3.5 Answers to 3.4 Exercise

Answer 1

(i) $arsinh(1) = ln(1 + \sqrt{2})$

[1 mark]

(ii) $arsinh(\sqrt{3}) = ln(\sqrt{3} + 2)$

[1 mark]

(iii) $arsinh(2\sqrt{2}) = ln(2\sqrt{2} + 3)$

[2 marks]

Answer 2

$$\begin{aligned} arsinh\left(\frac{3}{4}\right) &= ln\left(\frac{3}{4} + \sqrt{\left(\frac{3}{4}\right)^2 + 1}\right) \\ &= ln\left(\frac{3}{4} + \sqrt{\frac{9}{16} + \frac{16}{16}}\right) \\ &= ln\left(\frac{3}{4} + \sqrt{\frac{25}{16}}\right) \\ &= ln\left(\frac{3}{4} + \frac{5}{4}\right) \\ &= ln\,2 \end{aligned}$$

[2 marks]

Answer 3

$$\begin{aligned} arsinh\left(\frac{1}{\sqrt{3}}\right) &= ln\left(\frac{1}{\sqrt{3}} + \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}\right) \\ &= ln\left(\frac{1}{\sqrt{3}} + \sqrt{\frac{1}{3} + \frac{3}{3}}\right) \\ &= ln\left(\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\right) \\ &= ln\left(\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) \\ &= ln\sqrt{3} \quad \text{or} \quad \frac{1}{2}\,ln\,3 \end{aligned}$$

[2 marks]

Answer 4

$$2\,cosh^2 x - 5\,sinh\,x = 5$$

$$2\left(1 + sinh^2 x\right) - 5\,sinh\,x = 5$$

$$2\,sinh^2 x - 5\,sinh\,x - 3 = 0$$

$$\left(2\,sinh\,x + 1\right)\left(sinh\,x - 3\right) = 0$$

Either $sinh\,x = -\dfrac{1}{2}$ or $sinh\,x = 3$

From which is deduced that

$$x = arsinh\left(-\frac{1}{2}\right) \qquad \text{or} \quad x = arsinh(3)$$

$$x = ln\left(-\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{4}{4}}\right) \qquad x = ln\left(3 + \sqrt{10}\right)$$

$$= ln\left(-\frac{1}{2} + \frac{\sqrt{5}}{2}\right)$$

[6 marks]

Answer 5

$$2\,cosh^2 x - 3\,sinh\,x = 1$$

$$2\left(1 + sinh^2 x\right) - 3\,sinh\,x = 1$$

$$2\,sinh^2 x - 3\,sinh\,x + 1 = 0$$

$$\left(2\,sinh\,x - 1\right)\left(sinh\,x - 1\right) = 0$$

Either $sinh\,x = \dfrac{1}{2}$ or $sinh\,x = 1$

From which is deduced that

$$x = arsinh\left(\frac{1}{2}\right) \qquad \text{or} \quad x = arsinh(1)$$

$$x = ln\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{4}{4}}\right) \qquad x = ln\left(1 + \sqrt{2}\right)$$

$$= ln\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)$$

[6 marks]

Answer 6

The graphs will intersect when,

$$6\,cosh\,x \;=\; 9 \;-\; 2\,sinh\,x$$

$$6\left(\frac{e^{x} \;+\; e^{-x}}{2}\right) \;=\; 9 \;-\; 2\left(\frac{e^{x} \;-\; e^{-x}}{2}\right)$$

$$3\,e^{x} \;+\; 3\,e^{-x} \;=\; 9 \;-\; e^{x} \;+\; e^{-x}$$

$$4\,e^{x} \;-\; 9 \;+\; 2\,e^{-x} \;=\; 0$$

Multiply through by e^{x}

$$4\left(\,e^{x}\,\right)^{2} \;-\; 9\,e^{x} \;+\; 2 \;=\; 0$$

$$\left(\,4\,e^{x} \;-\; 1\,\right)\left(\,e^{x} \;-\; 2\,\right) \;=\; 0$$

Either $\quad e^{x} \;=\; \dfrac{1}{4} \qquad$ or $\qquad e^{x} \;=\; 2$

$$x \;=\; ln\left(\frac{1}{4}\right) \qquad\qquad x \;=\; ln\,(2)$$

$$=\; ln\,2^{-2}$$

$$=\; -2\,ln\,2$$

[6 marks]

Answer 7

Prove that $sinh\left(i\pi \;-\; \theta\right) \;=\; sinh\,\theta$

$$\text{LHS} \;=\; sinh\left(i\pi \;-\; \theta\right)$$

$$=\; \frac{1}{2}\left(\,e^{i\pi-\theta} \;-\; e^{-i\pi+\theta}\,\right)$$

$$=\; \frac{1}{2}\left(\,e^{i\pi}\,e^{-\theta} \;-\; e^{-i\pi}\,e^{\theta}\,\right)$$

$$=\; \frac{1}{2}\left(\;(-1)\;e^{-\theta} \;-\; (-1)\;e^{\theta}\,\right)$$

$$=\; \frac{1}{2}\left(\,e^{\theta} \;-\; e^{-\theta}\,\right)$$

$$=\; sinh\,\theta$$

$$=\; \text{RHS} \qquad\qquad \square$$

[4 marks]