

2.4 Answers to 2.3 Exercise

Further A-Level Pure Mathematics, Core 2 Polar Coordinates

Answer 1

$$\begin{aligned}(x^2 + y^2)^2 &= 100(x^2 - y^2) \\ (r^2)^2 &= 100(r^2 \cos^2 \theta - r^2 \sin^2 \theta) \\ r^4 &= 100r^2(\cos^2 \theta - \sin^2 \theta) \\ r^2 &= 100 \cos(2\theta) \quad \square\end{aligned}$$

[3 marks]

Answer 2

(i)

$$\begin{aligned}y^2(8 + x) &= x^2(8 - x) \\ r^2 \sin^2 \theta (8 + r \cos \theta) &= r^2 \cos^2 \theta (8 - r \cos \theta) \\ \sin^2 \theta (8 + r \cos \theta) &= \cos^2 \theta (8 - r \cos \theta) \\ 8 \sin^2 \theta + r \sin^2 \theta \cos \theta &= 8 \cos^2 \theta - r \cos^3 \theta \\ r \cos^3 \theta + r \sin^2 \theta \cos \theta &= 8 \cos^2 \theta - 8 \sin^2 \theta \\ r \cos \theta (\cos^2 \theta + \sin^2 \theta) &= 8(\cos^2 \theta - \sin^2 \theta) \\ r \cos \theta &= 8 \cos(2\theta) \\ r &= \frac{8 \cos(2\theta)}{\cos \theta} \quad \square\end{aligned}$$

[6 marks]

(ii)

$$\begin{aligned}\text{LHS} &= y^2(8 + x) \\ &= (-4\sqrt{3})^2(8 + (-4)) \quad \text{when } (-4, -4\sqrt{3}) \text{ is inserted} \\ &= 16 \times 3 \times 4 \\ &= 192\end{aligned}$$

$$\begin{aligned}\text{RHS} &= x^2(8 - x) \\ &= (-4)^2 \times (8 - (-4)) \quad \text{when } (-4, -4\sqrt{3}) \text{ is inserted} \\ &= 16 \times 12 \\ &= 192 \\ &= \text{LHS}\end{aligned}$$

$$\therefore (-4, -4\sqrt{3}) \text{ is on the curve} \quad \square$$

[2 marks]

$$(iii) \quad y^2(8+x) = x^2(8-x)$$

$$y^2(8+x) = 8x^2 - x^3$$

Differentiating Implicitly

$$y^2(1) + 2y \frac{dy}{dx} (8+x) = 16x - 3x^2$$

$$\frac{dy}{dx} (16y + 2xy) = 16x - 3x^2 - y^2$$

$$\frac{dy}{dx} = \frac{16x - 3x^2 - y^2}{16y + 2xy} \quad \square$$

[4 marks]

(iv)

$$\begin{aligned} \frac{dy}{dx} &= \frac{16(-4) - 3(-4)^2 - (-4\sqrt{3})^2}{16(-4\sqrt{3}) + 2(-4)(-4\sqrt{3})} \\ &= \frac{-64 - 48 - 48}{-64\sqrt{3} + 32\sqrt{3}} \\ &= \frac{-160}{-32\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{5\sqrt{3}}{3} \end{aligned}$$

$$y = mx + c$$

$$-4\sqrt{3} = \frac{5\sqrt{3}}{3} \times (-4) + c$$

$$\begin{aligned} c &= -\frac{12\sqrt{3}}{3} + \frac{20\sqrt{3}}{3} \\ &= \frac{8\sqrt{3}}{3} \end{aligned}$$

$$\therefore \text{ Tangent is } y = \frac{5\sqrt{3}}{3}x + \frac{8\sqrt{3}}{3}$$

[3 marks]

Answer 3**(i)**

$$(x^2 + y^2)(y^2 + x(x + 10)) = 40xy^2$$

$$(x^2 + y^2)(y^2 + x^2 + 10x) = 40xy^2$$

$$r^2(r^2 + 10r \cos \theta) = 40r \cos \theta r^2 \sin^2 \theta$$

$$r^3(r + 10 \cos \theta) = 40r^3 \cos \theta \sin^2 \theta$$

$$r + 10 \cos \theta = 40 \cos \theta \sin^2 \theta$$

$$r = 40 \cos \theta \sin^2 \theta - 10 \cos \theta$$

$$r = 10 \cos \theta (4 \sin^2 \theta - 1)$$

□

[5 marks]**(ii)** When $\theta = 60^\circ$,

$$r = 10 \cos 60 (4 \sin^2 60 - 1)$$

$$= 10 \times \frac{1}{2} \left(4 \times \left(\frac{\sqrt{3}}{2} \right)^2 - 1 \right)$$

$$= 10 \times \frac{1}{2} \left(4 \times \frac{3}{4} - 1 \right)$$

$$= 10 \times \frac{1}{2} \times 2$$

$$= 10 \quad \text{This is the tip of the loop in the first quadrant}$$

[1 mark]**(iii)** When $\theta = 30^\circ$,

$$r = 10 \cos 30 (4 \sin^2 30 - 1)$$

$$= 10 \times \frac{\sqrt{3}}{2} \left(4 \times \left(\frac{1}{2} \right)^2 - 1 \right)$$

$$= 0$$

The implication is that this is the angle of the slope as the curve makes one of its passes through the origin.

$$m = \tan \theta$$

$$= \tan 30$$

$$= \frac{\sqrt{3}}{3}$$

$$\therefore \text{The tangent would be } y = \frac{\sqrt{3}}{3} x$$

[3 marks]

Answer 4**(i)**

$$y^2 = \frac{x^3}{72 - x}$$

$$y^2 (72 - x) = x^3$$

$$r^2 \sin^2 \theta (72 - r \cos \theta) = r^3 \cos^3 \theta$$

$$\sin^2 \theta (72 - r \cos \theta) = r \cos^3 \theta$$

$$72 \sin^2 \theta - r \sin^2 \theta \cos \theta = r \cos^3 \theta$$

$$r \cos^3 \theta + r \sin^2 \theta \cos \theta = 72 \sin^2 \theta$$

$$r \cos \theta (\cos^2 \theta + \sin^2 \theta) = 72 \sin^2 \theta$$

$$r \cos \theta = 72 \sin^2 \theta$$

$$r = 72 \times \frac{\sin \theta}{\cos \theta} \times \sin \theta$$

$$r = 72 \tan \theta \sin \theta \quad \square$$

[4 marks]**(ii)**

$$y^2 = \frac{x^3}{72 - x}$$

$$y^2 (72 - x) = x^3$$

$$\text{LHS} = y^2 (72 - x)$$

$$= (2\sqrt{2})^2 (72 - 8) \quad \text{when } (8, 2\sqrt{2}) \text{ is inserted}$$

$$= 4 \times 2 \times 64$$

$$= 512$$

$$\text{RHS} = x^3$$

$$= 8^3 \quad \text{when } (8, 2\sqrt{2}) \text{ is inserted}$$

$$= 512$$

$$= \text{LHS}$$

$$\therefore (8, 2\sqrt{2}) \text{ is on the curve} \quad \square$$

[2 marks]

(iii)

$$y^2 = \frac{x^3}{72 - x}$$

$$y^2 (72 - x) = x^3$$

Differentiate Implicitly

$$y^2 (-1) + 2y \frac{dy}{dx} (72 - x) = 3x^2$$

$$\frac{dy}{dx} (144y - 2xy) = 3x^2 + y^2$$

$$\frac{dy}{dx} = \frac{3x^2 + y^2}{144y - 2xy}$$

[4 marks]

(iv)

$$\begin{aligned} \frac{dy}{dx} &= \frac{3(8)^2 + (2\sqrt{2})^2}{144(2\sqrt{2}) - 2(8)(2\sqrt{2})} \\ &= \frac{192 + 8}{288\sqrt{2} - 32\sqrt{2}} \\ &= \frac{200}{256\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{25\sqrt{2}}{64} \end{aligned}$$

$$y = mx + c$$

$$2\sqrt{2} = \frac{25\sqrt{2}}{64} \times (8) + c$$

$$2\sqrt{2} = \frac{25\sqrt{2}}{8} + c$$

$$\begin{aligned} c &= \frac{16\sqrt{2}}{8} - \frac{25\sqrt{2}}{8} \\ &= -\frac{9\sqrt{2}}{8} \end{aligned}$$

$$\therefore \text{ Tangent is } y = \frac{25\sqrt{2}}{64} x - \frac{9\sqrt{2}}{8}$$

[3 marks]

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