7.1 Test

7.2 Useful Information

7.2.1 A Table of Functions and their Derivatives

$f(x)$	$f'(x)$	In Formula Book ?
x^n	$n\,x^{n-1}$	No
e^x	e^x	No
$\ln x$	$\frac{1}{x}$	No
$\sin x$	$\cos x$	No
$\cos x$	$-\sin x$	No
$\tan x$	$\sec^2 x$	Yes
$\csc x$	$-\csc x \cot x$	Yes
$\sec x$	$\sec x \tan x$	Yes
$\cot x$	$-\csc^2 x$	Yes
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	Yes
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$	Yes
$\arctan x$	$\frac{1}{1+x^2}$	Yes
$\text{arccot}\, x$	$-\frac{1}{1+x^2}$	No
$\text{arcsec}\, x$	$\frac{1}{x\sqrt{x^2-1}}$	No
$\text{arccsc}\, x$	$-\frac{1}{x\sqrt{x^2-1}}$	No

7.2.2 The Trigonometric Addition Formula

The Addition Formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

7.2.3 Standard Maclaurin Series

Quotable Maclaurin Series Expansions

- $e^x = 1 + x + \dfrac{x^2}{2!} + \ldots + \dfrac{x^r}{r!} + \ldots$ valid for all x
- $ln(1 + x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \ldots + (-1)^{r+1}\dfrac{x^r}{r} + \ldots$ $-1 < x \leqslant 1$
- $sin\, x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \ldots + (-1)^r \dfrac{x^{2r+1}}{(2r + 1)!} + \ldots$ valid for all x
- $cos\, x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \ldots + (-1)^r \dfrac{x^{2r}}{(2r)!} + \ldots$ valid for all x
- $arctan\, x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \ldots. + (-1)^r \dfrac{x^{2r+1}}{2r + 1} + \ldots$ $-1 < x \leqslant 1$

7.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 35

Question 1

Use the addition formula for $cos(A - B)$ and the series expansions of $sin\, x$ and $cos\, x$ to show that,

$$cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\left(1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \ldots\right)$$

[3 marks]

Question 2

Given that $f(x) = ln(\,sec\,x + tan\,x\,)$

(a) show that $f'(x) = sec\,x$

[2 marks]

(b) find the values of $f'(0)$, $f''(0)$, and $f'''(0)$

[2 marks]

(c) express $ln\,(\,sec\,x + tan\,x\,)$ as a series in ascending powers of x up to and including the term in x^3

[2 marks]

(d) show that using the first two non-zero terms of the Maclaurin series for $ln\,(sec\,x + tan\,x)$, with $x = \frac{\pi}{4}$, gives $ln(\sqrt{2} + 1) = \frac{\pi}{4}\left(1 + \frac{\pi^2}{96}\right)$

[2 marks]

Question 3

$$f(x) = \arcsin(4x)$$

(i) Write down $f'(x)$

[2 marks]

(ii) A special case of the binomial theorem is that,

$$(1 + x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \ldots \qquad -1 < x \leqslant 1$$

Show how to use this to expand $f'(x)$, and hence find the Maclaurin series for $f(x)$ in ascending powers of x up to and including the term in x^5

[4 marks]

Question 4

Further A-Level Examination Question from June 2017, FP2, Q1 (b) (MEI)

Obtain the first three terms of the Maclaurin series for $f(x) = arctan(1 + x)$

[5 marks]

Question 5

The Laurent series expansion of $\csc x$ begins

$$\frac{1}{x} + ax + bx^3$$

where a and b are constants.

Explain why, for sufficiently small x,

$$\left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5\right)\left(\frac{1}{x} + ax + bx^3\right) \approx 1$$

and hence find the values of a and b

[5 marks]

Question 6

Given that $f(x) = e^{2x} \sin x$

(a) Show that $f''(x) = p\,f(x) + q\,f'(x)$
where p and q are integers to be determined

[5 marks]

(b) Hence find the Maclaurin series for $f(x)$, in ascending powers of x, up to and including the term in x^5, giving each coefficient in its simplest form.

[3 marks]