1.5 Answers

Further A-Level Pure Mathematics, Core 2
Maclaurin Series

Answer 1

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

$$\frac{dy}{dx} = a_1 + 2 a_2 x + 3 a_3 x^2 + 4 a_4 x^3$$

$$\frac{d^2y}{dx^2} = 2 a_2 + 6 a_3 x + 12 a_4 x^2$$

$$\frac{d^3y}{dx^3} = 6 a_3 + 24 a_4 x$$

$$\frac{d^4y}{dx^4} = 24 a_4$$

[4 marks]

Answer 2

$y = \cos x$	when $x = 0$,	$y = 1$
$\frac{dy}{dx} = -\sin x$	when $x = 0$,	$\frac{dy}{dx} = 0$
$\frac{d^2y}{dx^2} = -\cos x$	when $x = 0$,	$\frac{d^2y}{dx^2} = -1$
$\frac{d^3y}{dx^3} = \sin x$	when $x = 0$,	$\frac{d^3y}{dx^3} = 0$
$\frac{d^4y}{dx^4} = \cos x$	when $x = 0$,	$\frac{d^4y}{dx^4} = 1$

[4 marks]

Answer 3

$$24\,a_4 = 1 \Rightarrow a_4 = \frac{1}{24}$$

$$6\,a_3 + 24\,a_4\,x = 0 \text{ but } x = 0 \text{ so } a_3 = 0$$

$$2\,a_2 + 6\,a_3\,x + 12\,a_4\,x^2 = -1 \text{ but } x = 0 \text{ so } a_2 = -\frac{1}{2}$$

$$a_1 + 2\,a_2\,x + 3\,a_3\,x^2 + 4\,a_4\,x^3 = 0 \text{ but } x = 0 \text{ so } a_1 = 0$$

$$a_0 + a_1\,x + a_2\,x^2 + a_3\,x^3 + a_4\,x^4 = 1 \text{ but } x = 0 \text{ so } a_0 = 1$$

$$y = 1 - \frac{x^2}{2} + \frac{x^4}{24} \quad \text{or} \quad y = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

[6 marks]

Answer 4

The working can be shortened, as suggested in section 1.3
Here, however, is the full detail,

$y = a_0 + a_1\,x + a_2\,x^2 + a_3\,x^3 + a_4\,x^4 + a_5\,x^5$
$\frac{dy}{dx} = a_1 + 2\,a_2\,x + 3\,a_3\,x^2 + 4\,a_4\,x^3 + 5\,a_5\,x^4$
$\frac{d^2y}{dx^2} = 2\,a_2 + 6\,a_3\,x + 12\,a_4\,x^2 + 20\,a_5\,x^3$
$\frac{d^3y}{dx^3} = 6\,a_3 + 24\,a_4\,x + 60\,a_5\,x^2$
$\frac{d^4y}{dx^4} = 24\,a_4 + 120\,a_5\,x$
$\frac{d^5y}{dx^5} = 120\,a_5$

[4 marks]

Answer 5

$y = \sin x$	when $x = 0$,	$y = 0$
$\frac{dy}{dx} = \cos x$	when $x = 0$,	$\frac{dy}{dx} = 1$
$\frac{d^2y}{dx^2} = -\sin x$	when $x = 0$,	$\frac{d^2y}{dx^2} = 0$
$\frac{d^3y}{dx^3} = -\cos x$	when $x = 0$,	$\frac{d^3y}{dx^3} = -1$
$\frac{d^4y}{dx^4} = \sin x$	when $x = 0$,	$\frac{d^4y}{dx^4} = 0$
$\frac{d^5y}{dx^5} = \cos x$	when $x = 0$,	$\frac{d^5y}{dx^5} = 1$

[5 marks]

Answer 6

$$120\,a_5 = 1 \Rightarrow a_5 = \frac{1}{120}$$

$$24\,a_4 + 120\,a_5 x = 0 \text{ but } x = 0 \text{ so } a_4 = 0$$

$$6\,a_3 + 24\,a_4 x + 60\,a_5 x^2 = -1 \text{ but } x = 0 \text{ so } a_3 = -\frac{1}{6}$$

$$2\,a_2 + 6\,a_3 x + 12\,a_4 x^2 + 20\,a_5 x^3 = 0 \text{ but } x = 0 \text{ so } a_2 = 0$$

$$a_1 + 2\,a_2 x + 3\,a_3 x^2 + 4\,a_4 x^3 + 5\,a_5 x^4 = 1 \text{ but } x = 0 \text{ so } a_1 = 1$$

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 = 0 \text{ but } x = 0 \text{ so } a_0 = 0$$

$$y = x - \frac{x^3}{6} + \frac{x^5}{120}$$

or, more memorably,

$$y = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

[6 marks]

Answer 7

$y = ln(1 - x)$	when $x = 0$,	$y = 0$
$\frac{dy}{dx} = -\frac{1}{1-x}$	when $x = 0$,	$\frac{dy}{dx} = -1$
$\frac{d^2y}{dx^2} = -\frac{1}{(1-x)^2}$	when $x = 0$,	$\frac{d^2y}{dx^2} = -1$
$\frac{d^3y}{dx^3} = -\frac{2}{(1-x)^3}$	when $x = 0$,	$\frac{d^3y}{dx^3} = -2$
$\frac{d^4y}{dx^4} = -\frac{6}{(1-x)^4}$	when $x = 0$,	$\frac{d^4y}{dx^4} = -6$
$\frac{d^5y}{dx^5} = -\frac{24}{(1-x)^5}$	when $x = 0$,	$\frac{d^5y}{dx^5} = -24$

[5 marks]

Answer 8

$$120\,a_5 = -24 \Rightarrow a_5 = -\frac{1}{5}$$

$$24\,a_4 + 120\,a_5 x = -6 \text{ but } x = 0 \text{ so } a_4 = -\frac{1}{4}$$

$$6\,a_3 + 24\,a_4 x + 60\,a_5 x^2 = -2 \text{ but } x = 0 \text{ so } a_3 = -\frac{1}{3}$$

$$2\,a_2 + 6\,a_3 x + 12\,a_4 x^2 + 20\,a_5 x^3 = -1 \text{ but } x = 0 \text{ so } a_2 = -\frac{1}{2}$$

$$a_1 + 2\,a_2 x + 3\,a_3 x^2 + 4\,a_4 x^3 + 5\,a_5 x^4 = -1 \text{ but } x = 0 \text{ so } a_1 = -1$$

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 = 0 \text{ but } x = 0 \text{ so } a_0 = 0$$

$$y = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5}$$

[6 marks]