8.8 Answers

8.8.1 Solution to 8.6 Example

(i) Rotation of 90° (anticlockwise) about the x-axis.

[3 marks]

(ii)
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$$

$$\therefore\ (3, -1, 4) \rightarrow (3, -4, -1)$$

[1 mark]

(iii)
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ -a \\ 2a - 1 \end{pmatrix} = \begin{pmatrix} a \\ a - 5 \\ -a \end{pmatrix}$$

$$\begin{pmatrix} a \\ -2a + 1 \\ -a \end{pmatrix} = \begin{pmatrix} a \\ a - 5 \\ -a \end{pmatrix}$$

$$-2a + 1 = a - 5$$

$$a = 2$$

[3 marks]

8.8.1 Solutions to 8.7 Exercise

Answer 1

(i) reflection in the plane $x = 0$

$x = 0$ is the (left hand) side wall

$$\mathbf{Ref}_{x=0} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

[2 marks]

(ii) rotation of 180° about the x-axis

$$\mathbf{Rot}_{180x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

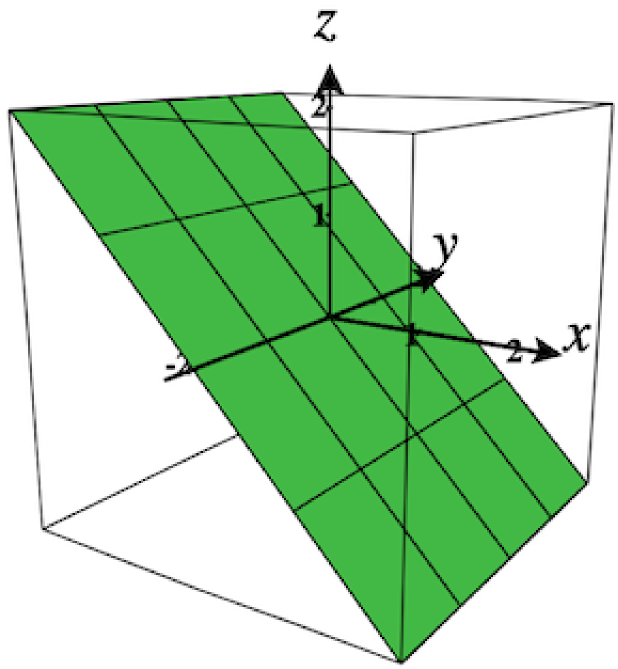
[2 marks]

(iii) rotation of 90° about the y-axis

$$\mathbf{Rot}_{90y} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

[2 marks]

Answer 2



$$\mathbf{Ref}_{z=-y} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

[2 marks]

Answer 3

Describe the transformations represented by the following matrices,

(i) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$: Reflection in the plane $y = 0$

[2 marks]

(ii) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$: Reflection in the plane $x = z$

[2 marks]

(iii) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$: Rotation of 180° about the z-axis

[2 marks]

Answer 4

(a) $$\mathbf{A}^4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \mathbf{I}$$

[1 mark]

(b) A rotation of 270° about the x-axis

[2 marks]

(c) $$\mathbf{B} = \mathbf{Ref}_{x=0} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

[2 marks]

(d) $$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix}$$

[1 mark]

Answer 5

(a) $\mathbf{M}_1 = \mathbf{Ref}_{y=0} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

[1 mark]

(b) $\mathbf{M}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$: Rotation 180° about the x-axis

[1 mark]

(c) **(i)** $\left|\mathbf{M}_1\right| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (1)(-1)(1) = -1$

$\left|\mathbf{M}_2\right| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = (1)(-1)(-1) = 1$

[2 marks]

(ii) The magnitude of the determinant has the geometric intepretation of “area scale factor”. Both of these matrices would not change the area of a shape being transformed by them.

The sign of the determinant gives the orientation of the image and so $\mathbf{M}_1$ will flip the orientation and $\mathbf{M}_2$ will not.

[2 marks]

(d) **(i)** $\mathbf{M}_3 = \mathbf{M}_1\mathbf{M}_2$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

[1 mark]

(ii) M_3 represents reflection in the plane $z = 0$

[2 marks]

Answer 6

(i) $\quad \mathbf{A} = \mathbf{Ref}_{x=0} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\mathbf{B} = \mathbf{Ref}_{y=0} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

[2 marks]

(ii) $\quad \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -a \\ b \\ c \end{pmatrix}$

[2 marks]

(iii) $\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -a \\ -b \\ c \end{pmatrix}$

[2 marks]

Answer 7

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

[2 marks]

Answer 8

(a) $\quad$ **(i)** $\quad \begin{vmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -(-1)(1)(1) = 1$

[1 mark]

(ii) The magnitude of the determinant has the geometric intepretation of "area scale factor". This matrix will not change the area of a shape being transformed by it. The sign of the determinant gives the orientation of the image and this matrix will preserve the orientation.

[2 marks]

(b) $\quad \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$: Rotation of $90°$ about the z-axis.

[1 mark]