

Lesson 6

Further A-Level Pure Mathematics : Core 1

Matrix Systems of Equations

6.1 Test

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 30

Question 1

Matrix $\mathbf{A} = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 4 \\ -4 & 2 & 1 \end{pmatrix}$ and matrix $\mathbf{B} = \begin{pmatrix} 3 & 1 & 2 \\ k & 4 & 5 \\ 0 & 2 & 3 \end{pmatrix}$ where k is a constant.

(i) Evaluate the determinant of $\mathbf{A}$

[2 marks]

Given that the determinant of $\mathbf{B}$ is 2,

(ii) find the value of k

[3 marks]

Question 2

Given that $\mathbf{A} = \begin{pmatrix} a & 2 \\ 3 & 2a \end{pmatrix}$ where a is a real constant

(i) find $\mathbf{A}^{-1}$ in terms of a

[3 marks]

(ii) write down two values of a for which $\mathbf{A}^{-1}$ does not exist

[1 mark]

Question 3

If $\mathbf{A}^{-1} = \mathbf{A}$, then the matrix $\mathbf{A}$ is said to be self-inverse.

(It's its own opposite)

Give an example of a matrix that is self-inverse other than an identity matrix.

[1 mark]

Question 4

The matrix $\mathbf{A} = \begin{pmatrix} 2 & 0 & 3 \\ k & 1 & 1 \\ 1 & 1 & 4 \end{pmatrix}$

(i) Show that $\det \mathbf{A} = 3(k + 1)$

[3 marks]

(ii) Given that $k \neq -1$, find $\mathbf{A}^{-1}$

[4 marks]

Question 5

A colony of 1000 mole-rats is made up of adult males, adult females and youngsters. Originally there were 100 more adult females than adult males.

After one year;

- the number of adult males had increased by 2 %
- the number of adult females had increased by 3 %
- the number of youngsters had decreased by 4 %
- the total number of mole-rats had decreased by 20

Form and solve a matrix equation to find out how many of each type of mole-rat were in the original colony.

[6 marks]

Question 6

Further A-Level Examination Question, June 2019, Core 2, Q7 (Edexcel)

$$\mathbf{M} = \begin{pmatrix} 2 & -1 & 1 \\ 3 & k & 4 \\ 3 & 2 & -1 \end{pmatrix} \text{ where } k \text{ is a constant}$$

- (a) Find the values of k for which $\mathbf{M}$ has an inverse

[2 marks]

- (b) Find, in terms of p , the coordinates of the point where the following planes intersect,

$$2x - y + z = p$$

$$3x - 6y + 4z = 1$$

$$3x + 2y - z = 0$$

[5 marks]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School

It may be freely duplicated and distributed, unaltered, for non-profit educational use

In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

© 2023 Number Wonder

Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk