

Lesson 4

Further A-Level Pure Mathematics, Core 2 Telescoping Series

4.1 Test

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 40

Question 1

(i) Prove that, $\frac{1}{r^2} - \frac{1}{(r + 1)^2} = \frac{2r + 1}{r^2(r + 1)^2}$

[1 marks]

(ii) Hence, using the method of differences, find a formula for,

$$\sum_{r=1}^n \frac{2r + 1}{r^2(r + 1)^2}$$

[5 marks]

(iii) State if the sum to infinity, $\sum_{r=1}^{\infty} \frac{2r + 1}{r^2(r + 1)^2}$, is convergent or divergent.

Give a reason for your answer.

[2 marks]

Question 2

Further A-Level Examination Question from June 2014, FP2, Q1

(a) Express $\frac{2}{(r+2)(r+4)}$ in partial fractions.

[1 mark]

(b) Hence show that $\sum_{r=1}^n \frac{2}{(r+2)(r+4)} = \frac{n(7n+25)}{12(n+3)(n+4)}$

[5 marks]

Question 3

- (i) Use the method of differences to obtain a formula for,

$$\sum_{r=1}^n \left(1024^{\frac{1}{r}} - 1024^{\frac{1}{r+1}} \right)$$

[3 marks]

- (ii) Hence, or otherwise, determine the exact value of,

$$\sum_{r=5}^9 \left(1024^{\frac{1}{r}} - 1024^{\frac{1}{r+1}} \right)$$

[3 marks]

Question 4

Further A-Level Examination Question from June 2016, FP2, Q2

(a) Show that, for $r > 0$

$$r - 3 + \frac{1}{r + 1} - \frac{1}{r + 2} = \frac{r^3 - 7r - 5}{(r + 1)(r + 2)}$$

[2 marks]

(b) Hence prove, using the method of differences, that

$$\sum_{r=1}^n \frac{r^3 - 7r - 5}{(r + 1)(r + 2)} = \frac{n(n^2 + an + b)}{2(n + 2)}$$

where a and b are constants to be found

[5 marks]

Question 5

(i) Prove that $\frac{2^k}{k+1} - \frac{2^{k-1}}{k} = \frac{(k-1)2^{k-1}}{k(k+1)}$

[3 marks]

(ii) Use the method of differences to obtain a formula for,

$$\sum_{r=1}^n \left(\frac{(k-1)2^{k-1}}{k(k+1)} \right)$$

[4 marks]

Question 6

Prove that,

$$\frac{2}{0! + 1! + 2!} + \frac{3}{1! + 2! + 3!} + \dots + \frac{n}{(n-2)! + (n-1)! + n!} = 1 - \frac{1}{n!}$$

[6 marks]

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