3.5 Answers

3.5.1 Solution to 3.2 Example

$$\begin{pmatrix} 4 \\ 7 \\ -14 \end{pmatrix} \bullet \begin{pmatrix} 7 \\ 12 \\ 8 \end{pmatrix} = 4 \times 7 + 7 \times 12 + (-14) \times 8$$

$$= 28 + 84 - 112$$

$$= 0$$

$\therefore$ $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are mutually perpendicular ☐

[3 marks]

3.5.2 Solution to 3.4

Answer 1

$$\begin{pmatrix} 5 \\ 3 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 7 \\ -9 \\ 2 \end{pmatrix} = 5 \times 7 + 3 \times (-9) + (-4) \times 2$$

$$= 35 - 27 - 8$$

$$= 0$$

$\therefore$ $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are mutually perpendicular ☐

[3 marks]

Answer 2

As the lines are perpendicular, $\begin{pmatrix} a \\ 4 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 2a \\ 5 \end{pmatrix} = 0$

$$a \times 2 + 4 \times 2a + (-3) \times 5 = 0$$

$$2a + 8a - 15 = 0$$

$$10a = 15$$

$$a = \frac{3}{2}$$

[3 marks]

Answer 3

As the lines are perpendicular, $\begin{pmatrix} p \\ 6 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ -3 \\ -7 \end{pmatrix} = 0$

$$p \times 5 + 6 \times (-3) + (-4) \times (-7) = 0$$

$$5p - 18 + 28 = 0$$

$$5p = -10$$

$$p = -2$$

[3 marks]

Answer 4

(a) **(i)** $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a} = \begin{pmatrix} -3 \\ 8 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \\ 7 \end{pmatrix} = \begin{pmatrix} -9 \\ 5 \\ -2 \end{pmatrix}$

[1 mark]

(ii) $\overrightarrow{AC} = \boldsymbol{c} - \boldsymbol{a} = \begin{pmatrix} -4 \\ -13 \\ 12 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \\ 7 \end{pmatrix} = \begin{pmatrix} -10 \\ -16 \\ 5 \end{pmatrix}$

[1 mark]

(b) $\begin{pmatrix} -9 \\ 5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} -10 \\ -16 \\ 5 \end{pmatrix} = (-9) \times (-10) + 5 \times (-16) + (-2) \times 5$

$= 90 - 80 - 10$

$= 0$ $\quad \therefore \angle BAC$ is a right angle ☐

[2 marks]

(c) Area $= \frac{1}{2} b\, h$

$= \frac{1}{2} \times |AB| \times |AC|$

$= \frac{1}{2} \times \sqrt{(-9)^2 + 5^2 + (-2)^2} \times \sqrt{(-10)^2 + (-16)^2 + 5^2}$

$= \frac{1}{2} \times \sqrt{110} \times \sqrt{381}$

$= 102.4$

[3 marks]

Answer 5

$$\begin{pmatrix} 1 \\ m \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -\frac{1}{m} \end{pmatrix} = 1 \times 1 + m \times \left(-\frac{1}{m}\right)$$

$$= 1 - 1$$

$$= 0$$

[1 mark]

$\therefore$ The line which is perpendicular to a line with gradient m has a gradient which is the sign changed reciprocal, that is $-\frac{1}{m}$

[1 mark]

Answer 6

$$r_{AB} = \overrightarrow{OA} + \lambda \overrightarrow{AB}$$

$$= \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + \lambda \left(\begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \right)$$

$$= \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix} \qquad \text{for example}$$

[2 marks]

Answer 7

$$\boldsymbol{s} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 8 \\ 1 \end{pmatrix}$$

[2 marks]

Answer 8

$$\boldsymbol{u} \bullet \boldsymbol{v} = 0$$

$$\begin{pmatrix} \lambda + 1 \\ 4 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} \lambda - 5 \\ \lambda \\ -11 \end{pmatrix} = 0$$

$$(\lambda + 1)(\lambda - 5) + 4\lambda - 11 = 0$$

$$\lambda^2 - 4\lambda - 5 + 4\lambda - 11 = 0$$

$$\lambda^2 - 16 = 0$$

$$\lambda^2 = 16$$

$$\lambda = \pm 4$$

[2 marks]

Answer 9

(a) $$\begin{pmatrix}1\\P\\Q\end{pmatrix} \bullet \begin{pmatrix}3\\1\\2\end{pmatrix} = 0 \Rightarrow 3 + P + 2Q = 0 \Rightarrow P + 2Q = -3 \qquad (1)$$

[1 mark]

(b) $$\begin{pmatrix}1\\P\\Q\end{pmatrix} \bullet \begin{pmatrix}6\\-2\\-7\end{pmatrix} = 0 \Rightarrow 6 - 2P - 7Q = 0 \Rightarrow 2P + 7Q = 6 \qquad (2)$$

[1 mark]

(c)

$$2P + 7Q = 6 \qquad (2)$$
$$2P + 4Q = -6 \qquad (1) \times 2$$

$$3Q = 12$$
$$Q = 4 \quad \Rightarrow P = -11$$

The vector perpendicular to both r_1 and r_2 is $\begin{pmatrix}1\\-11\\4\end{pmatrix}$

[2 marks]

Answer 10

$$\begin{pmatrix}1\\P\\Q\end{pmatrix} \bullet \begin{pmatrix}4\\10\\-3\end{pmatrix} = 0 \Rightarrow 4 + 10P - 3Q = 0 \Rightarrow 10P - 3Q = -4 \qquad (1)$$

[1 mark]

$$\begin{pmatrix}1\\P\\Q\end{pmatrix} \bullet \begin{pmatrix}5\\-9\\7\end{pmatrix} = 0 \Rightarrow 5 - 9P + 7Q = 0 \Rightarrow -9P + 7Q = -5 \qquad (2)$$

[1 mark]

$$70P - 21Q = -28 \qquad (1) \times 7$$
$$-27P + 21Q = -15 \qquad (2) \times 3$$

$$43P = -43$$
$$P = -1 \quad \Rightarrow Q = -2$$

The vector perpendicular to both r_1 and r_2 is $\begin{pmatrix}1\\-1\\-2\end{pmatrix}$

[2 marks]