

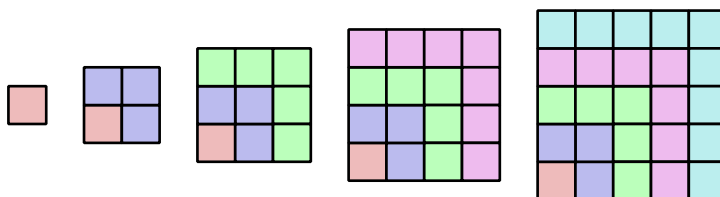
Lesson 6

Further A-Level Pure Mathematics : Core 1 Series and Visual Proof

6.1 Test

*Any solution based entirely on graphical
or numerical methods is not acceptable*
Marks Available : 40

Question 1



The partitioning of the squares from the “sum of squares” proof is an illustration that the sum of the first n odd numbers is the square number, $\square_n$.

Prove this result using the standard results for $\sum_1^n 1$ and $\sum_1^n r$

[4 marks]

Question 2

Further A-Level Examination Question from June 2018, IAL, F1, Q1

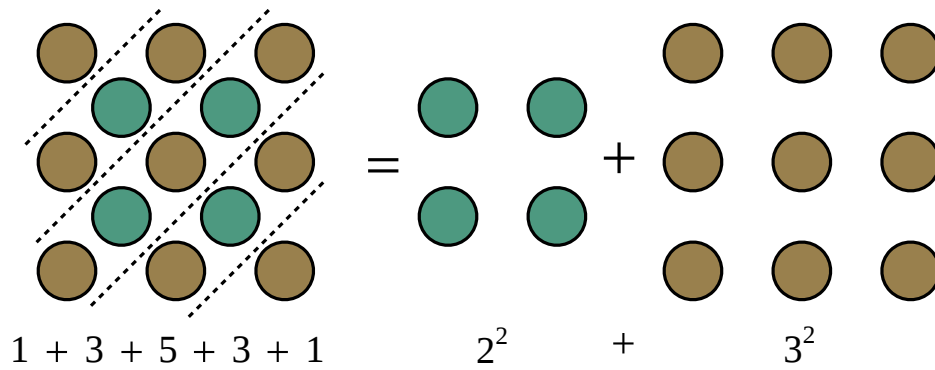
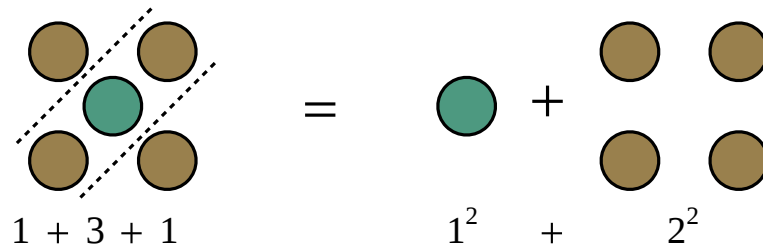
Use the standard results for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$ to show that, for all positive integers n ,

$$\sum_{r=1}^n r (r + 3) = \frac{n}{a} (n + 1) (n + b)$$

where a and b are integers to be found.

[4 marks]

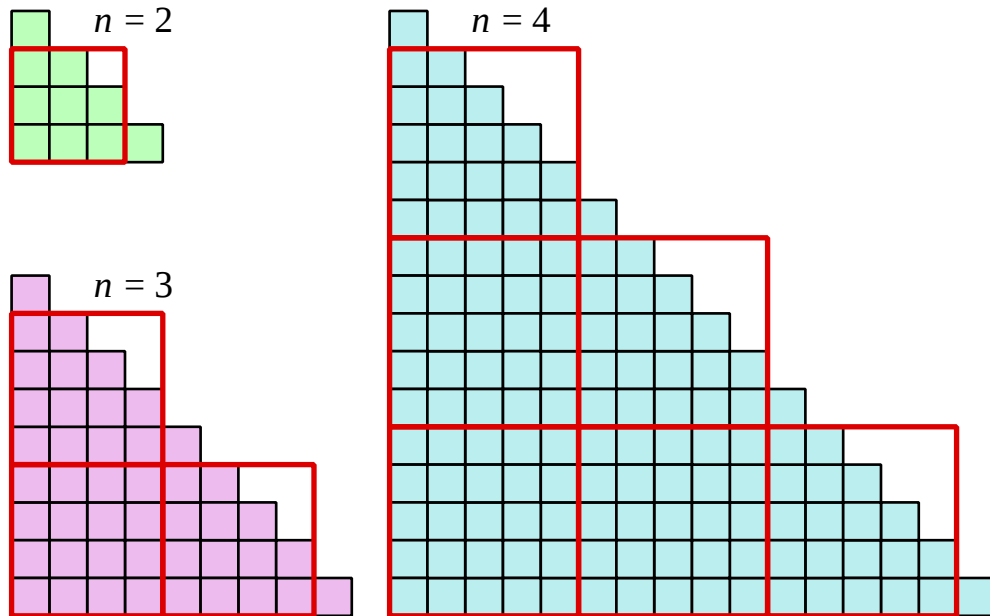
Question 3



State and prove the result suggested by the diagram

[5 marks]

Question 4



The diagrams suggest the following relationship between triangular numbers,

$$T_{n^2} = (n + 1)^2 T_{n-1} - (n - 1) T_{n-1} + n \quad n \geq 2$$

Prove this relationship by starting with the RHS and manipulating this into the LHS

[5 marks]

Question 5

Further A-Level Examination Question from May 2017. IAL, FP1, Q8

- (a) Use the standard results for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$ to show that,

$$\sum_{r=1}^n (3r^2 + 8r + 3) = \frac{1}{2} n (2n + 5) (n + 3)$$

for all positive integers n

[5 marks]

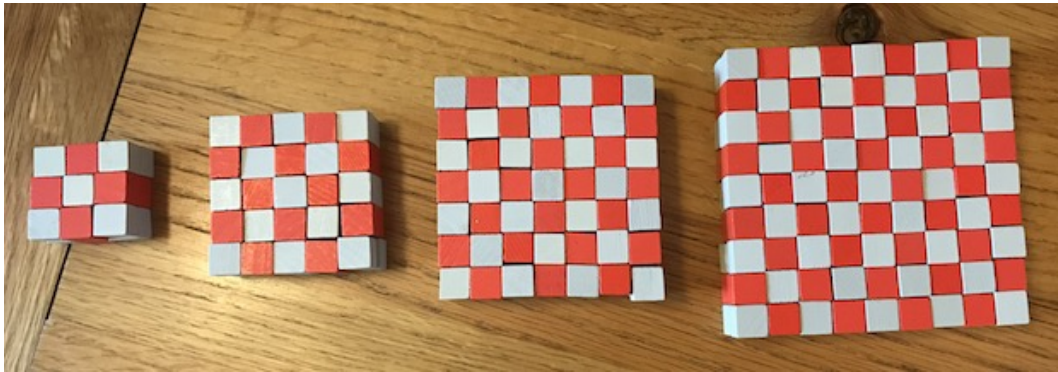
Given that $\sum_{r=1}^{12} (3r^2 + 8r + 3 + k(2^{r-1})) = 3520$

- (b) find the exact value of the constant k

[4 marks]

Question 6

The first few terms of the chequered board sequence are shown in the photograph.



- (i) Find an expression for the number of small red squares in the n^{th} term in this chequered board sequence.

[3 marks]

- (ii) Find an expression for the cumulative number of small red squares in the first n terms of the chequered board sequence.

[3 marks]

- (iii) Show that the cumulative number of small red squares in the first n terms of the chequered board sequence, when thought of as being unit cubes, can be assembled into a tower with a 2×2 base and a height that is the sum of the first n triangular numbers.

[4 marks]

- (iv) Now that you know such a tower can always be built, explain how the result can be visually proven from the photograph at the start of this question.

[3 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk