11.1 Test

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 35

Question 1

Given that $g(x) = \dfrac{f(x) + f(-x)}{2}$ and $h(x) = \dfrac{f(x) - f(-x)}{2}$ prove that,

(i) $g(x)$ is an even function

[2 marks]

(ii) $h(x)$ is an odd function

[2 marks]

(iii) $g(x) + h(x) = f(x)$

[1 mark]

(iv) Taking all of parts (i), (ii) and (iii) into account, what has been proved is true for every function $f(x)$?

[1 mark]

Question 2

Show that $\dfrac{\cos 2\theta + \mathrm{i}\sin 2\theta}{\cos 9\theta - \mathrm{i}\sin 9\theta}$ can be expressed in the form $\cos n\theta + \mathrm{i}\sin n\theta$,

where n is an integer to be found

[4 marks]

Question 3

Prove that $\dfrac{1 + e^{\mathrm{i}\theta}}{1 - e^{\mathrm{i}\theta}} = \mathrm{i}\cot\left(\dfrac{\theta}{2}\right)$

[5 marks]

Question 4

Further A-Level Examination Question, 2017, SAM, Core 2, Q4 (Edexcel)

A complex number z has modulus 1 and argument θ

(a) Show that,

$$z^n + \frac{1}{z^n} = 2\cos n\theta, \qquad n \in \mathbb{Z}^+$$

[2 marks]

(b) Hence, show that,

$$\cos^4\theta = \frac{1}{8}\left(\cos 4\theta + 4\cos 2\theta + 3 \right)$$

[5 marks]

Question 5

Further A-Level Examination Question from June 2014, FP2, Q2(a) (MEI)

The infinite series C and S are defined as follows,

$$C = a \cos \theta + a^2 \cos 2\theta + a^3 \cos 3\theta + \ldots$$

$$S = a \sin \theta + a^2 \sin 2\theta + a^3 \sin 3\theta + \ldots$$

where a is a real number and $|a| < 1$

By considering $C + \mathrm{i}S$, show that $S = \dfrac{a \sin \theta}{1 - 2a \cos \theta + a^2}$

Find a corresponding expression for C

[8 marks]

Question 6

Given that $z = e^{\frac{\pi}{n}i}$ where n is a positive integer, prove that,

$$1 + z + z^2 + ... + z^{2n-1} = 0$$

[5 marks]