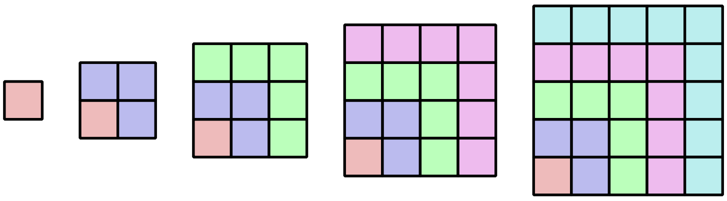
6.1 Test

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 40

Question 1



The partitioning of the squares from the “sum of squares” proof is an illustration that the sum of the first n odd numbers is the square number, $\square_n$.

Prove this result using the standard results for $\sum_{1}^{n} 1$ and $\sum_{1}^{n} r$

[4 marks]

Question 2

Further A-Level Examination Question from June 2018, IAL, F1, Q1

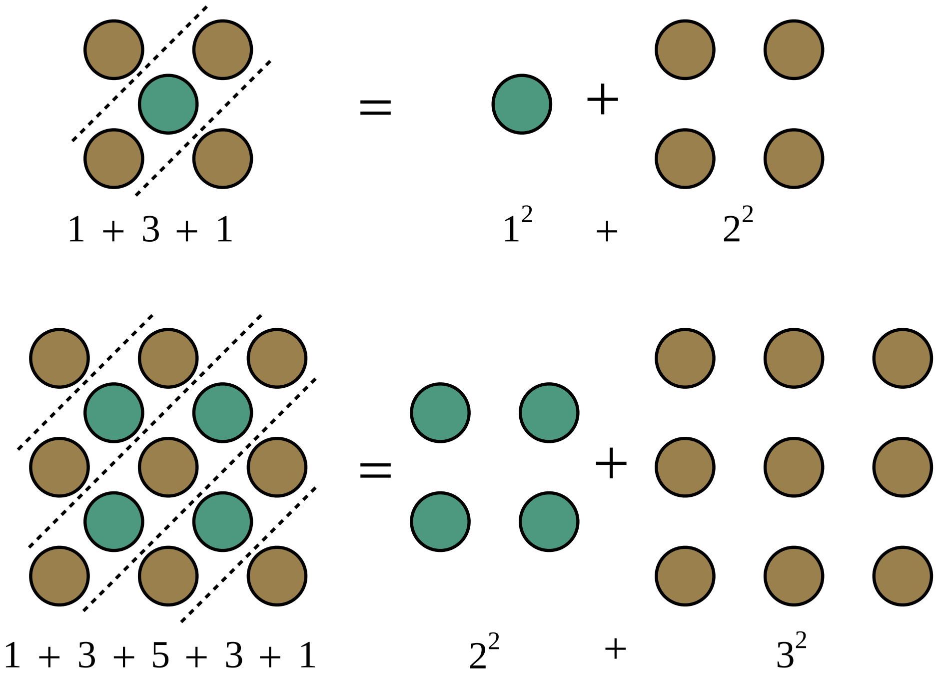
Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r+3) = \frac{n}{a}(n+1)(n+b)$$

where a and b are integers to be found.

[4 marks]

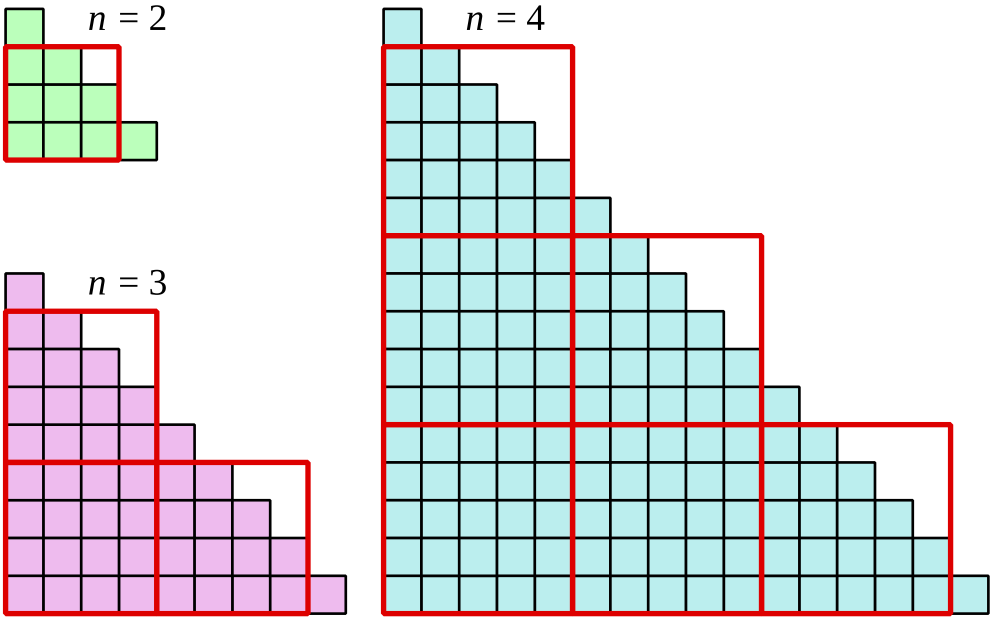
Question 3



State and prove the result suggested by the diagram

[5 marks]

Question 4



The diagrams suggest the following relationship between triangular numbers,

$$T_{n^2} = (n+1)^2\, T_{n-1} - (n-1)\, T_{n-1} + n \qquad n \geqslant 2$$

Prove this relationship by starting with the RHS and manipulating this into the LHS

[5 marks]

Question 5

Further A-Level Examination Question from May 2017. IAL, FP1, Q8

(a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that,

$$\sum_{r=1}^{n} \left(3r^2 + 8r + 3 \right) = \frac{1}{2} n \, (2n + 5)(n + 3)$$

for all positive integers n

[5 marks]

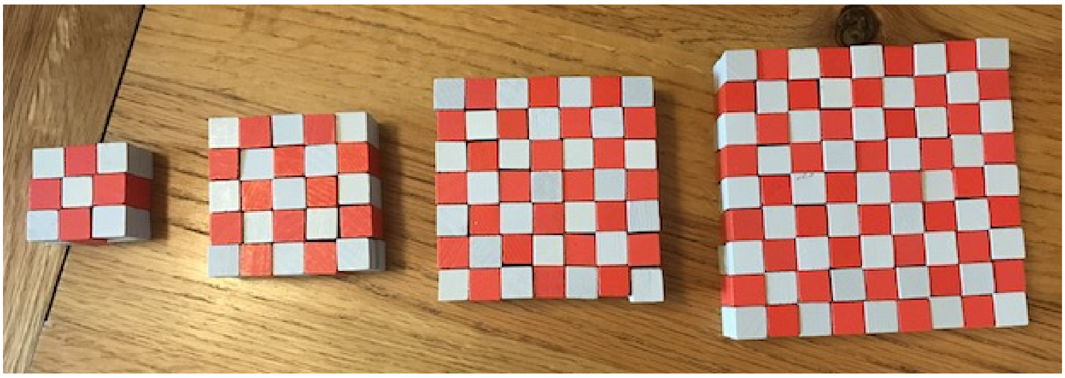
Given that $\sum_{r=1}^{12} \left(3r^2 + 8r + 3 + k \left(2^{r-1}\right) \right) = 3520$

(b) find the exact value of the constant k

[4 marks]

Question 6

The first few terms of the chequered board sequence are shown in the photograph.



(i) Find an expression for the number of small red squares in the n^{th} term in this chequered board sequence.

[3 marks]

(ii) Find an expression for the cumulative number of small red squares in the first n terms of the chequered board sequence.

[3 marks]

(iii) Show that the cumulative number of small red squares in the first *n* terms of the chequered board sequence, when thought of as being unit cubes, can be assembled into a tower with a 2×2 base and a height that is the sum of the first *n* triangular numbers.

[4 marks]

(iv) Now that you know such a tower can always be built, explain how the result can be visually proven from the photograph at the start of this question.

[3 marks]