

Lesson 7

Further A-Level Pure Mathematics, Core 2

Improper Integrals

7.1 Test

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 33

Question 1

Further A-Level Examination Question from June 2011, FP2, Q1 (b) (ii) (OCR)

Find, in exact form, the value of the integral,

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{(1 + 4x^2)^{\frac{3}{2}}} dx$$

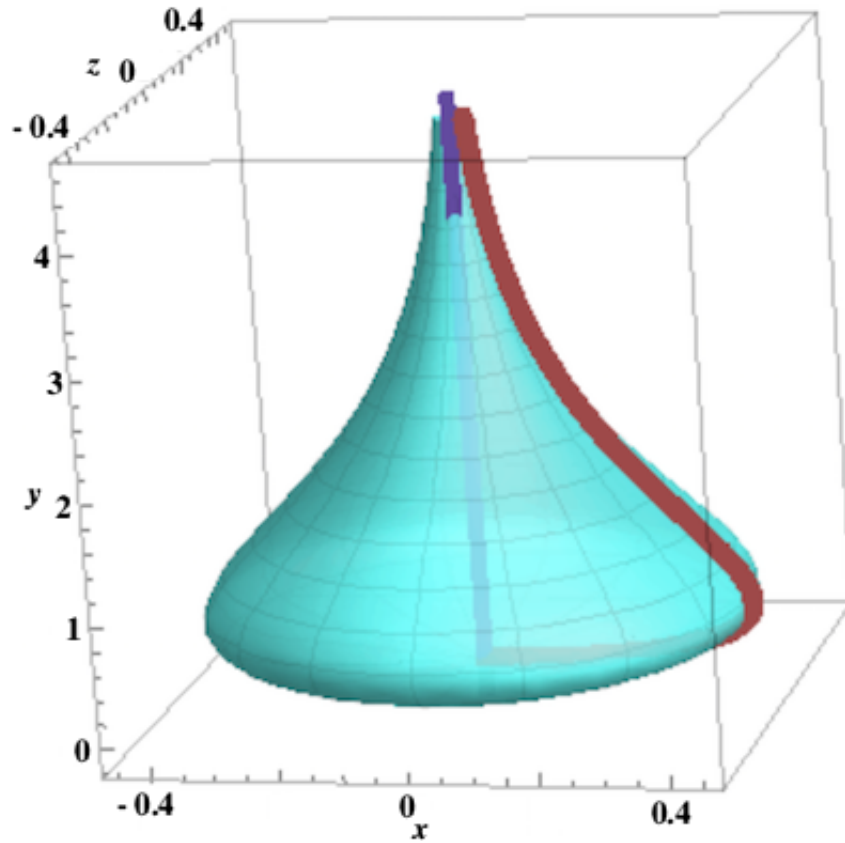
Hint : Use the substitution $x = \frac{1}{2} \tan \theta$

[6 marks]

Question 2

Find the volume swept out when the profile curve $x = \sqrt{y} e^{-y}$ is rotated 2π about the y -axis, $0 \leq y < \infty$. The lower end of the solid is depicted below.

You may use without proof the fact that $\lim_{t \rightarrow \infty} t e^{-2t} = 0$



[6 marks]

Question 3

Further A-Level Examination Question from June 2019, Paper 1, Q2 (Edexcel)

Show that,

$$\int_0^{\infty} \frac{8x - 12}{(2x^2 + 3)(x + 1)} dx = \ln k$$

where k is a rational number to be found

[7 marks]

Question 4

Further A-Level Examination Question from January 2010, FP2, Q1(a) (OCR)

- (i) Given that $y = \arctan \sqrt{x}$ find $\frac{dy}{dx}$ giving your answer in terms of x

[3 marks]

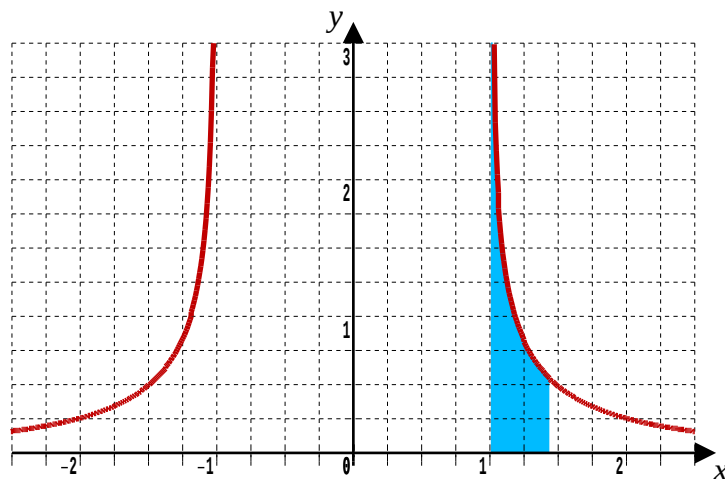
- (ii) Hence show that $\int_0^1 \frac{1}{\sqrt{x}(x+1)} dx = \frac{\pi}{2}$

[3 marks]

Question 5

The area shaded blue, and extending upwards indefinitely, is represented by the

improper integral $\int_1^{\sqrt{2}} \frac{1}{\sqrt{x^4 - 1}} dx$



Use the substitution $x^2 = \sec \theta$ to show that the improper integral can be converted to a proper one with limits that show the original integral is convergent.

Note : You are not asked to find the value of the improper integral, which is difficult.

[8 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk