5.5 Answers to 5.4 Exercise

Answer 1

(i) $arcosh(3) = ln(3 + 2\sqrt{2})$

[1 mark]

(ii) $arcosh(\sqrt{3}) = ln(\sqrt{3} + \sqrt{2})$

[1 mark]

(iii) $arcosh(3\sqrt{2}) = ln(3\sqrt{2} + \sqrt{17})$

[2 marks]

Answer 2

$$\begin{aligned} arcosh\left(\frac{13}{5}\right) &= ln\left(\frac{13}{5} + \sqrt{\left(\frac{13}{5}\right)^2 - 1}\right) \\ &= ln\left(\frac{13}{5} + \sqrt{\frac{169}{25} - \frac{25}{25}}\right) \\ &= ln\left(\frac{13}{5} + \sqrt{\frac{144}{25}}\right) \\ &= ln\left(\frac{13}{5} + \frac{12}{5}\right) \\ &= ln\left(\frac{25}{5}\right) \\ &= ln\,5 \end{aligned}$$

[3 marks]

Answer 3

$$\begin{aligned} \cosh w &= 12 \\ w &= arcosh(12) \qquad \text{i.e. } x = 12 \\ &= ln\left(x \pm \sqrt{x^2 - 1}\right) \\ &= ln\left(12 \pm \sqrt{143}\right) \\ &= \pm 3.176 \end{aligned}$$

[3 marks]

Answer 4

(a) Prove $\cosh 2x = 2\cosh^2 x - 1$

$$\text{RHS} = 2\cosh^2 x - 1$$

$$= 2\left(\frac{e^x + e^{-x}}{2}\right)^2 - 1$$

$$= \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{2} - \frac{2}{2}$$

$$= \frac{e^{2x} + 2 + e^{-2x} - 2}{2}$$

$$= \frac{e^{2x} + e^{-2x}}{2}$$

$$= \cosh 2x$$

$$= \text{LHS} \qquad \square$$

[3 marks]

(b)

$$29\cosh x - 3\cosh 2x = 38$$

$$29\cosh x - 3\left(2\cosh^2 x - 1\right) = 38$$

$$29\cosh x - 6\cosh^2 x + 3 = 38$$

$$6\cosh^2 x - 29\cosh x + 35 = 0$$

$$\left(3\cosh x - 7\right)\left(2\cosh x - 5\right) = 0$$

Either $\cosh x = \frac{7}{3}$ or $\cosh x = \frac{5}{2}$

$$x = ln\left(\frac{7}{3} \pm \sqrt{\frac{49}{9} - \frac{9}{9}}\right) \qquad x = ln\left(\frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{4}{4}}\right)$$

$$= ln\left(\frac{7}{3} \pm \frac{2\sqrt{10}}{3}\right) \qquad = ln\left(\frac{5}{2} \pm \frac{\sqrt{21}}{2}\right)$$

[6 marks]

Answer 5

$$8\cosh x - 4\sinh x = 13$$

$$8\left(\frac{e^x + e^{-x}}{2}\right) - 4\left(\frac{e^x - e^{-x}}{2}\right) = 13$$

$$4\left(e^x + e^{-x}\right) - 2\left(e^x - e^{-x}\right) = 13$$

$$4e^x + 4e^{-x} - 2e^x + 2e^{-x} = 13$$

$$2e^x - 13 + 6e^{-x} = 0$$

Multiply through by e^x

$$2\left(e^x\right)^2 - 13\left(e^x\right) + 6 = 0$$

$$\left(2e^x - 1\right)\left(e^x - 6\right) = 0$$

Either $e^x = \frac{1}{2}$ or $e^x = 6$

$$x = ln\left(\frac{1}{2}\right) \quad \text{or} \quad x = ln(6)$$

[6 marks]

Answer 6

$$csch\, x - 2\coth x = 2$$

$$\frac{1}{\sinh x} - \frac{2\cosh x}{\sinh x} = 2$$

Multiply through by $\sinh x$

$$1 - 2\cosh x = 2\sinh x$$

$$2\sinh x + 2\cosh x - 1 = 0$$

$$2\left(\frac{e^x - e^{-x}}{2}\right) + 2\left(\frac{e^x + e^{-x}}{2}\right) - 1 = 0$$

$$2e^x - 1 = 0$$

$$e^x = \frac{1}{2}$$

$$x = ln\left(\frac{1}{2}\right)$$

[5 marks]