2.4 Answers

2.4.1 Solution to Example #1 (Teaching Video)

The minor of the element 6 is given by,

$$M_{13} = \begin{vmatrix} 2 & 7 \\ 8 & 4 \end{vmatrix}$$

$$= 2 \times 4 - 8 \times 7$$

$$= 8 - 8 \times 7$$

$$= 8(1 - 7)$$

$$= 8 \times (-6)$$

$$= -48$$

[3 marks]

Solution to Spot Check

$$\mathbf{A} = \begin{pmatrix} 5 & -3 & 6 \\ 2 & 7 & -4 \\ 8 & 4 & 9 \end{pmatrix}$$

$$M_{21} = \begin{vmatrix} -3 & 6 \\ 4 & 9 \end{vmatrix}$$

$$= (-3) \times 9 - 4 \times 6$$

$$= -27 - 24$$

$$= -51 \qquad \square$$

[3 marks]

Solution to Example #2 (Teaching Video)

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -1 & 4 & 3 \end{vmatrix} = 1 \times \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} - 2 \times \begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} + 4 \begin{vmatrix} 3 & 2 \\ -1 & 4 \end{vmatrix}$$

$$= 1(2 \times 3 - 4 \times 1)$$

$$-2(3 \times 3 - (-1) \times 1)$$

$$+4(3 \times 4 - (-1) \times 2)$$

$$= 2 - 20 + 56$$

$$= 38$$

[4 marks]

2.4.2 Solutions to 2.3 Exercise

Answer 1

$$\begin{vmatrix} -3 & 1 & 9 \\ 2 & 5 & 3 \\ -1 & 6 & 8 \end{vmatrix} = (-3)\begin{vmatrix} 5 & 3 \\ 6 & 8 \end{vmatrix} - 1\begin{vmatrix} 2 & 3 \\ -1 & 8 \end{vmatrix} + 9\begin{vmatrix} 2 & 5 \\ -1 & 6 \end{vmatrix}$$

$$\begin{aligned} &= -3(5 \times 8 - 6 \times 3) \\ &\quad -1(2 \times 8 - (-1) \times 3) \\ &\quad +9(2 \times 6 - (-1) \times 5) \\ &= -66 - 19 + 153 \\ &= 68 \qquad \square \end{aligned}$$

[4 marks]

Answer 2

$$\mathbf{M} = \begin{pmatrix} 1 & k & 0 \\ 2 & -2 & 1 \\ -4 & 1 & -1 \end{pmatrix}, \quad k \in \mathbb{R}, \quad k \neq \frac{1}{2}$$

$$\begin{aligned} |\mathbf{M}| &= 1\begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} - k\begin{vmatrix} 2 & 1 \\ -4 & -1 \end{vmatrix} + 0\begin{vmatrix} 2 & -2 \\ -4 & 1 \end{vmatrix} \\ &= 1\,(\,(-2) \times (-1) - 1 \times 1\,) - k\,(\,2 \times (-1) - (-4) \times 1\,) + 0 \\ &= 1 - 2k \qquad \square \end{aligned}$$

[2 marks]

Answer 3

It tells you that the determinant is zero.

[1 mark]

Answer 4

As the matrix is singular, $det\,\mathbf{A} = 0$

$$\begin{vmatrix} -2 & 1 & -3 \\ k & 1 & 3 \\ 2 & -1 & k \end{vmatrix} = 0$$

$$(-2)\begin{vmatrix} 1 & 3 \\ -1 & k \end{vmatrix} - 1\begin{vmatrix} k & 3 \\ 2 & k \end{vmatrix} - 3\begin{vmatrix} k & 1 \\ 2 & -1 \end{vmatrix} = 0$$

$$\begin{aligned} (-2)(k + 3) - \left(k^2 - 6\right) - 3(-k - 2) &= 0 \\ -2k - 6 - k^2 + 6 + 3k + 6 &= 0 \\ -k^2 + k + 6 &= 0 \\ k^2 - k - 6 &= 0 \\ (k - 3)(k + 2) &= 0 \end{aligned}$$

Either $k = 3$ or $k = -2$

[4 marks]

Answer 5

$$\det A = 8$$

$$\begin{vmatrix} 2 & -1 & 3 \\ k & 2 & 4 \\ -2 & 1 & k+3 \end{vmatrix} = 8$$

$$2\begin{vmatrix} 2 & 4 \\ 1 & k+3 \end{vmatrix} + 1\begin{vmatrix} k & 4 \\ -2 & k+3 \end{vmatrix} + 3\begin{vmatrix} k & 2 \\ -2 & 1 \end{vmatrix} = 8$$

$$2(2(k+3) - 1 \times 4)$$

$$+ 1(k(k+3) - (-2) \times 4)$$

$$+ 3\,(k \times 1 - (-2) \times 2) = 8$$

$$4k + 12 - 8 + k^2 + 3k + 8 + 3k + 12 = 8$$

$$k^2 + 10k + 16 = 0$$

$$(k+8)(k+2) = 0$$

Either $k = -8$ or $k = -2$

[5 marks]

Answer 6

Top row expansion,

$$\begin{vmatrix} -7 & 3 & 5 \\ 0 & 2 & 0 \\ 5 & 6 & 4 \end{vmatrix} = -7\begin{vmatrix} 2 & 0 \\ 6 & 4 \end{vmatrix} - 3\begin{vmatrix} 0 & 0 \\ 5 & 4 \end{vmatrix} + 5\begin{vmatrix} 0 & 2 \\ 5 & 6 \end{vmatrix}$$

$$= -7 \times 8 - 3 \times 0 + 5 \times (-10)$$

$$= -56 - 0 - 50$$

$$= -106$$

Middle row expansion,

$$\begin{vmatrix} -7 & 3 & 5 \\ 0 & 2 & 0 \\ 5 & 6 & 4 \end{vmatrix} = -0\begin{vmatrix} 3 & 5 \\ 6 & 4 \end{vmatrix} + 2\begin{vmatrix} -7 & 5 \\ 5 & 4 \end{vmatrix} - 0\begin{vmatrix} -7 & 3 \\ 5 & 6 \end{vmatrix}$$

$$= 0 + 2(\,(-7) \times 4 - 5 \times 5\,) - 0$$

$$= 2\,(-28 - 25)$$

$$= 2 \times (-53))$$

$= -106$ As before □

[5 marks]

Answer 7

$$\begin{vmatrix} 2 & -2 & 4 \\ 3 & x & -2 \\ -1 & 3 & x \end{vmatrix} = 2\begin{vmatrix} x & -2 \\ 3 & x \end{vmatrix} + 2\begin{vmatrix} 3 & -2 \\ -1 & x \end{vmatrix} + 4\begin{vmatrix} 3 & x \\ -1 & 3 \end{vmatrix}$$

$$= 2\left(x^2 + 6\right) + 2\left(3x - 2\right) + 4\left(9 + x\right)$$

$$= 2x^2 + 12 + 6x - 4 + 36 + 4x$$

$$= 2x^2 + 10x + 44$$

$$= 2\left[x^2 + 5x\right] + 44$$

$$= 2\left[\left(x + \frac{5}{2}\right)^2 - \frac{25}{4}\right] + 44$$

$$= 2\left(x + \frac{5}{2}\right)^2 - \frac{25}{2} + \frac{88}{2}$$

$$= 2\left(x + \frac{5}{2}\right)^2 + \frac{63}{2}$$

This expression is always positive.

∴ There is no real value for x that will cause the matrix to be singular.

That is, the matrix is always non singular. □

[4 marks]