1.3 Answers to 1.2 Exercise

Answers 1

(i) $\cosh 2 = x \quad \Rightarrow \quad x = 3.762$

[1 mark]

(ii) $\sinh (0.7) = 2x \quad \Rightarrow \quad x = 0.379$

[1 mark]

(iii) $\tanh (-1) = x \quad \Rightarrow \quad x = -0.762$

[1 mark]

(iv) $\cosh x = 4 \quad \Rightarrow \quad x = 2.063$

[1 mark]

(v) $\sinh x = 5 \quad \Rightarrow \quad x = 2.312$

[1 mark]

(vi) $\tanh 3x = 0.9 \quad \Rightarrow \quad x = 0.491$

[1 mark]

Answer 2

(i) Let $f(x) = \cosh(x)$ in which case,

$$\begin{aligned} f(-x) &= \cosh(-x) \\ &= \frac{e^{(-x)} + e^{-(-x)}}{2} \\ &= \frac{e^{x} + e^{-x}}{2} \\ &= \cosh(x) \\ &= f(x) \end{aligned}$$

which is the definition of an even function □

[2 marks]

(ii) Let $f(x) = \sinh(x)$ in which case,

$$\begin{aligned} f(-x) &= \sinh(-x) \\ &= \frac{e^{(-x)} - e^{-(-x)}}{2} \\ &= \frac{-(e^{x} - e^{-x})}{2} \\ &= -\sinh(x) \\ &= -f(x) \end{aligned}$$

which is the definition of an odd function □

[2 marks]

(iii) Let $f\,(x) \;=\; tanh\,(x)$ in which case,

$$
\begin{aligned}
f\,(-\,x) \;&=\; tanh\,(-\,x) \\
&=\; \frac{e^{2(-\,x)} \;-\; 1}{e^{2(-\,x)} \;+\; 1} \;\times\; \frac{e^{2x}}{e^{2x}} \\
&=\; \frac{e^{0} \;-\; e^{2x}}{e^{0} \;+\; e^{2x}} \\
&=\; \frac{1 \;-\; e^{2x}}{1 \;+\; e^{2x}} \\
&=\; -\,\frac{e^{2x} \;-\; 1}{e^{2x} \;+\; 1} \\
&=\; -\,tanh\,(x) \\
&=\; -\,f\,(x) \qquad \text{which is the definition of an odd function} \qquad \square
\end{aligned}
$$

[2 marks]

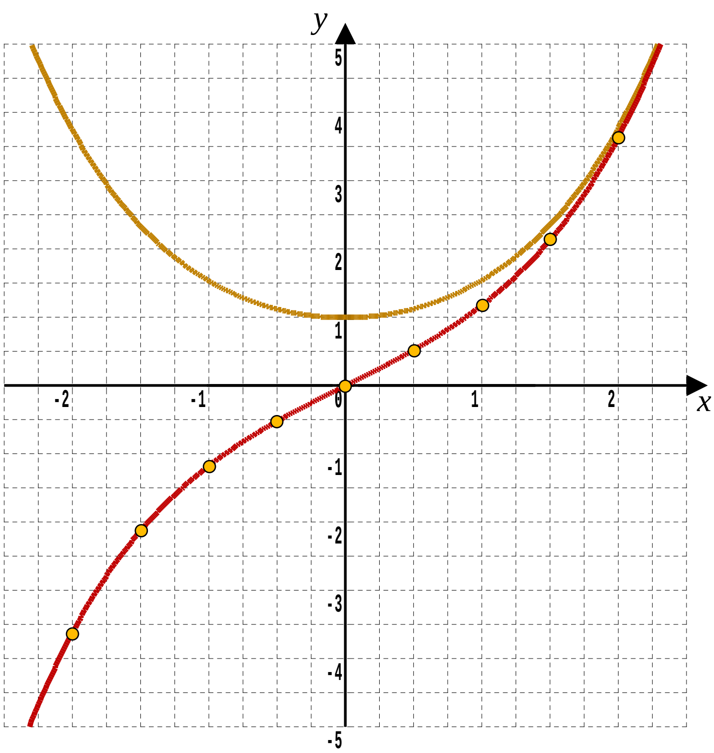
Answer 3

(i)

x	0	0.5	1	1.5	2	2.5
sinh x	0	0.52	1.18	2.13	3.63	6.05

[3 marks]

(ii)



[3 marks]

Answer 4

(i) Firstly, for $sinh\ x$

As x becomes large positive, e^{-x} becomes insignificant in comparison to e^{x}

Thus $sinh\ x = \dfrac{e^{x} - e^{-x}}{2}$ becomes, in effect, $\dfrac{e^{x}}{2}$

Secondly, for $cosh\ x$

As x becomes large positive, e^{-x} becomes insignificant in comparison to e^{x}

Thus $cosh\ x = \dfrac{e^{x} + e^{-x}}{2}$ becomes, in effect, $\dfrac{e^{x}}{2}$

$\therefore$ The two functions $cosh\ x$ and $sinh\ x$ are effectively the same for sufficiently large enough values of positive x

[2 marks]

(ii) Firstly, for $sinh\ x$

As x becomes large negative, e^{x} becomes insignificant in comparison to e^{-x}

Thus $sinh\ x = \dfrac{e^{x} - e^{-x}}{2}$ becomes, in effect, $-\dfrac{e^{-x}}{2}$

Secondly, for $cosh\ x$

As x becomes large negative, e^{x} becomes insignificant in comparison to e^{-x}

Thus $cosh\ x = \dfrac{e^{x} + e^{-x}}{2}$ becomes, in effect, $\dfrac{e^{-x}}{2}$

$\therefore$ Each of the functions $cosh\ x$ and $sinh\ x$ is effectively a reflection of the other in the x-axis for sufficiently large enough values of negative x

[3 marks]

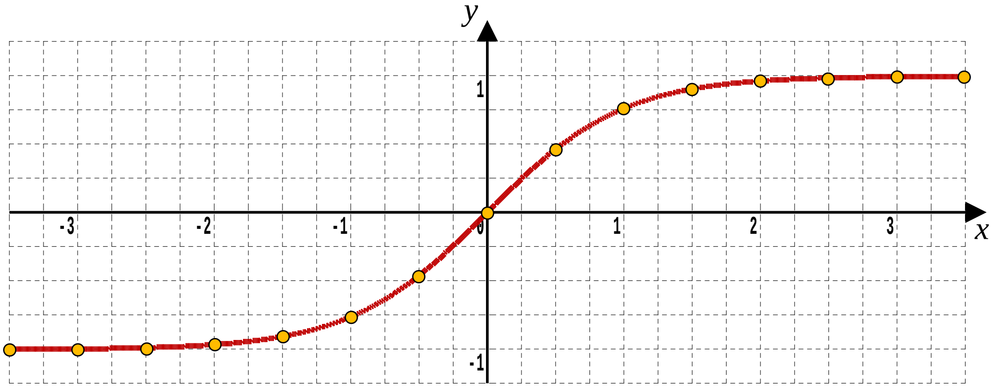
Answer 5

(i)

x	0	0.5	1	1.5	2	2.5	3	3.5
$tanh\ x$	0	0.46	0.76	0.91	0.96	0.99	1.00	1.00

[3 marks]

(ii)



[2 marks]

Answer 6

$$\underset{x \to \infty}{limit}\ tanh\,x \;=\; \underset{x \to \infty}{limit}\ \frac{e^{2x} - 1}{e^{2x} + 1} \;=\; \underset{x \to \infty}{limit}\ \frac{e^{2x}}{e^{2x}} \;=\; \underset{x \to \infty}{limit}\ 1 \;=\; 1$$

$\therefore$ For large positive values of x the graph of $y = tanh\ x$ is effectively the same as the graph of $y = 1$ ☐

[2 marks]