6.4 Answers to 6.3 Exercise

Answer 1

(i) $y = \sinh(5x) \Rightarrow \dfrac{dy}{dx} = 5\cosh(5x)$

[1 mark]

(ii) $y = \tanh\left(\dfrac{x}{3}\right) \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{3}\operatorname{sech}^2\left(\dfrac{x}{3}\right)$

[1 mark]

(iii) $y = \operatorname{arcosh}(2x) \Rightarrow \dfrac{dy}{dx} = \dfrac{2}{\sqrt{4x^2 - 1}}$

[1 mark]

(iv) $y = \operatorname{arsinh}\left(\dfrac{x}{3}\right) \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{3}\dfrac{1}{\sqrt{\left(\frac{x}{3}\right)^2 + 1}}$

$$= \frac{1}{3} \times \frac{3}{\sqrt{x^2 + 9}}$$

$$= \frac{1}{\sqrt{x^2 + 9}}$$

[2 mark]

Answer 2

$$y = a\cosh(nx) + b\sinh(nx)$$

$$\frac{dy}{dx} = an\sinh(nx) + bn\cosh(nx)$$

$$\frac{d^2y}{dx^2} = an^2\cosh(nx) + bn^2\sinh(nx)$$

$$= n^2\left(a\cosh(nx) + b\sinh(nx)\right)$$

$$= n^2 y \qquad \square$$

[4 marks]

Answer 3

$$y = (arcosh\, x)^2$$

$$\frac{dy}{dx} = 2\,(arcosh\, x)^1 \times \frac{1}{\sqrt{x^2 - 1}}$$

multiply both sides by $\sqrt{x^2 - 1}$

$$\sqrt{x^2 - 1}\,\frac{dy}{dx} = 2\, arcosh\, x$$

square both sides

$$\left(x^2 - 1\right)\left(\frac{dy}{dx}\right)^2 = 4\,(arcosh\, x)^2$$

$$\left(x^2 - 1\right)\left(\frac{dy}{dx}\right)^2 = 4y \qquad \square$$

[5 marks]

Answer 4

(i) $y = sinh(2x)\, cosh(3x)$

$$\frac{dy}{dx} = 3\, sinh(2x) sinh(3x) + 2\, cosh(2x) cosh(3x)$$

[2 marks]

(ii) $y = \dfrac{cosh\, x}{4x}$

$$\frac{dy}{dx} = \frac{4x\, sinh\, x - 4\, cosh\, x}{16\, x^2}$$

$$= \frac{x\, sinh\, x - cosh\, x}{4\, x^2}$$

[2 marks]

(iii) $y = x^2\, arcosh\, x$

$$\frac{dy}{dx} = \frac{x^2}{\sqrt{x^2 - 1}} + 2x\, arcosh\, x$$

[2 marks]

Answer 5

$$y = x\, arsinh(2x)$$

$$\frac{dy}{dx} = \frac{2x}{\sqrt{4\, x^2 + 1}} + arsinh(2x)$$

[3 marks]

Answer 6

$$\begin{aligned} y &= sech(2x) \\ &= (\cosh(2x))^{-1} \\ \frac{dy}{dx} &= (-1)(\cosh(2x))^{-2}(\sinh(2x))\ 2 \\ &= -\frac{2\sinh(2x)}{\cosh^2(2x)} \\ &= -2\,sech(2x)\tanh(2x) \end{aligned}$$

[3 marks]

Answer 7

$$y = artanh\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{1-\left(\frac{x}{\sqrt{1+x^2}}\right)^2} \times \frac{\sqrt{1+x^2} \times 1 - \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \times 2x \times x}{1+x^2}$$

$$= \frac{1}{1-\frac{x^2}{1+x^2}} \times \frac{1+x^2-x^2}{(1+x^2)\sqrt{1+x^2}}$$

$$= \frac{1+x^2}{1+x^2-x^2} \times \frac{1}{(1+x^2)\sqrt{1+x^2}}$$

$$= \frac{1+x^2}{1} \times \frac{1}{(1+x^2)\sqrt{1+x^2}}$$

$$= \frac{1}{\sqrt{1+x^2}} \qquad \square$$

[4 marks]