n^2 \times \frac{1}{2} \times (n-1)n + n \times \frac{1}{2} \times n(n+1) \\
&= \left(\frac{1}{2} n^2 \right) ((n-1)n + (n+1)) \\
&= \left(\frac{1}{2} n^2 \right) (n^2 - n + n + 1) \\
&= \left(\frac{1}{2} n^2 \right) (n^2 + 1) \\
&= \frac{1}{2} n^2 (n^2 + 1) \\
&= T_{n^2} \\
&= \text{LHS} \quad \square
\end{aligned}$$

[5 marks]**Answer 6**

(i) $a = 1$

[1 mark]

(ii) $d = 1$

[1 mark]

(iii)

$$\begin{aligned}
S_n &= \frac{n}{2} \{ 2a + (n-1)d \} \\
\therefore \sum_{r=1}^n r &= \frac{n}{2} \{ 2 \times 1 + (n-1) \times 1 \} \\
&= \frac{n}{2} \{ 2 + n - 1 \} \\
&= \frac{n}{2} \{ n + 1 \} \\
&= \frac{1}{2} n (n + 1)
\end{aligned}$$

[3 marks]

Answer 7

$$\sum_{r=n}^{3n} (80 - 3r) = 54$$

$$\sum_1^{3n} (80 - 3r) - \sum_1^{n-1} (80 - 3r) = 54$$

$$80 \sum_1^{3n} 1 - 3 \sum_1^{3n} r - 80 \sum_1^{n-1} 1 + 3 \sum_1^{n-1} r = 54$$

$$80 \times 3n - \frac{3}{2} 3n (3n + 1) - 80 (n - 1) + \frac{3}{2} (n - 1) n = 54$$

Multiply through by 2

$$160 \times 3n - 9n (3n + 1) - 160 (n - 1) + 3 (n - 1) n = 108$$

$$480n - 27n^2 - 9n - 160n + 160 + 3n^2 - 3n - 108 = 0$$

$$-24n^2 + 308n + 52 = 0$$

Divide through by (-4)

$$6n^2 - 77n - 13 = 0$$

$$(6n + 1) (n - 13) = 0$$

$$\text{Either } n = -\frac{1}{6} \text{ (reject) or } n = 13$$

Position	1	2	3	4	5	6	7	8	9	10
Term	77	74	71	68	65	62	59	56	53	50

Position	11	12	13	14	15	16	17	18	19	20
Term	47	44	41	38	35	32	29	26	23	20

Position	21	22	23	24	25	26	27	28	29	30
Term	17	14	11	8	5	2	-1	-4	-7	-10

Position	31	32	33	34	35	36	37	38	39	40
Term	-13	-16	-19	-22	-25	-28	-31	-34	-37	-40

$$\text{Between } n = 13 \text{ and } n = 39 \text{ (positives - negatives) } = 301 - 247 = 54$$

[8 marks]

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