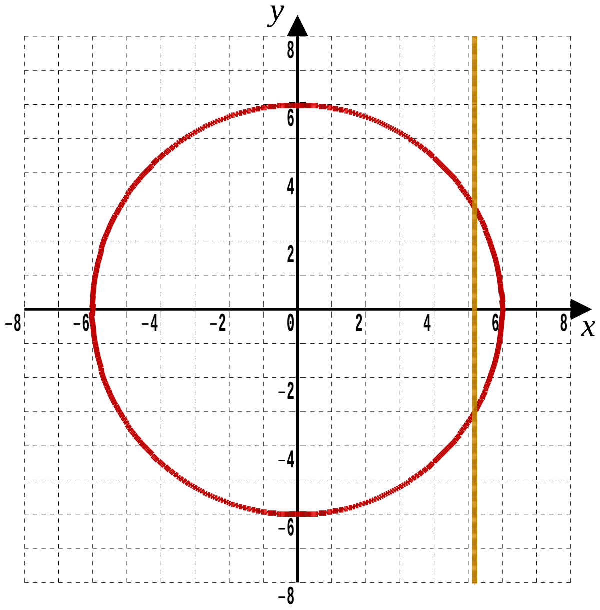
4.5 Answers to 4.4 Exercise

Answer 1

(i)



$r = 6$ is a circle, centre origin, radius 6

$r = 3\sqrt{3}\ sec\ \theta$ is the vertical line $x = 3\sqrt{3}$

(ii) At points of intersection,

$$3\sqrt{3}\ sec\ \theta = 6$$

$$\frac{1}{cos\ \theta} = \frac{6}{3\sqrt{3}}$$

$$cos\ \theta = \frac{\sqrt{3}}{2}$$

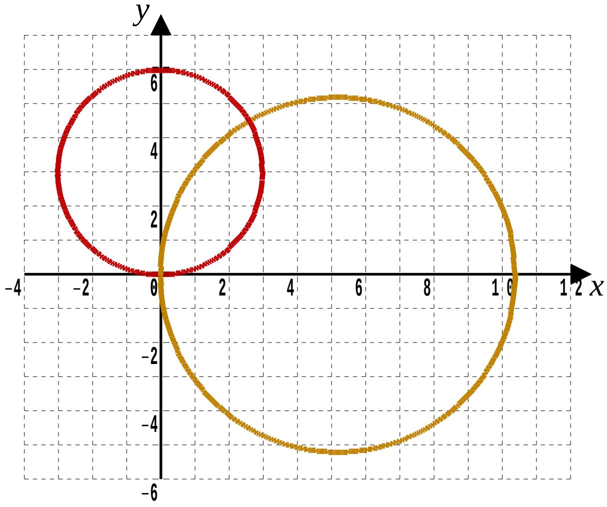
$$\theta = 30° \text{ or } 330°$$

So the polar coordinates of intersection are $(6,\ 30°)$, $(6.\ 330°)$

[4 marks]

Answer 2

(i)



$$r_1 = 6 \sin \theta_1 \text{ is a circle, centre } (0,\ 3) \text{ radius } 3$$

$$r_2 = 6\sqrt{3} \cos \theta_2 \text{ is a circle, centre } (3\sqrt{3},\ 0) \text{ and radius } 3\sqrt{3}$$

(ii) At the point of intersection,

$$6 \sin \theta = 6\sqrt{3} \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \sqrt{3}$$

$$\tan \theta = \sqrt{3}$$

$$\theta = 60°,\ 240°$$

When $\theta = 60^0$, $r_1 = 6 \sin(60)$

$= 3\sqrt{3} \quad \therefore (3\sqrt{3},\ 60°)$

When $\theta = 240°$, $r_1 = 6 \sin(240)$

$= -3\sqrt{3} \quad \therefore (-3\sqrt{3},\ 240°)$

But $(-3\sqrt{3},\ 240°) = (3\sqrt{3},\ 60°)$ so this is the same point twice.

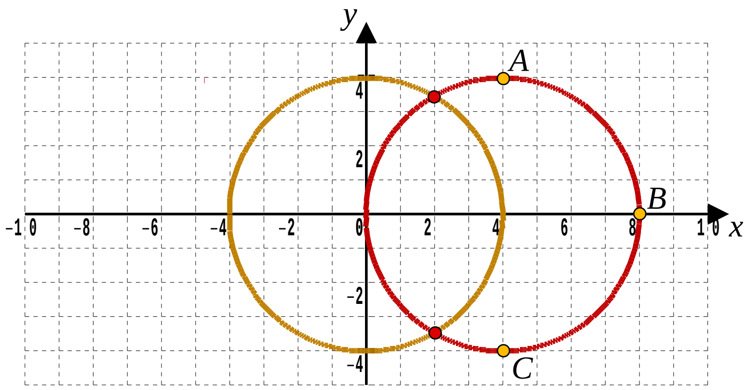
However, there is another point of intersection at the pole !

Note: At the pole the graphs intersect, not because $\theta_1 = \theta_2$ but because $r_1 = r_2 = 0$; Always check if the pole is a point of intersection !

The pole can be written as the polar coordinate $(0,\ \phi°)$ where ϕ is any angle at all !

Answer 3

(i), **(ii)** and **(iii)**



[3, 3, 1 marks)

(iv)

$$4 = 8 \cos\theta$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60°, \; 300°$$

The polar coordinates of intersection are;

$$(4, \; 60°), \; (4, \; 300°)$$

[4 marks]

Answer 4

(i)

$$\begin{aligned} r &= 4\cos(\theta - 30°) \\ &= 4\cos\theta\cos 30 + 4\sin\theta\sin 30 \\ &= 4\cos\theta\,\frac{\sqrt{3}}{2} + 4\sin\theta \times \frac{1}{2} \\ &= 2\sqrt{3}\cos\theta + 2\sin\theta \qquad \square \end{aligned}$$

[2 marks]

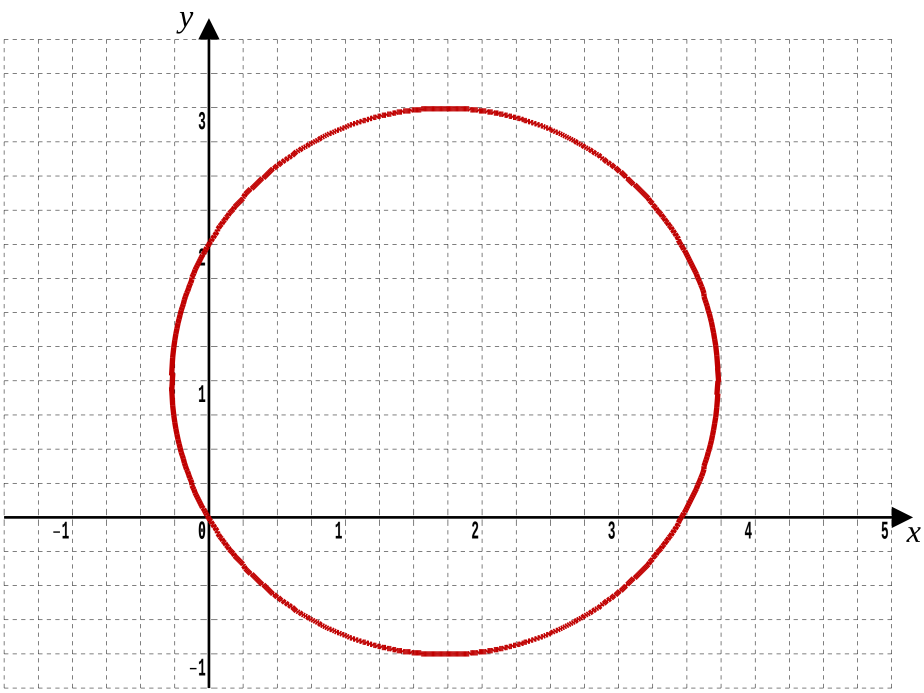
(ii) It is known that a circle,

Centre $(a,\ b)$, radius r ($\therefore$ passing through the pole)

: Cartesian $(x - a)^2 + (y - b)^2 = r^2$ with $r^2 = a^2 + b^2$

: Polar $r = 2a\cos\theta + 2b\sin\theta$

Thus $r = 2\sqrt{3}\cos\theta + 2\sin\theta$ has centre $(\sqrt{3},\ 1)$, radius 2



[3 marks]

(iii) $(x - \sqrt{3})^2 + (y - 1)^2 = 2^2$

[2 marks]

(iv) Rotation about the pole of 30° (anticlockwise)

[2 marks]

Answer 5

(i)

$$r = 5\,sec\,(\theta - 60°)$$
$$= \frac{5}{cos\,(\theta - 60)}$$
$$= \frac{5}{cos\,\theta\,cos\,60 + sin\,\theta\,sin\,60}$$
$$= \frac{5}{cos\,\theta \times \left(\frac{1}{2}\right) + sin\,\theta \times \left(\frac{\sqrt{3}}{2}\right)} \times \frac{2}{2}$$
$$= \frac{10}{cos\,\theta + \sqrt{3}\,sin\,\theta} \qquad \square$$

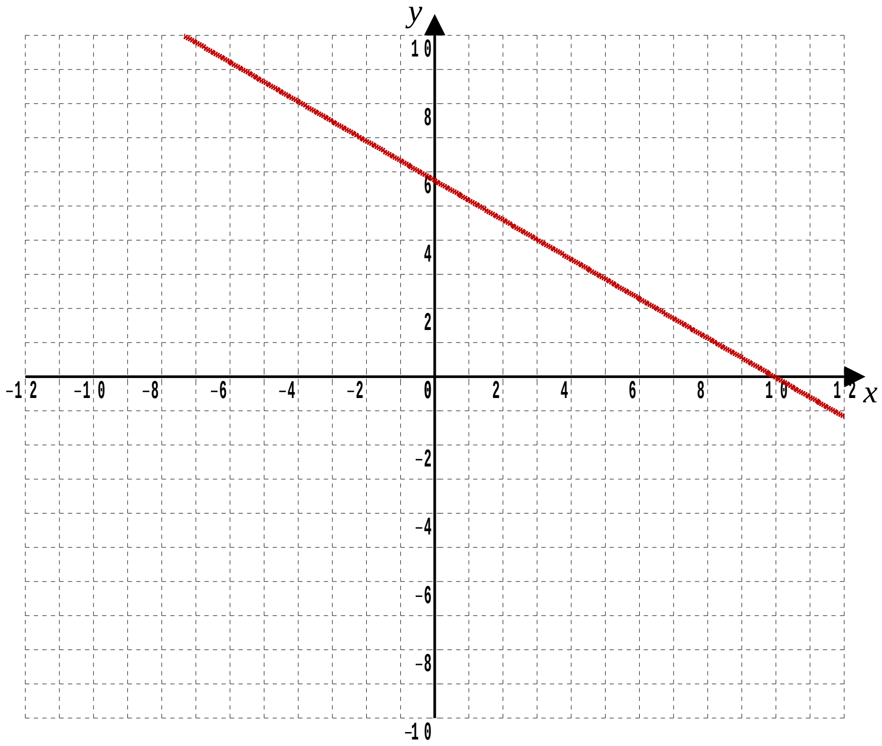
[2 marks]

(ii) It is known that for lines,

: Cartesian $y = mx + c$: $ax + by = c$

: Polar $r = \dfrac{c}{sin\,\theta - m\,cos\,\theta}$: $r = \dfrac{c}{a\,cos\,\theta + b\,sin\,\theta}$

Thus $r = \dfrac{10}{cos\,\theta + \sqrt{3}\,sin\,\theta}$ is the line $x + \sqrt{3}\,y = 10$



[3 marks]

(iii) $y = -\dfrac{1}{\sqrt{3}}x + \dfrac{10}{\sqrt{3}}$

[2 marks]

(iv) Rotation about the pole of 60° (anticlockwise)

[2 marks]

Answer 6

$$(x - 5)^2 + (y - 12)^2 = 169$$
$$x^2 - 10x + 25 + y^2 - 24y + 144 = 169$$
$$x^2 - 10x + y^2 - 24y = 0$$
$$x^2 + y^2 = 10x + 24y$$
$$r^2 = 10\, r \cos\theta + 24\, r \sin\theta$$
$$r = 10 \cos\theta + 24 \sin\theta \qquad \square$$

[6 marks]

Answer 7

$$y = x \tan\left(\sqrt{x^2 + y^2}\right)$$
$$y = x \tan\left(\sqrt{r^2}\right)$$
$$r \sin\theta = r \cos\theta \tan(r)$$
$$\sin\theta = \cos\theta \tan(r)$$
$$\frac{\sin\theta}{\cos\theta} = \tan(r)$$
$$\tan\theta = \tan(r)$$
$$\theta = \pm r$$

If r is restricted to being positive

$$\theta = r \qquad \square$$

[3 marks]