

Lesson 12

Further A-Level Pure Mathematics, Core 2 Hyperbolic Functions

12.1 Test

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 40

Provided : Examination Mathematical Formulae Book

Question 1

Find $\int \cos^5 x \, dx$

[3 marks]

Question 2

Given that $y = \operatorname{artanh}(\sin x)$ find a simplified expression for $\frac{dy}{dx}$

[3 marks]

Question 3

Use the substitution $x = 2 \cosh u$ to show that, for some constant c ,

$$\int \sqrt{x^2 - 4} \, dx = \frac{1}{2} x \sqrt{x^2 - 4} - 2 \operatorname{arcosh}\left(\frac{x}{2}\right) + c$$

[5 marks]

Question 4

Show that the solutions to the equation,

$$2 \tanh^2 x - \operatorname{sech} x = 1$$

can be expressed in the form,

$$x = \pm \ln(a + \sqrt{b})$$

where a and b are integers to be found.

You may use without proof the result $\operatorname{arcosh} y = \ln \{ y + \sqrt{y^2 - 1} \}$

[5 marks]

Question 5

Given that $y = \operatorname{arsinh} x$, show that,

$$(x^2 + 1) \frac{d^3 y}{dx^3} + 3x \frac{d^2 y}{dx^2} + 1 = 0$$

[7 marks]

Question 6

- (i) Use the substitution $x = \sinh^2 u$ to show that, for $x \geq 0$,

$$\int \sqrt{\frac{x}{x+1}} dx = \sqrt{x} \sqrt{x+1} - \operatorname{arsinh}(\sqrt{x}) + c$$

for some unknown constant, c

[6 marks]

- (ii) Hence, or otherwise, determine the exact value of $\int_0^1 \sqrt{\frac{x}{x+1}} dx$

[4 marks]

Question 7

FM A-Level Examination Question from June 2015, Paper FP3, Q4 (Edexcel)

The curve C has equation $y = \frac{1}{\sqrt{x^2 + 2x - 3}}, x > 1$

(a) Find $\int y \, dx$

[3 marks]

The region R is bounded by the curve C , the x -axis and the lines with equations $x = 2$ and $x = 3$. The region R is rotated through 2π radians about the x -axis.

(b) Find the volume of the solid generated. Give your answer in the form $p\pi \ln q$, where p and q are rational numbers to be found.

[4 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com