## 3.5 Answers



### 3.5.1 Solution to 3.2 Exercise

**Answer 1**

**( i )**

$$r(90) = \frac{8}{2\,cos\,90 + sin\,90}$$

$$= 8 \qquad \text{yielding the polar coordinate } (8,\ 90°)$$

$$r(270) = \frac{8}{2\,cos\,270 + sin\,270}$$

$$= -8 \qquad \text{yielding the polar coordinate } (-8,\ 270°)$$

However, $(8,\ 90°)$ and $(-8,\ 270°)$ are the same point □

**[ 2 marks ]**

**( ii )**

| $\theta$ (in degrees) | 0 | 45 | 90 | 100 | [redacted] | 135 | 165 |
|---|---|---|---|---|---|---|---|
| $r = \dfrac{8}{2\cos\theta + \sin\theta}$ | 4 | 3.8 | 8 | 12.5 | [redacted] | –11.3 | – 4.8 |

| $\theta$ (in degrees) | 180 | 225 | 270 | 280 | [redacted] | 315 | 345 |
|---|---|---|---|---|---|---|---|
| $r = \dfrac{8}{2\cos\theta + \sin\theta}$ | – 4 | – 3.8 | – 8 | – 12.5 | [redacted] | 11.3 | 4.8 |

**[ 4 marks ]**

**( iii )** See the graph on the following page
The graph looks suspiciously like a straight line which the next part of the question will prove is indeed the case.

**[ 4 marks ]**

**( iv )**

$$r = \frac{c}{a\cos\theta + b\sin\theta}$$

$$r(a\cos\theta + b\sin\theta) = c$$

$$a\,r\cos\theta + b\,r\sin\theta = c$$

$$ax + by = c$$

The Cartesian equation of a straight line !

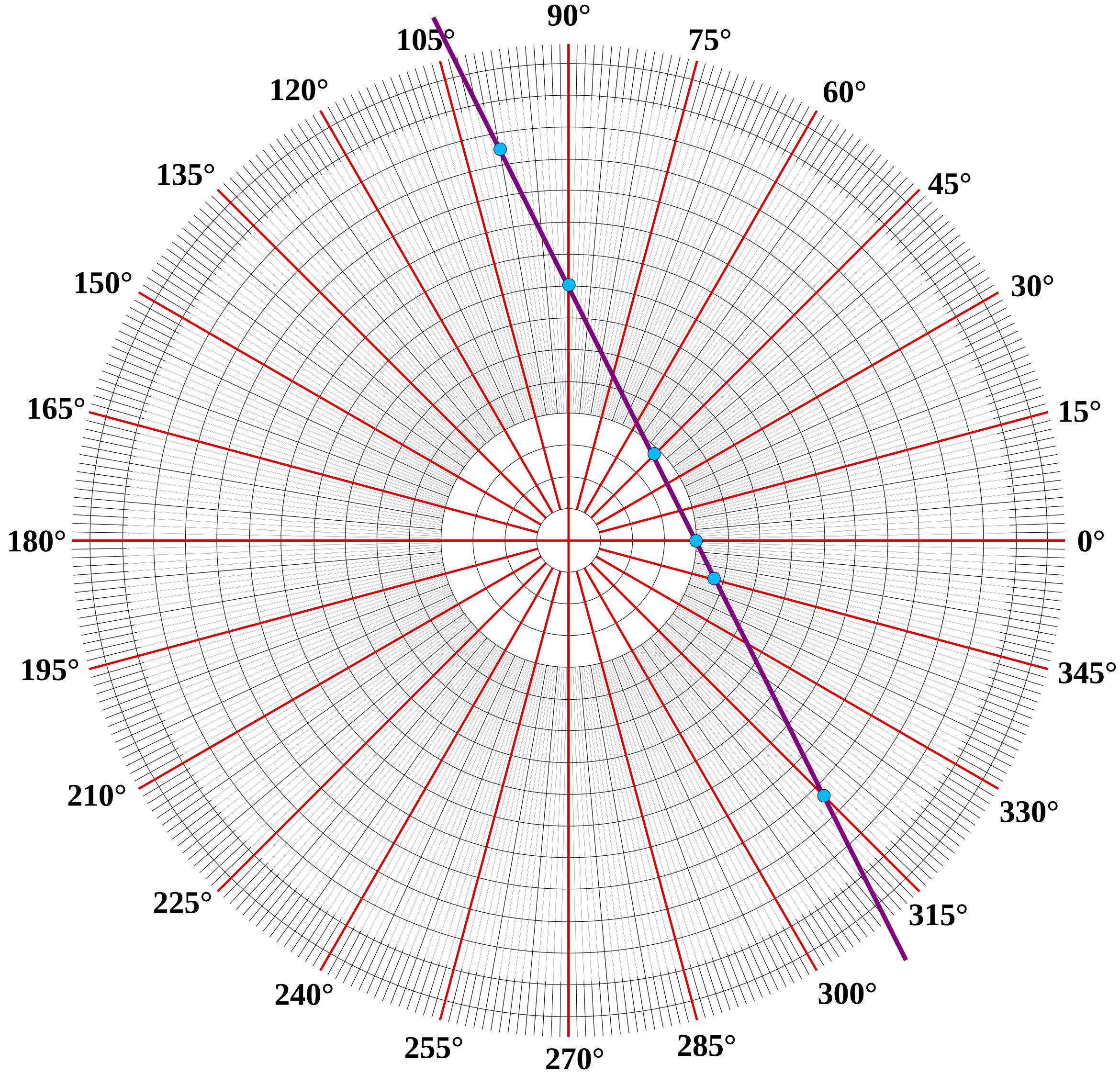
Dividing through by $b$

$$\frac{a}{b}x + y = \frac{c}{b}$$

$$y = -\frac{a}{b}x + \frac{c}{b}$$

$$y = mx + k \qquad \text{with } m = -\frac{a}{b},\quad k = \frac{c}{b}$$ □

**[ 4 marks ]**



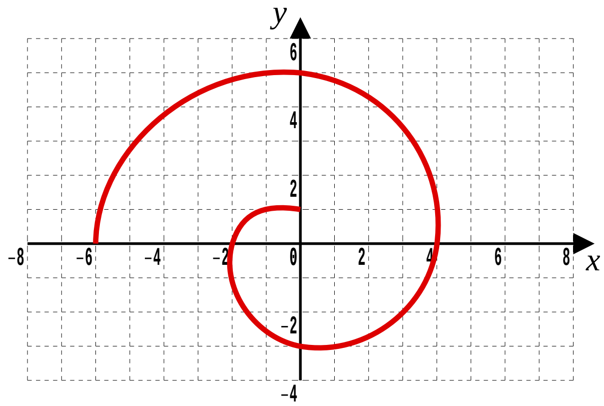

**Answer 2**

**( i )**

| $\theta$ (in degrees) | 90 | 180 | 270 | 360 | 450 | 540 |
|---|---|---|---|---|---|---|
| $r = \dfrac{\theta}{90}$ | 1 | 2 | 3 | 4 | 5 | 6 |

**[ 2 marks ]**

**( ii )**




**[ 3 marks ]**

**Answer 3**

**( i )** $r(0) = 6\,cos\,0 + 8\,sin\,0$

$= 6$ yielding the polar coordinate $(6,\ 0°)$

$r(180) = 6\,cos\,180 + 8\,sin\,180$

$= -6$ yielding the polar coordinate $(-6,\ 270°)$

However, $(6,\ 0°)$ and $(-6,\ 180°)$ are the same point ☐

**[ 2 marks ]**

**( ii )**

| $\theta$ (in degrees) | 0 | 10 | 20 | 30 | 45 | 75 | 90 |
|---|---|---|---|---|---|---|---|
| $r = 6\,cos\,\theta + 8\,sin\,\theta$ | 6 | 7.3 | 8.3 | 9.2 | 9.9 | 9.2 | 8 |

| $\theta$ (in degrees) | 100 | 110 | 120 | 140 | 180 | 210 | 270 |
|---|---|---|---|---|---|---|---|
| $r = 6\,cos\,\theta + 8\,sin\,\theta$ | 6.8 | 5.5 | 3.9 | 0.5 | – 6 | – 9.2 | – 8 |

**[ 5 marks ]**

**( iii )** See the graph on the following page.
The graph looks suspiciously like a circle which the next part of the question will prove is indeed the case.

**( iv )** $r$ is the variable in the polar equation
$R$ is the constant radius of a given circle

$$r = 2a\,cos\,\theta + 2b\,sin\,\theta$$

Multiply both sides by $r$

$$r^2 = 2a\,r\,cos\,\theta + 2b\,r\,sin\,\theta$$

$$x^2 + y^2 = 2a\,x + 2b\,y$$

$$x^2 - 2ax + y^2 - 2ay = 0$$

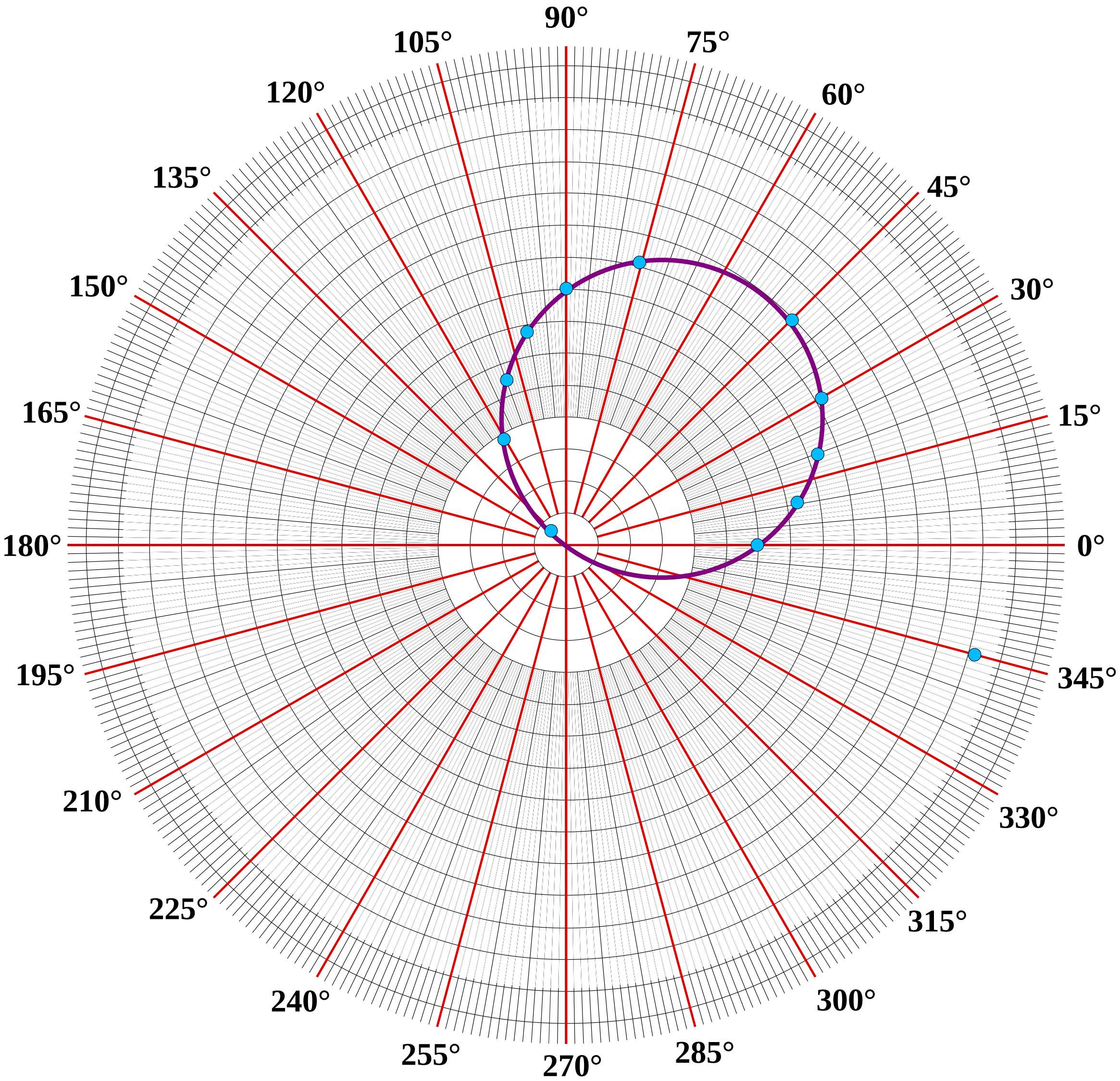
$$(x - a)^2 - a^2 + (y - b)^2 - b^2 = 0$$

$$(x - a)^2 + (y - b)^2 = a^2 + b^2$$

$$(x - a)^2 + (y - b)^2 = R^2$$

which is the Cartesian equation of a circle, centre $(a,\ b)$, Radius $R$

**[ 5 marks ]**




**[ 5 marks ]**

**Answer 4**

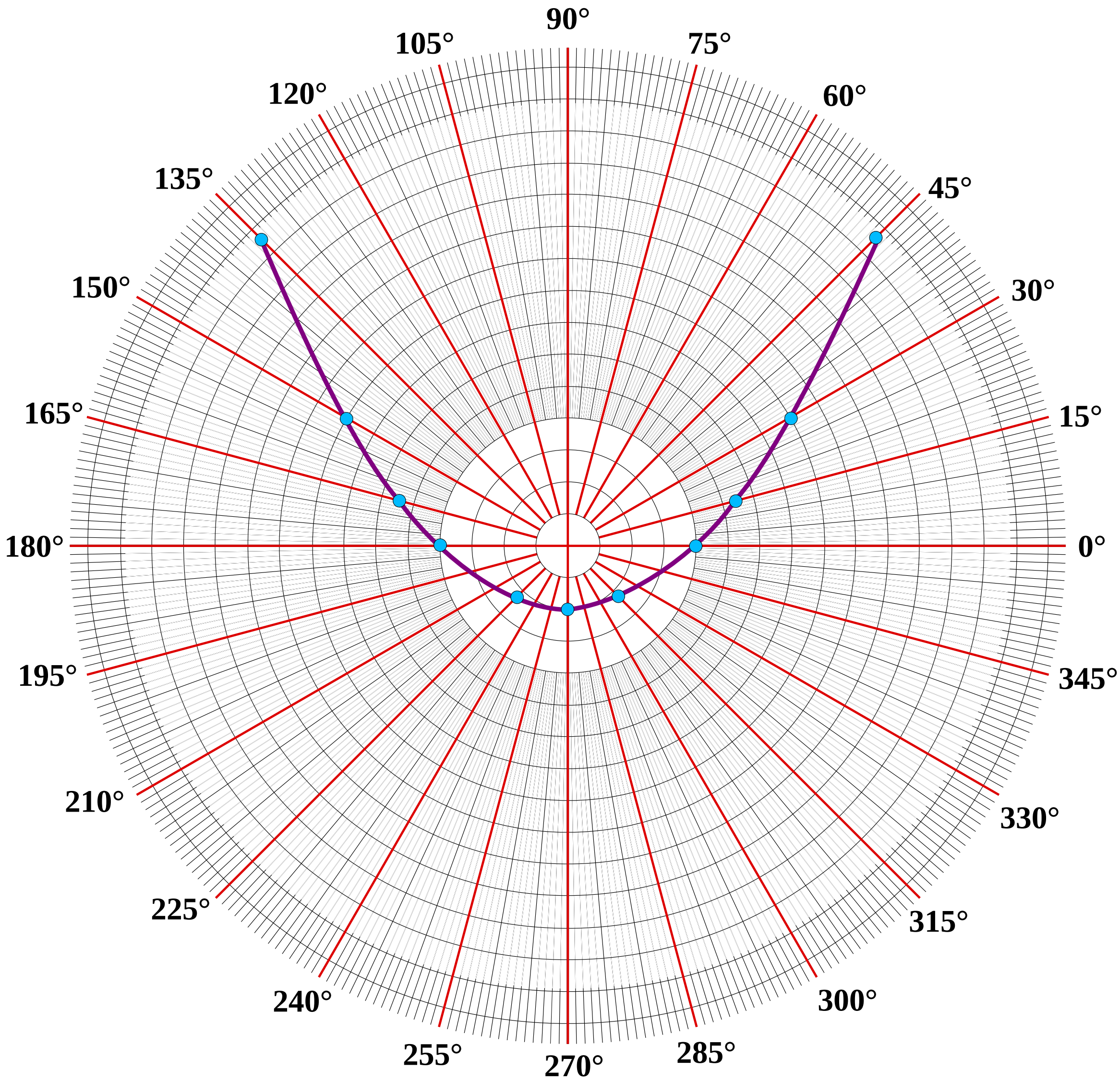
**( i )**

| $\theta$ (in degrees) | 0 | 15 | 30 | 45 | [redacted] | 135 |
|---|---|---|---|---|---|---|
| $r = \dfrac{4}{1-\sin\theta}$ | 4 | 5.4 | 8 | 13.7 | [redacted] | 13.7 |

| $\theta$ (in degrees) | 150 | 165 | 180 | 225 | 270 | 315 |
|---|---|---|---|---|---|---|
| $r = \dfrac{4}{1-\sin\theta}$ | 8 | 5.4 | 4 | 2.3 | 2 | 2.3 |

**[ 4 marks ]**

**( ii )**




**[ 4 marks ]**

This looks suspiciously like a parabola.
The next part of the question will prove that this is indeed the case !

**( iii )**

$$r = \frac{p}{1 - \sin\theta}$$

$$r\left(1 - \sin\theta\right) = p$$

$$r - r\sin\theta = p$$

$$r - y = p$$

$$r = p + y$$

square both sides

$$r^2 = (p + y)^2$$

$$x^2 + y^2 = p^2 + 2py + y^2$$

$$x^2 = p^2 + 2py$$

$$2py = x^2 - p^2$$

$$y = \frac{1}{2p}x^2 - \frac{p}{2}$$

Which is the form of a parabola

**[ 6 marks ]**