5.4 Answers

5.4.1 Solution to 5.2 Example (Teaching Video)

The words give rise to the following three equations;

$$\begin{aligned} x + y + z &= 24500 \\ 0.04x + 0.055y + 0.06z &= 1300 \\ x - 4y + 0z &= 0 \end{aligned}$$

The middle equation can be multiplied through by 100 to make the numbers a little easier to handle,

$$\begin{aligned} x + y + z &= 24500 \\ 4x + 5.5y + 6z &= 130000 \\ x - 4y + 0z &= 0 \end{aligned}$$

Translating this into a matrix equation gives,

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 5.5 & 6 \\ 1 & -4 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 24500 \\ 130000 \\ 0 \end{pmatrix}$$

Using a calculator to get the inverse of the leftmost matrix results in,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{17} \begin{pmatrix} 48 & -8 & 1 \\ 12 & -2 & -4 \\ -43 & 10 & 3 \end{pmatrix} \begin{pmatrix} 24500 \\ 130000 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 8000 \\ 2000 \\ 14500 \end{pmatrix}$$

$\therefore$ Miranda invested £8000 in account X,
£2000 in account Y
and £14500 in account Z

[5 marks]

5.4.2 Solutions to 5.3 Exercise

Answer 1

Let a represent the number of inflatable space aliens.
Let f represent the number of boxes of indoor fireworks.
Let p represent the number of bags of party poppers.
Thus the system of equations becomes,

$$3a + 4f + 5p = 244$$
$$6a + 5f + 2p = 304$$
$$3a + 2f + p = 134$$

As a matrix equation this will be,

$$\begin{pmatrix} 3 & 4 & 5 \\ 6 & 5 & 2 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} a \\ f \\ p \end{pmatrix} = \begin{pmatrix} 244 \\ 304 \\ 134 \end{pmatrix}$$

Using a calculator to get the inverse of the leftmost matrix results in,

$$\begin{pmatrix} a \\ f \\ p \end{pmatrix} = \frac{1}{12}\begin{pmatrix} -1 & -6 & 17 \\ 0 & 12 & -24 \\ 3 & -6 & 9 \end{pmatrix}\begin{pmatrix} 244 \\ 304 \\ 134 \end{pmatrix}$$
$$= \begin{pmatrix} 17.50 \\ 36.00 \\ 9.50 \end{pmatrix}$$

The unit cost of each item is thus,

1 inflatable space alien : £17.50

1 box of indoor fireworks : £36.00

1 bag of party poppers : £9.50

[6 marks]

Answer 2

Let s represent the initial number of Suri Alpacas.
Let h represent the initial number of Huacaya Alpacas.
Let d represent the initial number of Domestic Llamas.

The words give rise to the following three equations;

$$\begin{aligned} s + h + d &= 2810 \\ 0.05s + 0.03h - 0.04d &= 46 \\ s - h + 0d &= 160 \end{aligned}$$

The middle equation can be multiplied through by 100 to make the numbers easier to handle,

$$\begin{aligned} s + h + d &= 2810 \\ 5s + 3h - 4d &= 4600 \\ s - h + 0d &= 160 \end{aligned}$$

Translating this into a matrix equation gives,

$$\begin{pmatrix} 1 & 1 & 1 \\ 5 & 3 & -4 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} s \\ h \\ d \end{pmatrix} = \begin{pmatrix} 2810 \\ 4600 \\ 160 \end{pmatrix}$$

Using a calculator to get the inverse of the leftmost matrix results in,

$$\begin{aligned} \begin{pmatrix} s \\ h \\ d \end{pmatrix} &= \frac{1}{16} \begin{pmatrix} 4 & 1 & 7 \\ 4 & 1 & -9 \\ 8 & -2 & 2 \end{pmatrix} \begin{pmatrix} 2810 \\ 4600 \\ 160 \end{pmatrix} \\ &= \begin{pmatrix} 1060 \\ 900 \\ 850 \end{pmatrix} \end{aligned}$$

$\therefore$ Initially there were 1060 Suri Alpacas, 900 Huacaya Alpacas and 850 Domestic Llamas.

[5 marks]

Answer 3

Let b represent the number of bananas.
Let o represent the number of oranges.
Let p represent the number of papayas.
Thus the system of equations becomes,

$$\begin{aligned} b + o + p &= 70 \\ 27b + 32o + 160p &= 3420 \\ 2b - o + 0p &= 0 \end{aligned}$$

As a matrix equation this will be,

$$\begin{pmatrix} 1 & 1 & 1 \\ 27 & 32 & 160 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} b \\ o \\ p \end{pmatrix} = \begin{pmatrix} 70 \\ 3420 \\ 0 \end{pmatrix}$$

Using a calculator to get the inverse of the leftmost matrix results in,

$$\begin{pmatrix} a \\ f \\ p \end{pmatrix} = \frac{1}{389} \begin{pmatrix} 160 & -1 & 128 \\ 320 & -2 & -133 \\ -91 & 3 & 5 \end{pmatrix} \begin{pmatrix} 70 \\ 3420 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 20 \\ 40 \\ 10 \end{pmatrix}$$

The number of each item is thus,

bananas : 20

oranges : 40

papaya : 10

[6 marks]

Answer 4

Let *brn* represent the number of brown bats.
Let *gry* represent the number of grey bats.
Let *blk* represent the number of black bats.
Thus the system of equations becomes,

$$\begin{aligned} brn + gry + blk &= 2000 \\ -0.01brn - 0.02gry + 0.04blk &= 40 \\ brn - gry + 0blk &= 250 \end{aligned}$$

The middle equation can be multiplied through by 100 to make the numbers a little easier to handle,

$$\begin{aligned} brn + gry + blk &= 2000 \\ -brn - 2gry + 4blk &= 4000 \\ brn - gry + 0blk &= 250 \end{aligned}$$

As a matrix equation this will be,

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & -2 & 4 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} brn \\ gry \\ blk \end{pmatrix} = \begin{pmatrix} 2000 \\ 4000 \\ 250 \end{pmatrix}$$

Using a calculator to get the inverse of the leftmost matrix results in,

$$\begin{pmatrix} brn \\ gry \\ blk \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 4 & -1 & 6 \\ 4 & -1 & -5 \\ 3 & 2 & -1 \end{pmatrix} \begin{pmatrix} 2000 \\ 4000 \\ 250 \end{pmatrix}$$

$$= \begin{pmatrix} 500 \\ 250 \\ 1250 \end{pmatrix}$$

The number of each type of bat is thus,

brown : 500

grey : 250

black : 1250

[7 marks]

Answer 5

The system of equations,

$$\begin{aligned} x + y + z + w &= 13 \\ 2x + 3y + 0z - w &= -1 \\ -3x + 4y + z + 2w &= 10 \\ x + 2y - z + w &= 1 \end{aligned}$$

converted into matrix form becomes,

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 0 & -1 \\ -3 & 4 & 1 & 2 \\ 1 & 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 13 \\ -1 \\ 10 \\ 1 \end{pmatrix}$$

Using a calculator to get the inverse of the leftmost matrix results in,

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \frac{1}{54} \begin{pmatrix} 15 & 3 & -9 & 6 \\ -4 & 10 & 6 & 2 \\ 25 & 5 & 3 & -26 \\ 18 & -18 & 0 & 18 \end{pmatrix} \begin{pmatrix} 13 \\ -1 \\ 10 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 0 \\ 6 \\ 5 \end{pmatrix}$$

$$\therefore x = 2, \quad y = 0, \quad z = 6, \quad w = 5$$

[5 marks]