

Lesson 11

Further A-Level Pure Mathematics, Core 2 Complex Numbers II

11.1 Test

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 35

Question 1

Given that $g(x) = \frac{f(x) + f(-x)}{2}$ and $h(x) = \frac{f(x) - f(-x)}{2}$ prove that,

(i) $g(x)$ is an even function

[2 marks]

(ii) $h(x)$ is an odd function

[2 marks]

(iii) $g(x) + h(x) = f(x)$

[1 mark]

(iv) Taking all of parts (i), (ii) and (iii) into account, what has been proved
is true for every function $f(x)$?

[1 mark]

Question 2

Show that $\frac{\cos 2\theta + i \sin 2\theta}{\cos 9\theta - i \sin 9\theta}$ can be expressed in the form $\cos n\theta + i \sin n\theta$,
where n is an integer to be found

[4 marks]

Question 3

Prove that $\frac{1 + e^{i\theta}}{1 - e^{i\theta}} = i \cot\left(\frac{\theta}{2}\right)$

[5 marks]

Question 4

Further A-Level Examination Question, 2017, SAM, Core 2, Q4 (Edexcel)

A complex number z has modulus 1 and argument θ

(a) Show that,

$$z^n + \frac{1}{z^n} = 2 \cos n\theta, \quad n \in \mathbb{Z}^+$$

[2 marks]

(b) Hence, show that,

$$\cos^4 \theta = \frac{1}{8} (\cos 4\theta + 4 \cos 2\theta + 3)$$

[5 marks]

Question 5

Further A-Level Examination Question from June 2014, FP2, Q2(a) (MEI)

The infinite series C and S are defined as follows,

$$C = a \cos \theta + a^2 \cos 2\theta + a^3 \cos 3\theta + \dots$$

$$S = a \sin \theta + a^2 \sin 2\theta + a^3 \sin 3\theta + \dots$$

where a is a real number and $|a| < 1$

By considering $C + iS$, show that $S = \frac{a \sin \theta}{1 - 2a \cos \theta + a^2}$

Find a corresponding expression for C

[8 marks]

Question 6

Given that $z = e^{\frac{\pi i}{n}}$ where n is a positive integer, prove that,

$$1 + z + z^2 + \dots + z^{2n-1} = 0$$

[5 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk