7.1 Test

Any solution based entirely on graphical
or numerical methods is not acceptable.
Make the method used clear.

Marks available : 40

Question 1

Consider the equation $a\,x^2 + bx + c = 0$ whose roots are α and β.

What would $\alpha - \beta$ be in terms of a, b and c

[4 marks]

Question 2

The cubic equation $x^3 + mx + n = 0$ has roots $1, -4$ and γ

(i) State, with a reason, whether γ is real

[1 mark]

(ii) Find the values of m, n and γ

[4 marks]

Question 3

The roots of the equation $3x^3 - px + 11 = 0$ are α, β and γ

(i) Given that $\alpha\beta + \beta\gamma + \gamma\alpha = 4$, write down the value of p

[1 mark]

(ii) Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$

[2 marks]

(iii) Hence find the value of $(3 - \alpha)(3 - \beta)(3 - \gamma)$

[3 marks]

Question 4

It is given that α and β are roots of $3x^2 + 2x + 1$.

Without solving the equation, find the exact value of $\left(\frac{1-\alpha}{1+\alpha}\right)^3 + \left(\frac{1-\beta}{1+\beta}\right)^3$

[8 marks]

Question 5

Further A-Level Examination Question from SAM, 2018, Core 1, Q6 (Edexcel)

$$f(x) = kx^2 + 3x - 11 \qquad g(x) = mx^3 - 2x^2 + 3x - 9$$

where k and m are real constants.

Given that,

- the sum of the roots of f is equal to the product of the roots of g
- g has at least one root on the imaginary axis

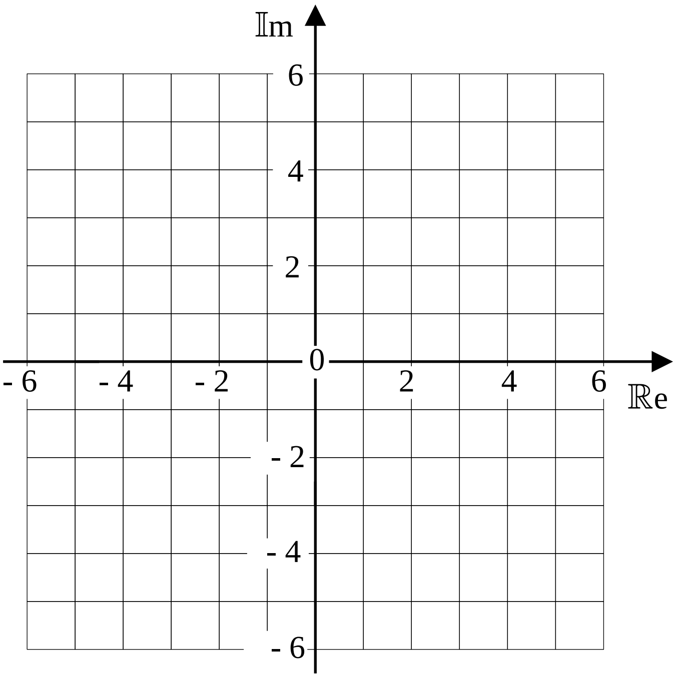
(a) Solve completely,

(i) $f(x) = 0$

(ii) $g(x) = 0$

[7 marks]

(b) Plot the roots of f and the roots of g on a single Argand diagram



[2 marks]

Question 6

If α, β, γ and δ are the roots of $x^4 + x^3 + px^2 + 4x - 2 = 0$, where p is a constant, then,

(i) find (in terms of p) an equation whose roots are $\frac{1}{\alpha}$, $\frac{1}{\beta}$, $\frac{1}{\gamma}$ and $\frac{1}{\delta}$

[3 marks]

(ii) Given that $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}$ find the value of p.

[5 marks]

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