Lesson 6

Further A-Level Pure Mathematics : Core 1

Proof by Induction

6.1 Test

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 36

Question 1

Further A-Level Examination Question, June 2011, IAL, FP1, Q9 (b) (Edexcel)

Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 7^{2n-1} + 5$$

is divisible by 12

[6 marks]

Question 2

Further A-Level Sample Assessment Materials, Core 2, Q3 (ii) (Edexcel)

Prove by induction that for all positive integers n

$$\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$$

[6 marks]

Question 3

Further AS-Level Examination Question, May 2018, Core 1, Q8 (ii) (Edexcel)

Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 4^{n+1} + 5^{2n-1}$$

is divisible by 21

[6 marks]

Question 4

Further A-Level Examination Question, May 2015, IAL, FP1, Q6 (ii) (Edexcel)

Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} (2r - 1)^2 = \frac{1}{3} n\left(4n^2 - 1\right)$$

[6 marks]

Question 5

Further A-Level Examination Question, January 2009, IAL, FP1, Q6 (Edexcel)

A series of positive integers u_1, u_2, u_3, ... is defined by

$$u_1 = 6 \text{ and } u_{n+1} = 6u_n - 5, \quad \text{for } n \geqslant 1$$

Prove by induction that $u_n = 5 \times 6^{n-1} + 1$ for $n \geqslant 1$

[5 marks]

Question 6

Further A-Level Examination Question, June 2009, IAL, FP1, Q8 (a) (Edexcel)

Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 5^n + 8n + 3 \text{ is divisible by 4}$$

[7 marks]