6.2 Answers

Answer 1

(i) $\overrightarrow{VW} = \boldsymbol{w} - \boldsymbol{v} = \begin{pmatrix} 11 \\ 11 \\ -1 \end{pmatrix} - \begin{pmatrix} 9 \\ 8 \\ -7 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$

[2 marks]

(ii) $\left|\overrightarrow{VW}\right| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7$

[2 marks]

(iii) To show, $\overrightarrow{VW} \bullet \overrightarrow{VO}$

$$\begin{aligned} \text{LHS} &= \overrightarrow{VW} \bullet \overrightarrow{VO} \\ &= \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \bullet \begin{pmatrix} -9 \\ -8 \\ 7 \end{pmatrix} \\ &= -18 - 24 + 42 \\ &= 0 \\ &= \text{RHS} \qquad \square \end{aligned}$$

[2 marks]

(iv) $\triangle OVW$ is right angled at V

[1 mark]

Answer 2

(i) $\boldsymbol{r} = \begin{pmatrix} 2 \\ 8 \\ 3 \end{pmatrix} + \lambda\left(\begin{pmatrix} 7 \\ 5 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 8 \\ 3 \end{pmatrix} \right) \Rightarrow \boldsymbol{r} = \begin{pmatrix} 2 \\ 8 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -3 \\ -2 \end{pmatrix}$ for example

[3 marks]

(ii) Try $\lambda = -2$,

$$\begin{aligned} \boldsymbol{r} &= \begin{pmatrix} 2 \\ 8 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -3 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 8 \\ 3 \end{pmatrix} - 2 \begin{pmatrix} 5 \\ -3 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} -8 \\ 14 \\ 7 \end{pmatrix} \end{aligned}$$

$\therefore$ $(-8,\ 2,\ 7)$ is not on the line, l

[2 marks]

Answer 3

(i) $\overrightarrow{LM} = \boldsymbol{m} - \boldsymbol{l} = \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$

[2 marks]

(ii) $\left|\overrightarrow{OL}\right| = \sqrt{1^2 + (-1)^2 + 3^2} = \sqrt{11}$

$\left|\overrightarrow{LM}\right| = \sqrt{1^2 + (-3)^2 + (-1)^2} = \sqrt{11}$

$$\therefore \left|\overrightarrow{OL}\right| = \left|\overrightarrow{LM}\right| \qquad \square$$

[2 marks]

(iii) Making use of the scalar product,

$$\overrightarrow{LO} \bullet \overrightarrow{LM}$$

$$\begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$$

$$-1 - 3 + 3 = \sqrt{11}\ \sqrt{11}\ cos\ \theta$$

$$cos\ \theta = -\frac{1}{11}$$

$$\theta = 95.2°$$

[3 marks]

Answer 4

(a) $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

[2 marks]

(b) $l_1 : \boldsymbol{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ for example

[2 marks]

(c) Making use of the scalar product,

$$\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$1 + 0 + 2 = \sqrt{1^2 + (-2)^2 + 2^2}\ \sqrt{1^2 + 0^2 + 1^2}\ cos\theta$$

$$cos\,\theta = \frac{3}{\sqrt{9}\ \sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\theta = 45°$$

[3 marks]

(d)

$$\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Middle row : $6 - 2\lambda = 0 \Rightarrow \lambda = 3$

To get point C, $\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$$= \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} \text{ i.e. } C\,(\,5,\ 0,\ 5\,)$$

[4 marks]