

2.4 Answers

Further A-Level Pure Mathematics, Core 2

Maclaurin Series

2.4.1 Solution to 2.2 Example (Teaching Video)

$$f(x) = e^{\sin x} \Rightarrow f(0) = 1$$

$$f'(x) = e^{\sin x} \cos x \Rightarrow f'(0) = 1$$

$$\begin{aligned} f''(x) &= e^{\sin x}(-\sin x) + e^{\sin x} \cos x \cos x \\ &= e^{\sin x}(\cos^2 x - \sin x) \Rightarrow f''(0) = 1 \end{aligned}$$

By Maclaurin,

$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2$$

$$f(x) \approx 1 + x + \frac{x^2}{2}$$

[6 marks]

2.4.2 Solutions to 2.3 Exercise

Answer 1

$$f(x) = e^{\cos x} \Rightarrow f(0) = e$$

$$f'(x) = e^{\cos x}(-\sin x) \Rightarrow f'(0) = 0$$

$$\begin{aligned} f''(x) &= e^{\cos x}(-\cos x) + e^{\cos x}(-\sin x)(-\sin x) \\ &= e^{\cos x}(\sin^2 x - \cos x) \Rightarrow f''(0) = -e \end{aligned}$$

By Maclaurin,

$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2$$

$$f(x) \approx e - \frac{ex^2}{2}$$

[6 marks]

Answer 2

$$f(x) = \cos(\pi e^x) \Rightarrow f(0) = -1$$

$$f'(x) = -\sin(\pi e^x) \pi e^x \Rightarrow f'(0) = 0$$

$$\begin{aligned} f''(x) &= -\sin(\pi e^x) \pi e^x - \cos(\pi e^x) \pi e^x \pi e^x \\ &= \pi e^x(-\sin(\pi e^x) - \pi e^x \cos(\pi e^x)) \Rightarrow f''(0) = \pi^2 \end{aligned}$$

By Maclaurin,

$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2$$

$$f(x) \approx -1 + \frac{\pi^2}{2}x^2$$

[6 marks]

Answer 3

$$\begin{aligned}
f(x) &= \sin(\pi \cos x) & \Rightarrow f(0) &= 0 \\
f'(x) &= \cos(\pi \cos x) \pi (-\sin x) & \Rightarrow f'(0) &= 0 \\
f''(x) &= \cos(\pi \cos x) \pi (-\cos x) + (-\sin(\pi \cos x) \pi (-\sin x) \pi (-\sin x)) \\
&= -\pi \cos x \cos(\pi \cos x) - \pi^2 \sin^2 x \sin(\pi \cos x) \Rightarrow f''(0) = \pi
\end{aligned}$$

By Maclaurin, $f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2$

$$f(x) \approx \frac{\pi}{2}x^2$$

[6 marks]**Answer 4**

$$\begin{aligned}
f(x) &= e^{1-x^2} & \Rightarrow f(0) &= e \\
f'(x) &= e^{1-x^2} (-2x) & \Rightarrow f'(0) &= 0 \\
f''(x) &= e^{1-x^2} (-2) + e^{1-x^2} (-2x)(-2x) \\
&= -2e^{1-x^2} + 4x^2 e^{1-x^2} \Rightarrow f''(0) = -2e
\end{aligned}$$

By Maclaurin, $f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2$

$$f(x) \approx e - ex^2$$

[6 mark]**Answer 5****(i)**

$f(x) = \ln(2+x)$	when $x = 0$,	$f(0) = \ln 2$
$f'(x) = \frac{1}{2+x}$	when $x = 0$,	$f'(0) = \frac{1}{2}$
$f''(x) = -\frac{1}{(2+x)^2}$	when $x = 0$,	$f''(0) = -\frac{1}{4}$
$f'''(x) = \frac{2}{(2+x)^3}$	when $x = 0$,	$f'''(0) = \frac{1}{4}$
$f^{(4)}(x) = -\frac{6}{(2+x)^4}$	when $x = 0$,	$f^{(4)}(0) = -\frac{3}{8}$
$f^{(5)}(x) = \frac{24}{(2+x)^5}$	when $x = 0$,	$f^{(5)}(0) = \frac{3}{4}$

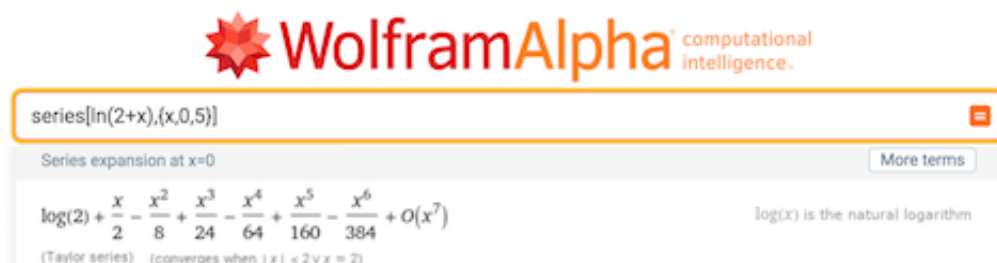
(ii)
$$f(x) \approx \ln 2 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{24} - \frac{x^4}{64} + \frac{x^5}{160}$$

[8 marks]

Wolfram Alpha can do Maclaurin series.

To check your answer to question 5,

`series [ ln(2 + x), { x, 0, 5} ]`



Answer 6

$f(x) = \ln(1 + \sin x)$	when $x = 0$,	$f(0) = 0$
$f'(x) = \frac{\cos x}{1 + \sin x}$	when $x = 0$,	$f'(0) = 1$
$f''(x) = -\frac{1}{1 + \sin x}$	when $x = 0$,	$f''(0) = -1$
$f'''(x) = -\frac{\cos x}{(1 + \sin x)^2}$	when $x = 0$,	$f'''(0) = -1$
$f^{(4)}(x) = \frac{\sin x - \sin^2 x + 2 \cos^2 x}{(1 + \sin x)^3}$	when $x = 0$,	$f^{(4)}(0) = 2$

$$f(x) \approx x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12}$$

[10 marks]

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