1.4 Answers

1.4.1 Solution to 1.2 Example (Teaching Video)

$$2x + 5y = 4$$

$$6x + 4y = 1$$

The representation in matrix form is then,

$$\begin{pmatrix} 2 & 5 \\ 6 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Let $\mathbf{M} = \begin{pmatrix} 2 & 5 \\ 6 & 4 \end{pmatrix}$ be the matrix of the coefficients, in which case,

$$\mathbf{M}^{-1} = \frac{1}{2 \times 4 - 5 \times 6}\begin{pmatrix} 4 & -5 \\ -6 & 2 \end{pmatrix}$$

$$= \frac{1}{22}\begin{pmatrix} -4 & 5 \\ 6 & -2 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{22}\begin{pmatrix} -4 & 5 \\ 6 & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$= \frac{1}{22}\begin{pmatrix} -4 \times 4 + 5 \times 1 \\ 6 \times 4 + (-2) \times 1 \end{pmatrix}$$

$$= \frac{1}{22}\begin{pmatrix} -11 \\ 22 \end{pmatrix}$$

$$= \begin{pmatrix} -0.5 \\ 1 \end{pmatrix}$$

That is, $x = -0.5$, $y = 1$

[5 marks]

1.4.2 Solutions to 1.3 Exercise

Answer 1

$$\begin{aligned}
\mathbf{PQ} &= \begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 6 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} 7 \times 6 + 5 \times 1 \\ 3 \times 6 + 2 \times 1 \end{pmatrix} \\
&= \begin{pmatrix} 47 \\ 20 \end{pmatrix}
\end{aligned}$$

[3 marks]

Answer 2

$$\begin{aligned}
\mathbf{M}^{-1}\begin{pmatrix} 2 & 5 \\ 6 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} &= \frac{1}{22}\begin{pmatrix} -4 & 5 \\ 6 & -2 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 6 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \\
&= \frac{1}{22}\begin{pmatrix} (-4) \times 2 + 5 \times 6 & (-4) \times 5 + 5 \times 4 \\ 6 \times 2 + (-2) \times 6 & 6 \times 5 + (-2) \times 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \\
&= \frac{1}{22}\begin{pmatrix} 22 & 0 \\ 0 & 22 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \\
&= \begin{pmatrix} x \\ y \end{pmatrix}
\end{aligned}$$

[4 marks]

Answer 3

$$\begin{aligned}
\mathbf{A} &= \begin{pmatrix} 10 & 4 \\ 7 & 3 \end{pmatrix} \\
\mathbf{A}^{-1} &= \frac{1}{10 \times 3 - 4 \times 7}\begin{pmatrix} 3 & -4 \\ -7 & 10 \end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix} 3 & -4 \\ -7 & 10 \end{pmatrix} \\
&= \begin{pmatrix} 1.5 & -2 \\ -3.5 & 5 \end{pmatrix}
\end{aligned}$$

[4 marks]

Answer 4

(i)

$$\mathbf{M} = \begin{pmatrix} 0.5 & 0.25 \\ 2 & 1.5 \end{pmatrix}$$

$$\begin{aligned} det\ (\mathbf{M}) &= 0.5 \times 1.5 - 0.25 \times 2 \\ &= 0.75 - 0.5 \\ &= 0.25 \end{aligned}$$

[2 marks]

(ii)

$$\begin{aligned} \mathbf{M}^{-1} &= \frac{1}{0.25} \begin{pmatrix} 1.5 & -0.25 \\ -2 & 0.5 \end{pmatrix} \\ &= 4 \begin{pmatrix} 1.5 & -0.25 \\ -2 & 0.5 \end{pmatrix} \\ &= \begin{pmatrix} 6 & -1 \\ -8 & 2 \end{pmatrix} \end{aligned}$$

[2 marks]

Answer 5

$$\begin{aligned} 3x + 4y &= 18 \\ 4x - \ \ y &= \ \ 5 \end{aligned}$$

The representation in matrix form is then,

$$\begin{pmatrix} 3 & 4 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 \\ 5 \end{pmatrix}$$

Let $\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 4 & -1 \end{pmatrix}$ be the matrix of the coefficients, in which case,

$$\begin{aligned} \mathbf{M}^{-1} &= \frac{1}{3 \times (-1) - 4 \times 4} \begin{pmatrix} 3 & 4 \\ 4 & -1 \end{pmatrix} \\ &= -\frac{1}{19} \begin{pmatrix} -1 & -4 \\ -4 & 3 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \therefore \begin{pmatrix} x \\ y \end{pmatrix} &= \frac{1}{19} \begin{pmatrix} 1 & 4 \\ 4 & -3 \end{pmatrix} \begin{pmatrix} 18 \\ 5 \end{pmatrix} \\ &= \frac{1}{19} \begin{pmatrix} 1 \times 18 + 4 \times 5 \\ 4 \times 18 - 3 \times 5 \end{pmatrix} \\ &= \frac{1}{19} \begin{pmatrix} 38 \\ 57 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 3 \end{pmatrix} \end{aligned}$$

That is, $x = 2, \ y = 3$

[5 marks]

Answer 6

$$5x - 2y = 16$$

$$7x + 6y = -4$$

The representation in matrix form is then,

$$\begin{pmatrix} 5 & -2 \\ 7 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 16 \\ -4 \end{pmatrix}$$

Let $\mathbf{M} = \begin{pmatrix} 5 & -2 \\ 7 & 6 \end{pmatrix}$ the matrix of the coefficients, in which case,

$$\mathbf{M}^{-1} = \frac{1}{5 \times 6 - (-2) \times 7}\begin{pmatrix} 6 & 2 \\ -7 & 5 \end{pmatrix}$$

$$= \frac{1}{44}\begin{pmatrix} 6 & 2 \\ -7 & 5 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{44}\begin{pmatrix} 6 & 2 \\ -7 & 5 \end{pmatrix}\begin{pmatrix} 16 \\ -4 \end{pmatrix}$$

$$= \frac{1}{44}\begin{pmatrix} 6 \times 16 + 2 \times (-4) \\ (-7) \times 16 + 5 \times (-4) \end{pmatrix}$$

$$= \frac{1}{44}\begin{pmatrix} 88 \\ -132 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

That is, $x = 2, \ y = -3$

[5 marks]

Answer 7

$$\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{1 \times 1 - (-3) \times 2}\begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix}$$

$$\mathbf{B} = \frac{1}{7}\begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 & 9 \\ 1 & 9 & 4 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 7 & 28 & 21 \\ -7 & 7 & -14 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & 3 \\ -1 & 1 & -2 \end{pmatrix}$$

[5 marks]