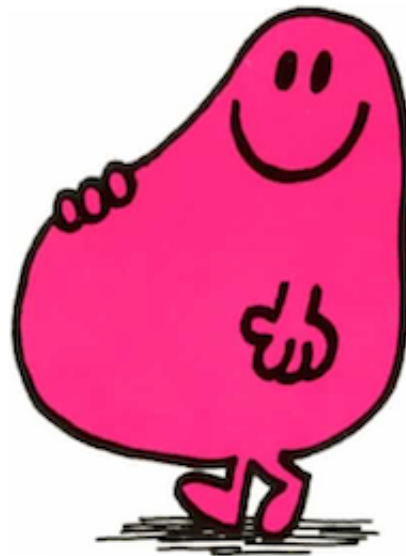


S E T T H E O R Y II



$$A \cap B$$



$$A \cup B$$

The images of Mr Fussy & Mr Greedy are by Roger Hargreaves
They help illustrate the difference between the intersection of two sets (Mr Fussy)
and the union of two sets (Mr Greedy)

Lesson 1

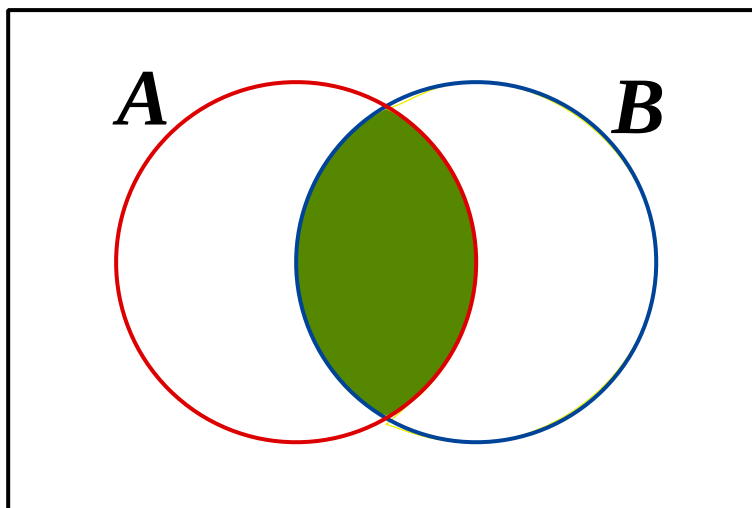
GCSE Mathematics Set Theory II

1.1 Mr Fussy

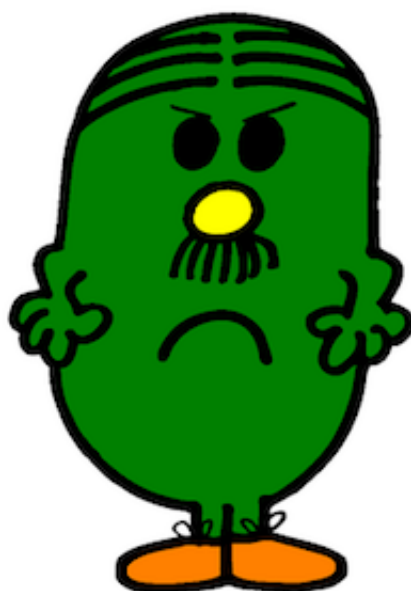
In Set Theory I, we considered the intersection of sets.

The symbol for an intersection is $\cap$ and when we write $A \cap B$ we are referring to that part of A that is also in B . Equivalently, this is the part of B that is in A .

An intersection can be thought of as Mr Fussy; the elements in an intersection have to be in both A and in B to be wanted by Mr Fussy



$$A \cap B$$



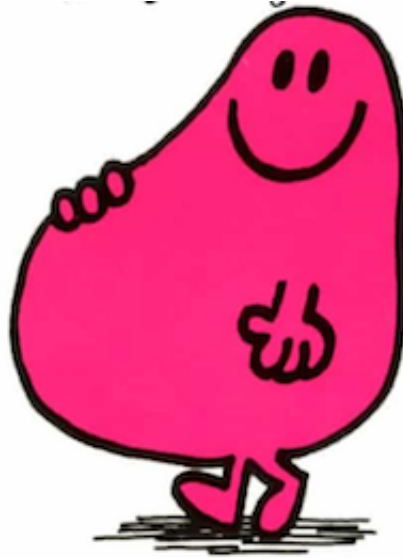
Notice that Mr Fussy has a mouth in the shape of the intersection symbol, $\cap$

1.2 Mr Greedy

Mr Fussy has a friend called Mr Greedy.

Mr Greedy grabs what he can as soon as he sees it and once he has grabbed something he never, ever, gives it back.

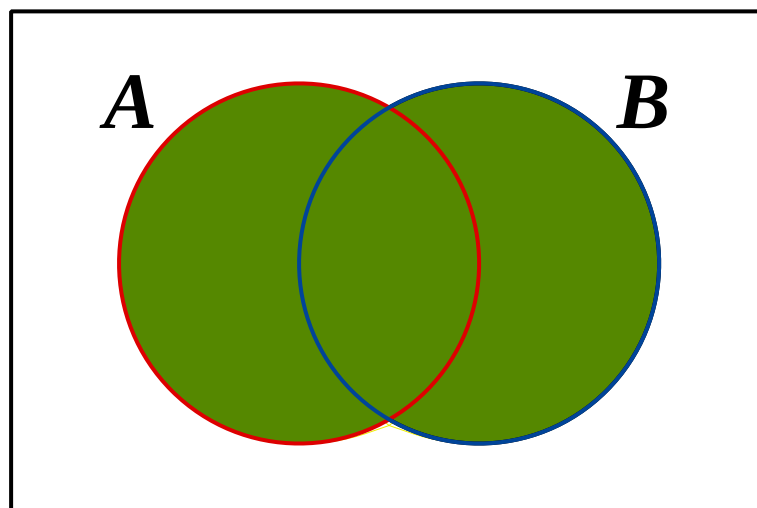
Mr Greedy has a mouth in the shape of the Set Theory symbol for union, $\cup$



The union of A and B is written $A \cup B$ and it is all the elements in set A merged with all the elements in set B .

When Mr Greedy looks at $A \cup B$ his eye first of all sees the A and he grabs all of that and will now never, ever, give any of it back. He then moves on to see more stuff; set B and he grabs all of that as well.

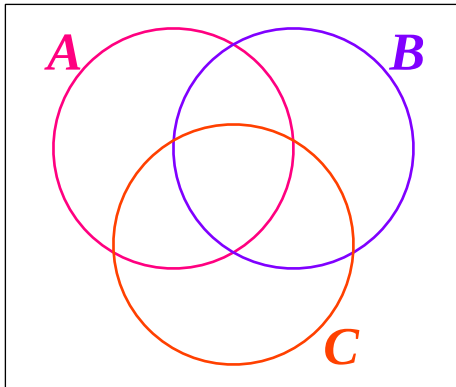
No wonder his tummy is so big !



$$A \cup B$$

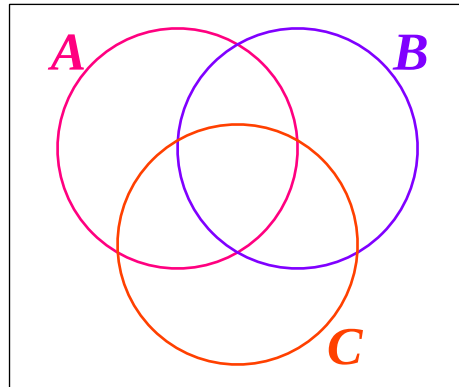
1.3 Example

(i)



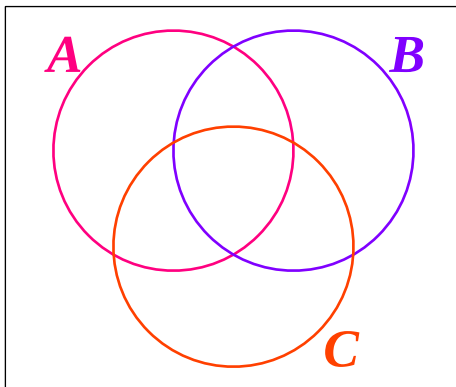
Shade $A \cap B$
The part of A that is also in B

(ii)



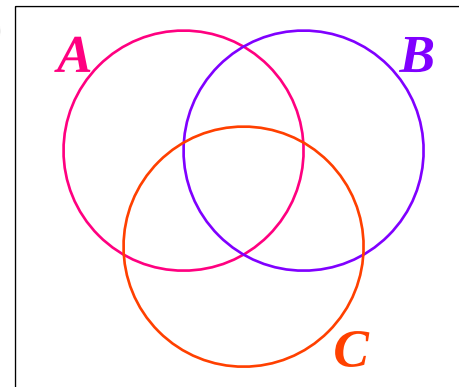
Shade $A \cup B$
Merge all A with all B

(iii)



Shade $A \cap B'$
The part of A not in B

(iv)

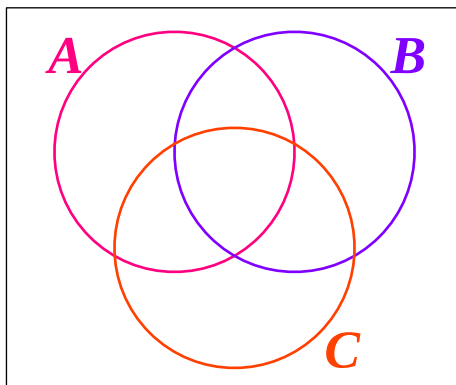


Shade $A \cup B'$
Merge all A with everything not in B

[4 marks]

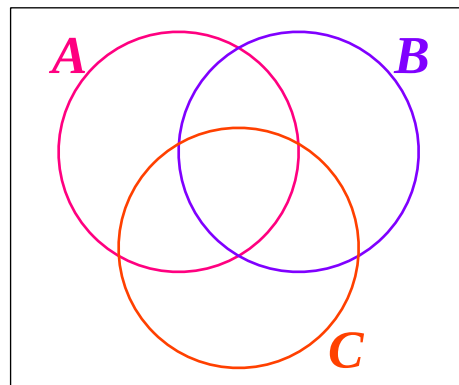
1.4 You Try

(i)



Shade $A \cap C$
The part of A that is also in C

(ii)

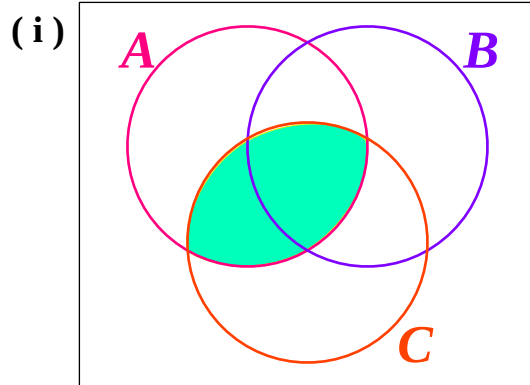


Shade $A \cup C$
Merge all A with all C

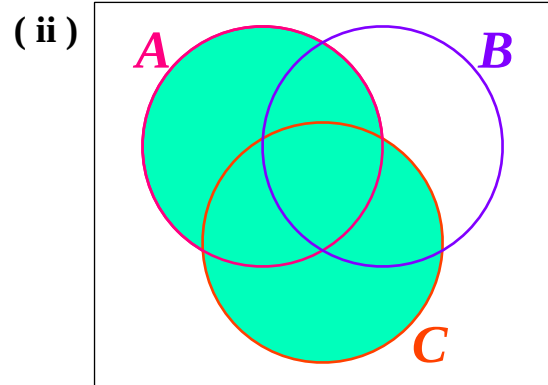
[2 marks]

The answer is over the page so that you can see if you got it right or not

1.5 You Try Answer



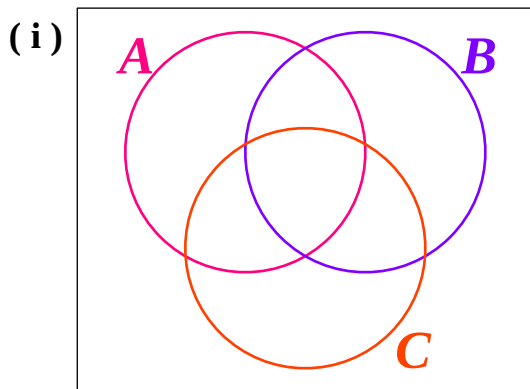
Shade $A \cap C$
The part of A that is also in C



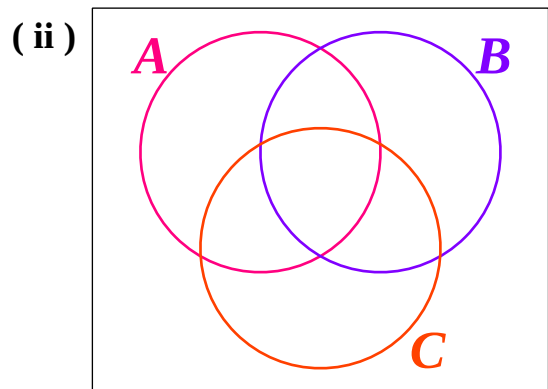
Shade $A \cup C$
Merge all A with all C

[2 marks]

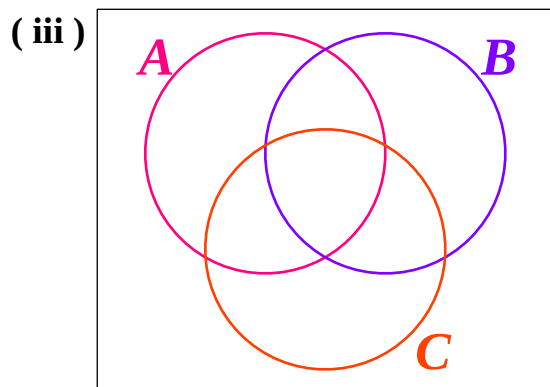
1.6 You Try Two



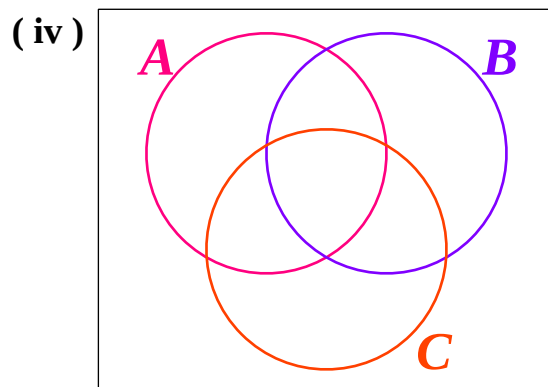
Shade $B \cap C$
The part of B that is also in C



Shade $B \cup C$
Merge all B with all C



Shade $B \cap C'$
The part of B not in C



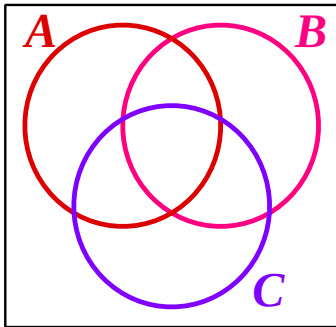
Shade $B \cup C'$
Merge all B with everything not in C

[4 marks]

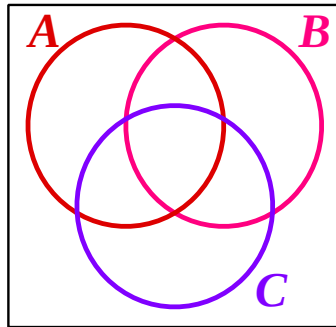
1.7 Exercise

Marks Available : 30

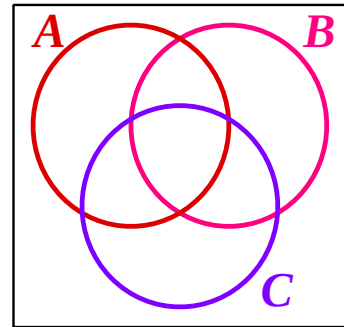
Question 1



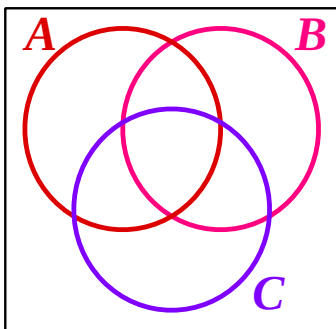
Shade : $A \cap B$



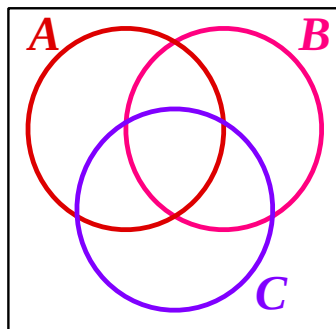
Shade : $A \cup B$



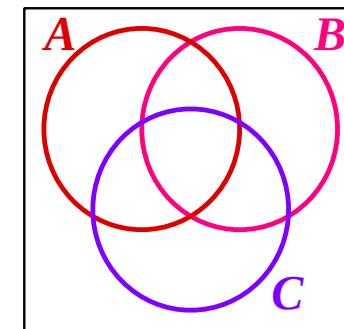
Shade : $A \cup C$



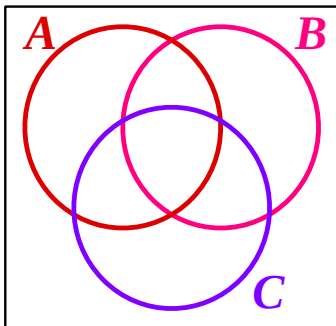
Shade : B



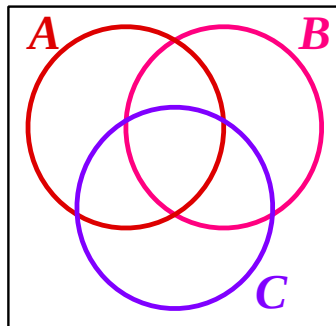
Shade : B'



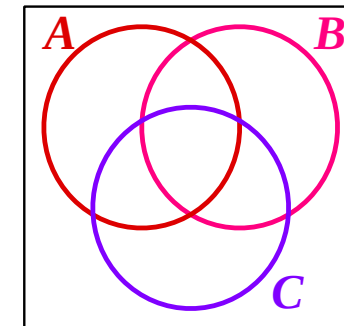
Shade : $(A \cup B)'$



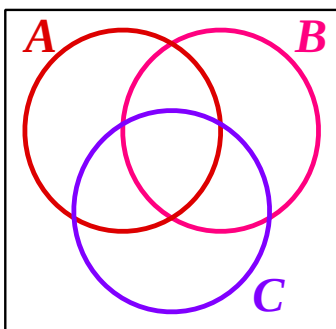
Shade : $B \cap C$



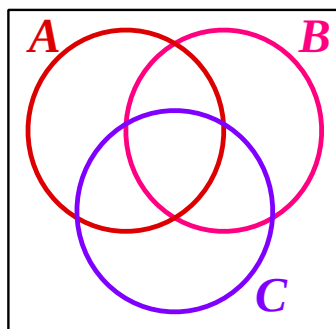
Shade : $(B \cap C)'$



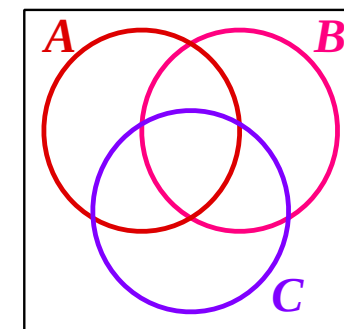
Shade : $B \cup C$



Shade : $(B \cup C)'$



Shade : $A \cap B \cap C$



Shade : $A \cup B \cup C$

[12 marks]

Question 2

Let the sets T and F be,

$$T = \{ \text{factors of } 21 \}$$

$$F = \{ \text{factors of } 14 \}$$

- (i) List the elements of the two sets below,

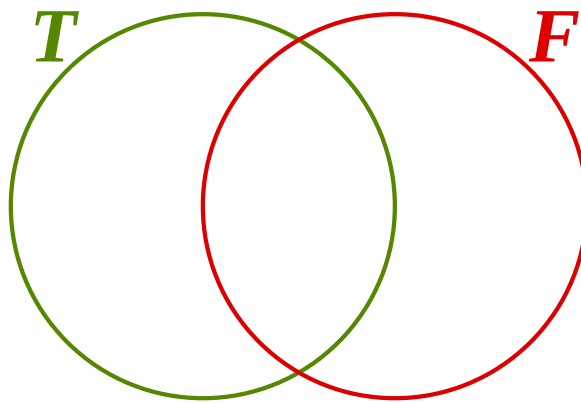
$$T = \{ \quad , \quad , \quad , \quad \}$$

[1 mark]

$$F = \{ \quad , \quad , \quad , \quad \}$$

[1 mark]

- (ii) On the Venn Diagram show the relationship between the sets T and F



[2 marks]

(iii) $T \cap F = \{ \quad , \quad \}$

[2 marks]

(iv) $T \cup F = \{ \quad , \quad , \quad , \quad , \quad , \quad \}$

[2 marks]

Question 3

Consider the two sets A and B where,

$$A = \{ \triangle , \times , O , \square \}$$

$$B = \{ \diamond , \square , \pi , \times \}$$

List the elements of,

(i) $A \cap B =$

[2 marks]

(ii) $A \cup B =$

[2 marks]

Question 4

Let the sets S and M be,

$$S = \{ \text{first 4 square numbers} \}$$

$$M = \{ \text{first four multiples of 4} \}$$

- (i) List the elements of the two sets below,

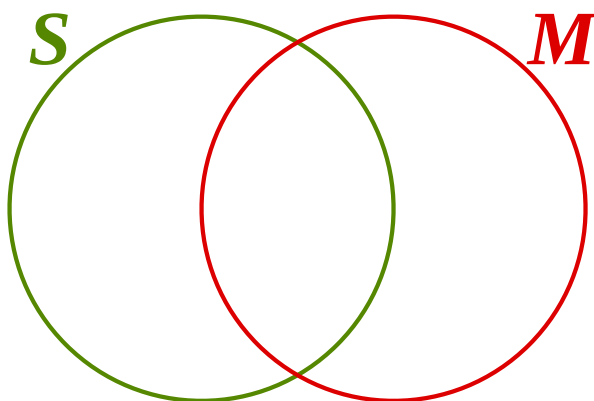
$$S = \{ \quad , \quad , \quad , \quad \}$$

[1 mark]

$$M = \{ \quad , \quad , \quad , \quad \}$$

[1 mark]

- (ii) On the Venn Diagram show the relationship between the sets S and M



(iii) $S \cap M = \{ \quad , \quad \}$

[2 marks]

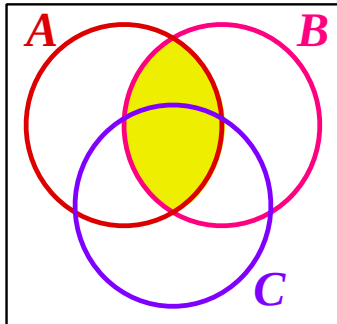
(iv) $S \cup M = \{ \quad , \quad , \quad , \quad , \quad , \quad \}$

[2 marks]

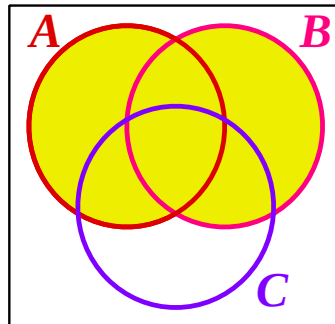
1.8 Answers

1.8.1 Solutions to Exercise 1.7

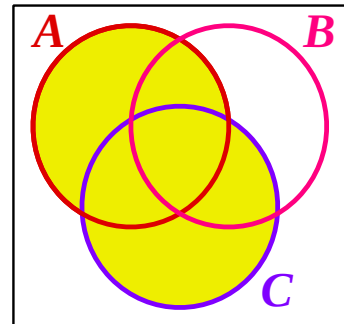
Answer 1



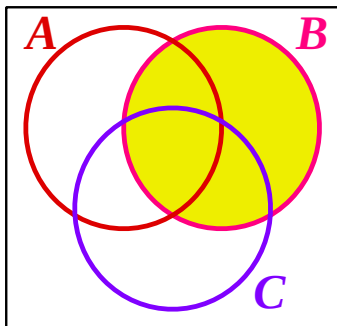
Shade : $A \cap B$



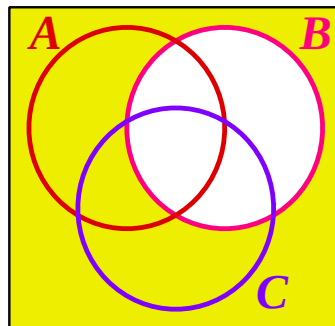
Shade : $A \cup B$



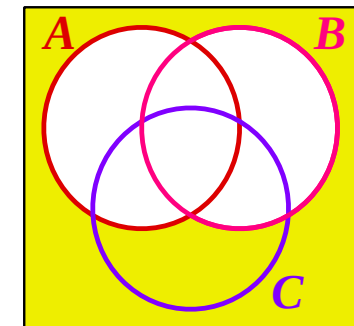
Shade : $A \cup C$



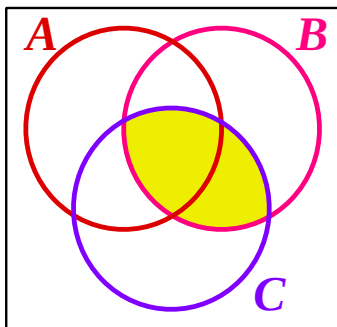
Shade : B



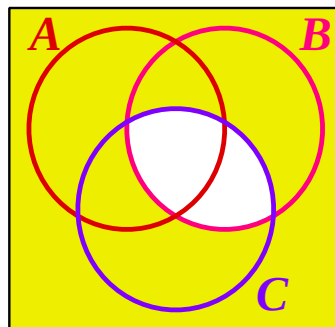
Shade : B'



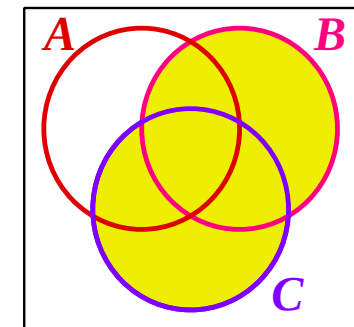
Shade : $(A \cup B)'$



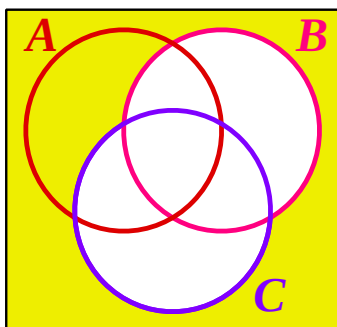
Shade : $B \cap C$



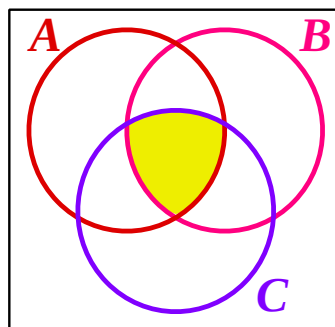
Shade : $(B \cap C)'$



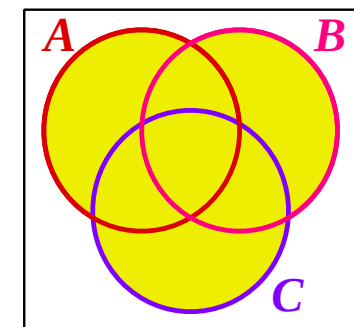
Shade : $B \cup C$



Shade : $(B \cup C)'$



Shade : $A \cap B \cap C$



Shade : $A \cup B \cup C$

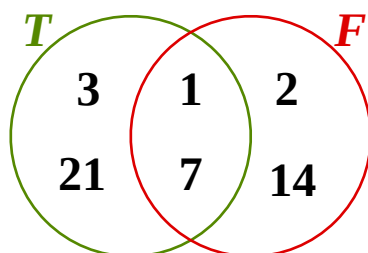
[12 marks]

Answer 2

(i) $T = \{1, 3, 7, 21\}$ $F = \{1, 2, 7, 14\}$

[1, 1 marks]

(ii)



[2 marks]

(iii) $T \cap F = \{1, 7\}$

[2 marks]

(iv) $T \cup F = \{1, 2, 3, 7, 14, 21\}$

[2 marks]

Answer 3

(i) $A \cap B = \{\square, \times\}$

[2 marks]

(ii) $A \cup B = \{\Delta, \times, O, \square, \diamond, \pi\}$

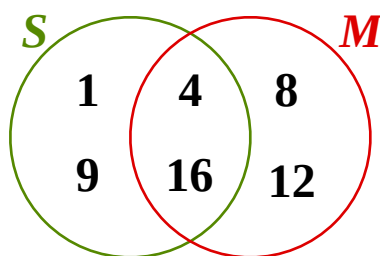
[2 marks]

Answer 4

(i) $S = \{1, 4, 9, 16\}$ $M = \{4, 8, 12, 16\}$

[1, 1 marks]

(ii)



[2 marks]

(iii) $S \cap M = \{4, 16\}$

[2 marks]

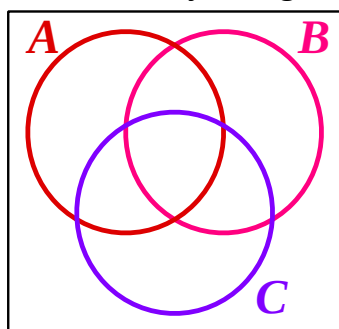
(iv) $S \cup M = \{1, 4, 8, 9, 12, 16\}$

[2 marks]

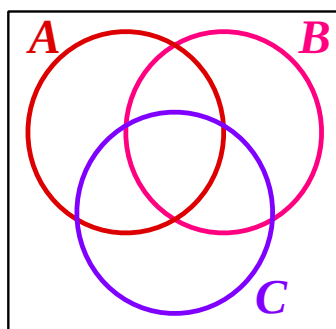
Lesson 2

GCSE Mathematics Set Theory II

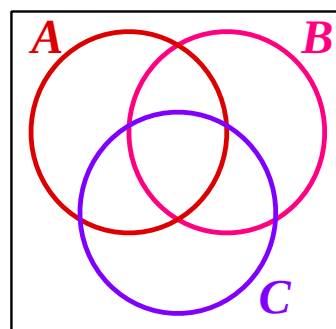
2.1 Shady Goings On



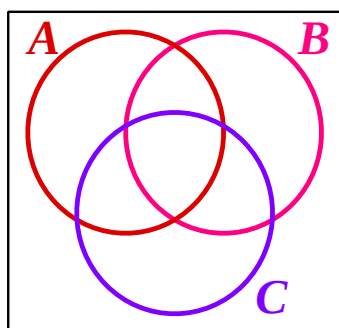
Shade : A



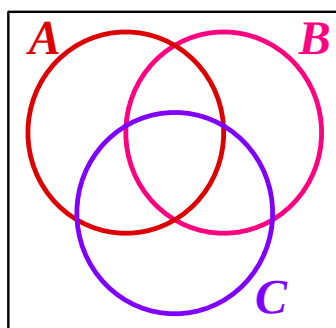
Shade : $A \cap B$



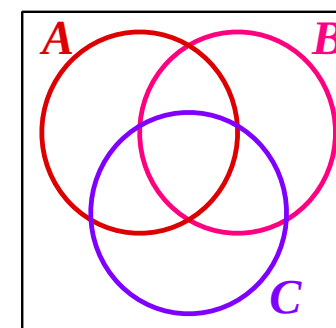
Shade : $A \cup B$



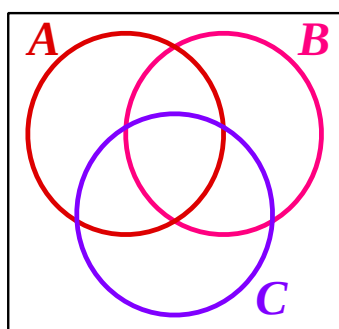
Shade : $A \cup B \cup C$



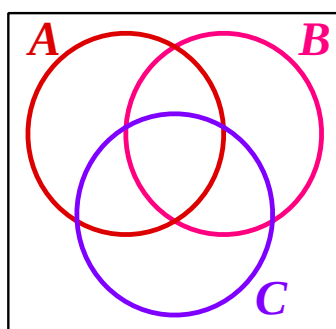
Shade : $(A \cup B \cup C)'$



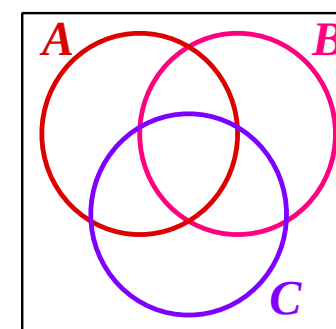
Shade : B'



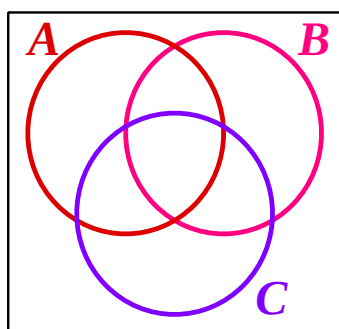
Shade : $B \cap C'$



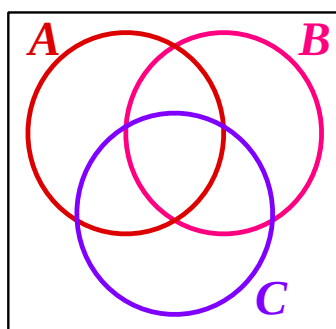
Shade : $(B \cap C')'$



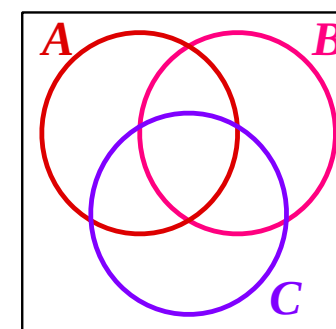
Shade : $B \cup C$



Shade : $(B \cup C)'$



Shade : $C \cap A'$



Shade : $C \cup A'$

[12 marks]

2.2 Exercise

Marks Available : 30

Question 1

Let : A be the set of numbers in the infinite sequence 1, 5, 9, 13, 17, ...

B be the set of numbers in the infinite sequence 3, 7, 11, 15, 19, ...

(i) List the first few members of the set $A \cup B$

[2 marks]

(ii) What is the special name given to the set $A \cup B$?

[1 mark]

(iii) Use a Set Theory Symbol to describe $A \cap B$

[1 mark]

Question 2

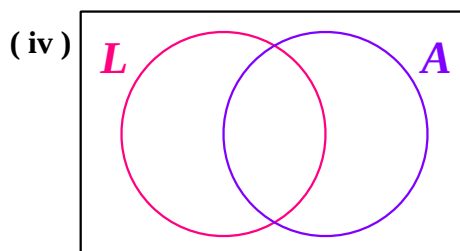
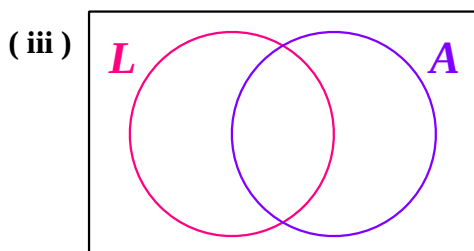
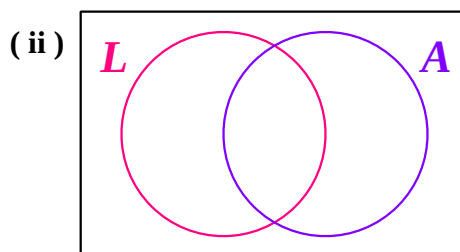
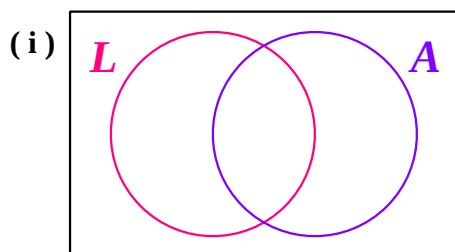
On the Venn Diagrams below, shade the part that represents;

(i) $L \cup A$

(ii) $L \cup A'$

(iii) $L' \cup A$

(iv) $L' \cup A'$



[4 marks]

Question 3

Sets A and B are :

$A = \{ \text{The multiples of 3 that are less than 20} \}$

$B = \{ \text{The multiples of 6 that are less than 20} \}$

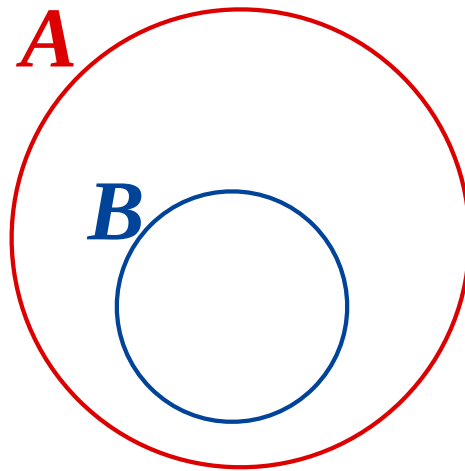
- (i) List the elements of set A

[1 mark]

- (ii) List the elements of set B

[1 mark]

- (iii) Complete the Venn Diagram to show the relationship between sets A and B



[2 marks]

- (iv) The Venn Diagram is drawn in an unusual way.
Explain why this has been done.

[2 marks]

Question 4

In this question,

$\varepsilon = \{ \text{all cars in the world} \}$

$P = \{ \text{pink cars} \}$

$R = \{ \text{Rolls-Royce cars} \}$

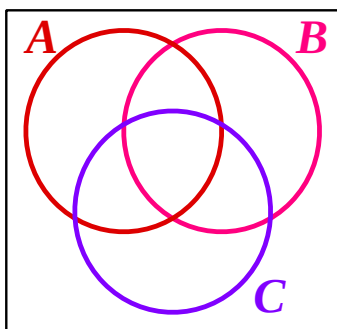
- (i) Describe the set $P \cap R$ in words

[2 marks]

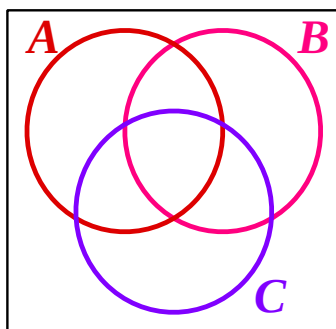
- (ii) George writes that $P \cap R = \emptyset$
Describe in words what George is claiming.
(Do you think George is correct ?)

[2 marks]

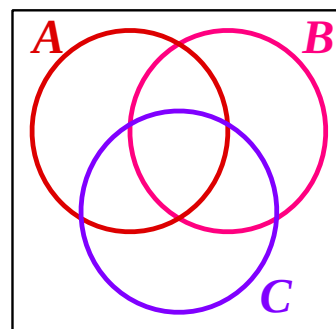
Question 5



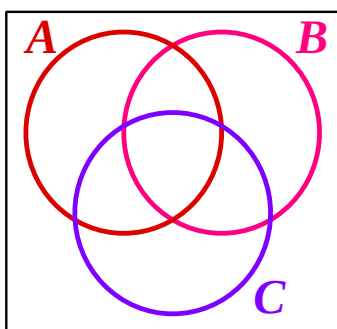
Shade : $B \cap C'$



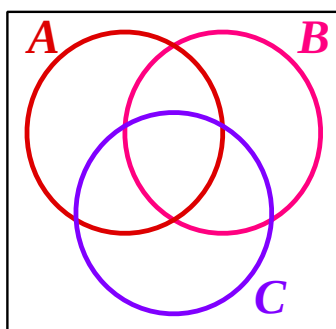
Shade : $B \cup B'$



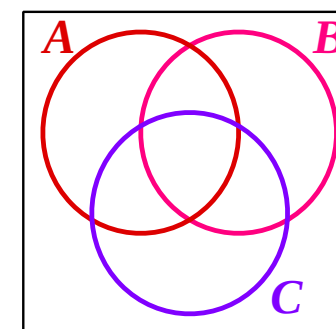
Shade : $B \cup C'$



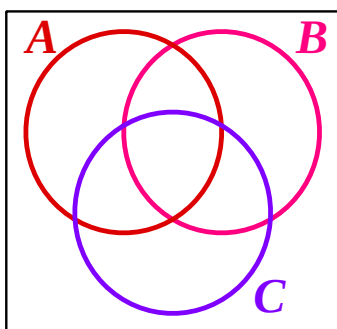
Shade : $A \cap C$



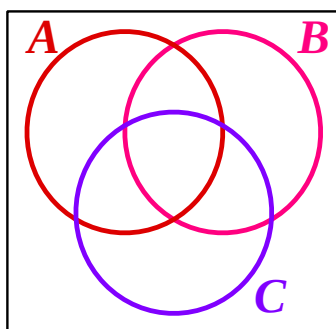
Shade : $(A \cap C)'$



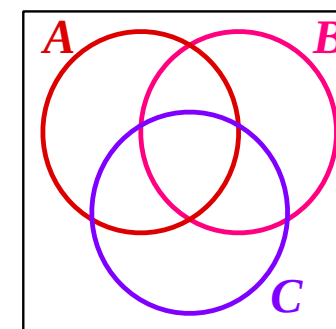
Shade : $(A \cup C)'$



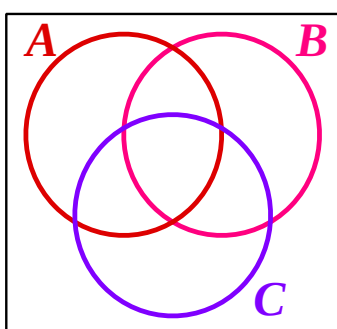
Shade : $B' \cap C'$



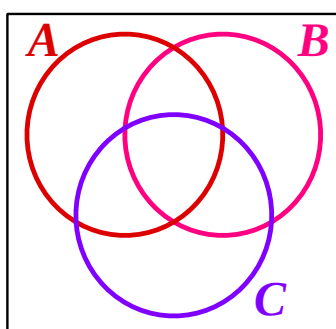
Shade : $A \cap A$



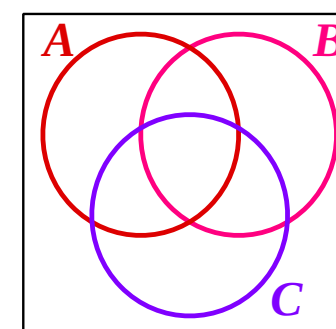
Shade : $A \cup A$



Shade : $B \cup C$



Shade : $(A \cap B \cap C)'$



Shade : $(A \cup B \cup C)'$

[12 marks]

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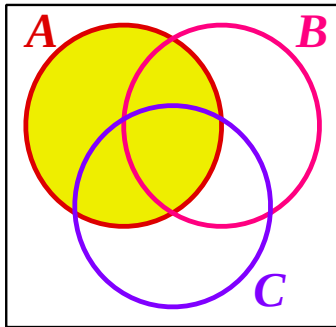
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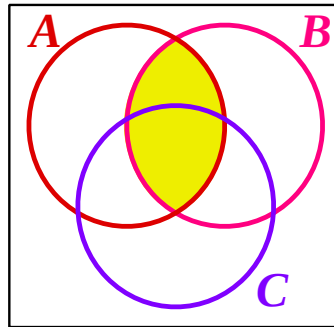
Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk

2.3 Answers

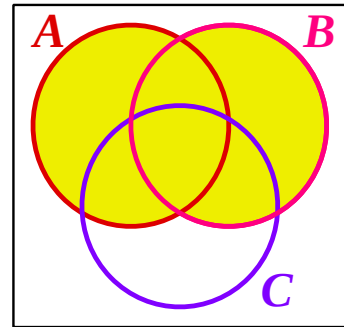
2.3.1 Solutions to 2.1 Starter



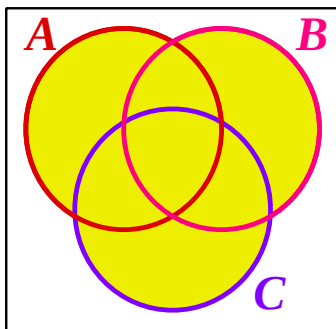
Shade : A



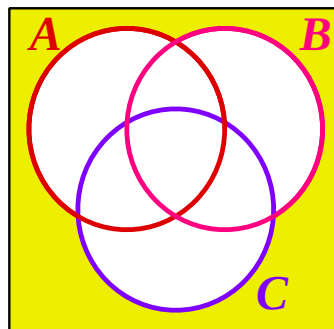
Shade : $A \cap B$



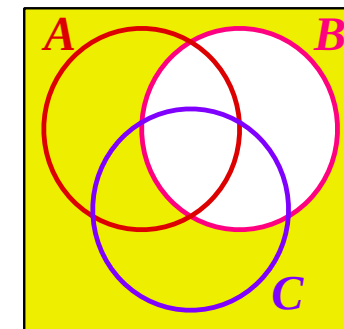
Shade : $A \cup B$



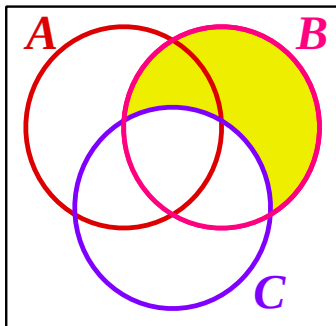
Shade : $A \cup B \cup C$



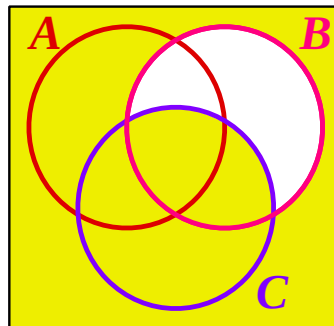
Shade : $(A \cup B \cup C)'$



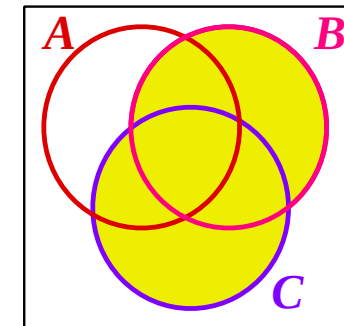
Shade : B'



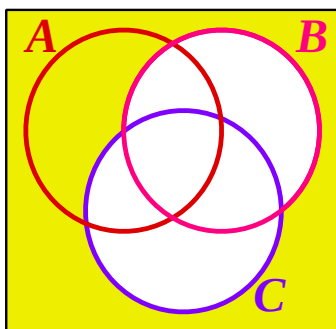
Shade : $B \cap C'$



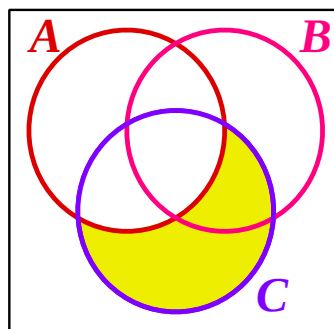
Shade : $(B \cap C')'$



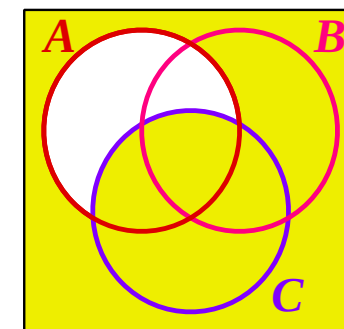
Shade : $B \cup C$



Shade : $(B \cup C)'$



Shade : $C \cap A'$



Shade : $C \cup A'$

[12 marks]

2.3.2 Solutions to 2.2 Exercise

Answer 1

(i) $A \cup B = \{ 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, \dots \}$

[2 marks]

(ii) The odd numbers

[1 mark]

(iii) $A \cap B = \emptyset$

[1 mark]

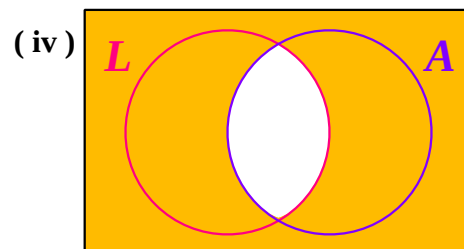
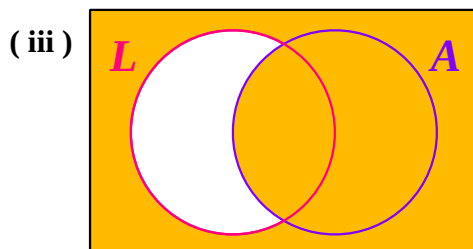
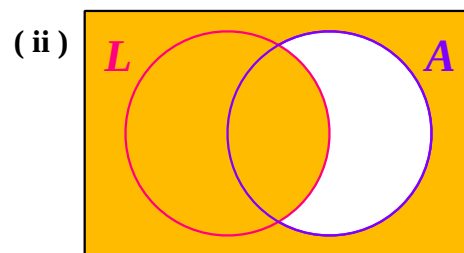
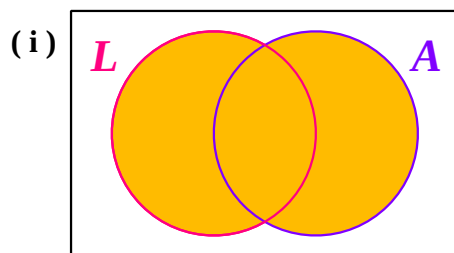
Answer 2

(i) $L \cup A$

(ii) $L \cup A'$

(iii) $L' \cup A$

(iv) $L' \cup A'$



[4 marks]

Answer 3

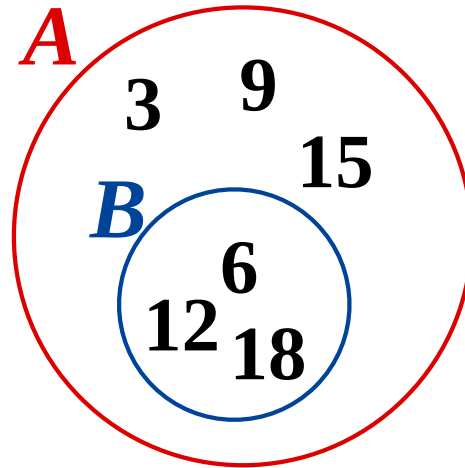
(i) $A = \{ 3, 6, 9, 12, 15, 18 \}$

[1 mark]

(ii) $B = \{ 6, 12, 18 \}$

[1 mark]

(iii)



[2 marks]

Answer 4

(i) $P \cap R = \{ \text{pink Rolls-Royce cars} \}$

[2 marks]

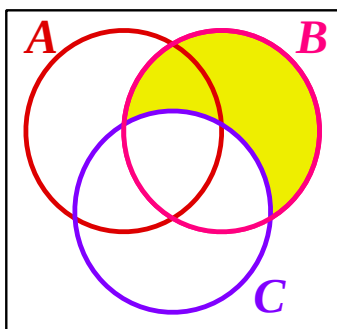
(ii) George is saying that there are no pink Rolls-Royce cars }



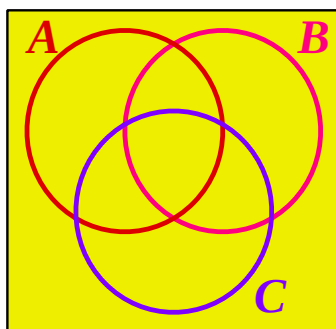
George isn't correct !

[2 marks]

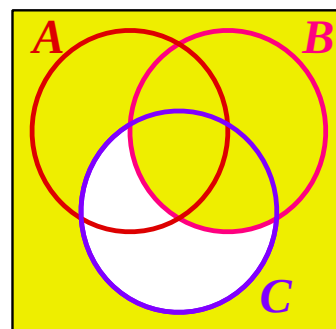
Answer 5



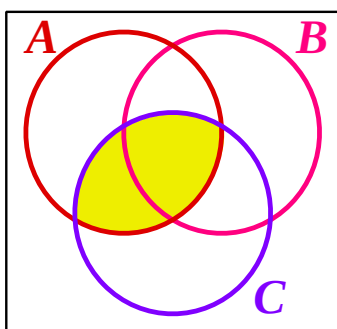
Shade : $B \cap C'$



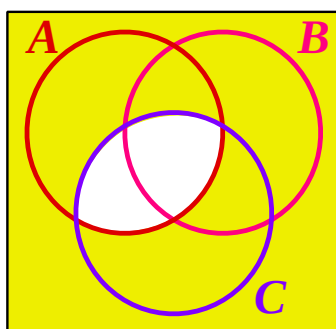
Shade : $B \cup B'$



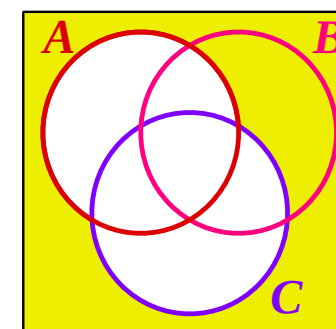
Shade : $B \cup C'$



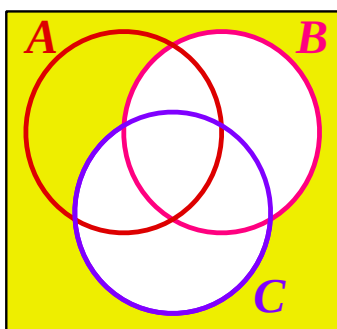
Shade : $A \cap C$



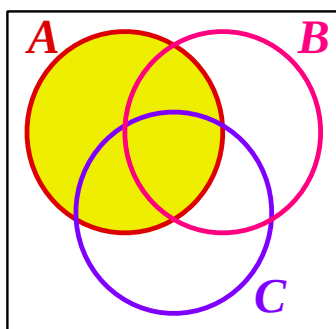
Shade : $(A \cap C)'$



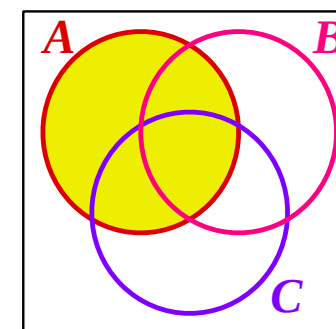
Shade : $(A \cup C)'$



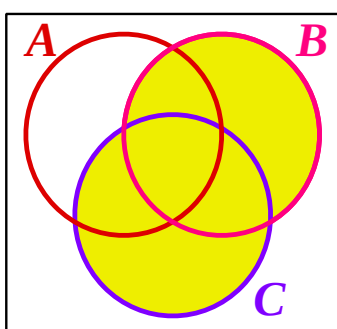
Shade : $B' \cap C'$



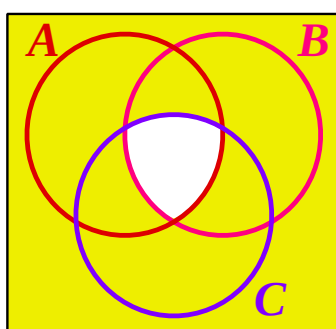
Shade : $A \cap A$



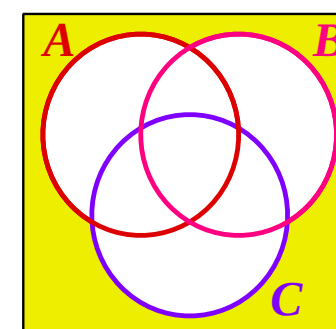
Shade : $A \cup A$



Shade : $B \cup C$



Shade : $(A \cap B \cap C)'$



Shade : $(A \cup B \cup C)'$

[12 marks]

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3.1 Counting The Elements

Given a set with a finite number of elements, it's useful to have an instruction to count the number of elements in the set.

The small lower case letter n is used to either state the number of elements in a set or to ask that they be counted.

For example, the statement $n\{\text{Factors of } 10\} = 4$ is observing that the number 10 has 4 factors.

3.2 Example

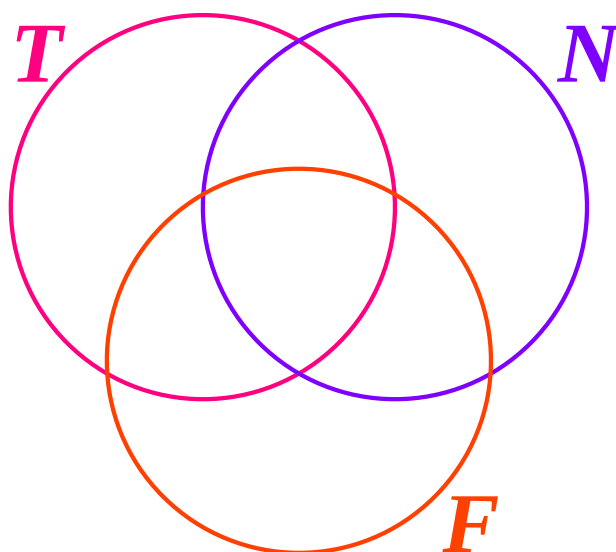
Let T , N and F be the following sets,

$$T = \{\text{Factors of } 10\}$$

$$N = \{\text{Factors of } 9\}$$

$$F = \{\text{Factors of } 15\}$$

- (i) On the Venn Diagram show the relationship between the sets T , N and F



- (ii) Shade in $N \cup F$

[3 marks]

- (iii) $N \cup F = \{ _, _, _, _, _ \}$

[1 mark]

[1 mark]

- (iv) $n(N \cup F) = _$

[1 mark]

3.3 Exercise

Marks Available : 47

Question 1

Let $T = \{\text{factors of } 22\}$

(i) List the elements of set T

[1 mark]

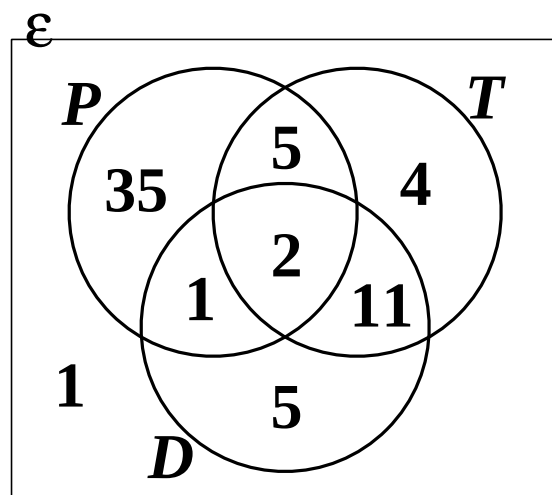
(ii) What is $n\{\text{factors of } 22\}$

[1 mark]

Question 2

In the Venn Diagram,

$\varepsilon = \{\text{people at a nightclub}\}$
 $P = \{\text{people who like *Pop* music}\}$
 $T = \{\text{people who like *Techno* music}\}$
 $D = \{\text{people who like *Drum \& Bass* music}\}$



(a) Explain why $n\{\text{people who like *Pop* music}\}$ is not 35

[1 mark]

(b) Determine the following,

(i) $n\{\text{people who like *Pop* music}\}$

[1 mark]

(ii) $n\{\text{people who like *Techno* music}\}$

[1 mark]

(iii) $n(D)$

[1 mark]

(iv) $n(P \cap D)$

[1 mark]

(v) $n(P \cup D)$

[1 mark]

Question 3

Let T , E , and W be the following sets,

$$T = \{\text{the factors of } 33\}$$

$$E = \{\text{the factors of } 18\}$$

$$W = \{\text{the factors of } 12\}$$

- (a) List the elements of sets T , E and W below,

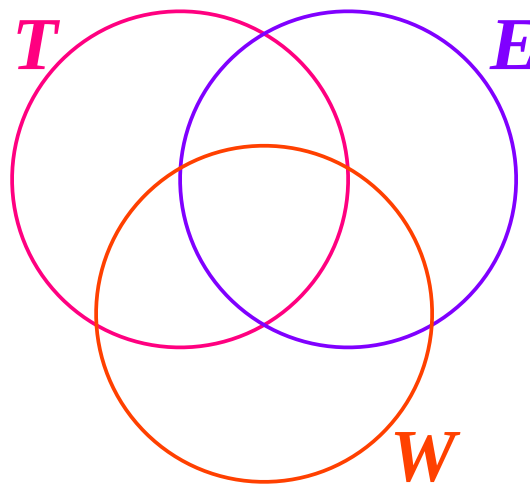
$$T = \{ \quad, \quad, \quad, \quad \}$$

$$E = \{ \quad, \quad, \quad, \quad, \quad, \quad \}$$

$$W = \{ \quad, \quad, \quad, \quad, \quad, \quad \}$$

[3 marks]

- (b) On the Venn diagram, show the relationship between T , E and W .



[3 marks]

- (c) Determine the following,

(i) $n(T) = \quad$ (ii) $n(E) = \quad$

(iii) $n(T \cap E) = \quad$ (iv) $n(T \cup E) = \quad$

(v) $n(E \cap W) = \quad$ (vi) $n(E \cup W) = \quad$

(vii) $n(T \cap E \cap W) = \quad$ (viii) $n(T \cup E \cup W) = \quad$

[8 marks]

- (d) Does $n(T) + n(E) = n(T \cup E)$?

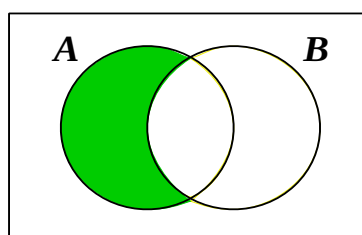
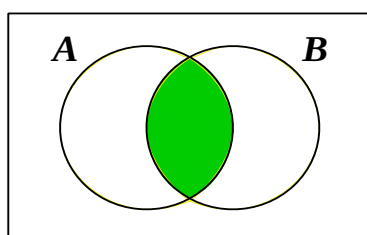
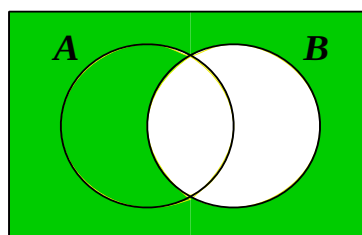
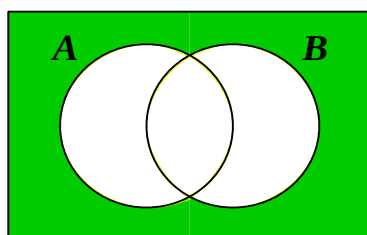
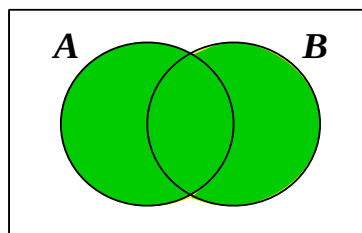
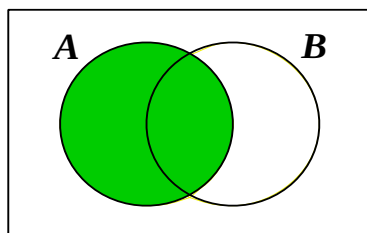
[1 mark]

- (e) What is the HCF of 33, 18 and 12 ?

[1 mark]

Question 4

For each of the Venn diagrams use set notation to describe the regions shaded green,



[6 marks]

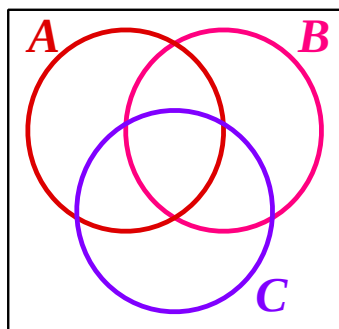
Question 5

TRUE or FALSE ?

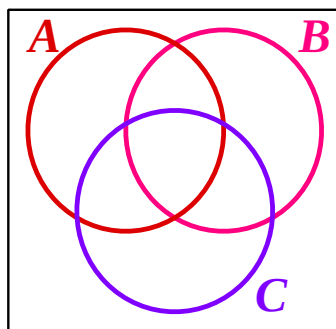
- (i) $n \{ \text{factors } 14 \} = 4$
- (ii) $n \{ \text{factors of } 19 \} = 3$
- (iii) $n \{ \text{factors of a prime number} \} = 2$
- (iv) $n \{ \text{factors of square number} \} = 3$
- (v) $n \{ \text{common factors of } 12 \text{ and } 16 \} = 4$

[5 marks]

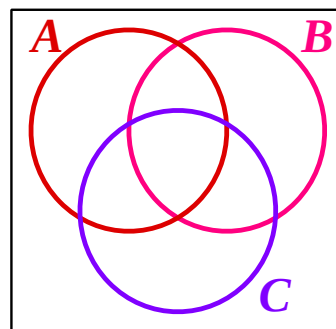
Question 6



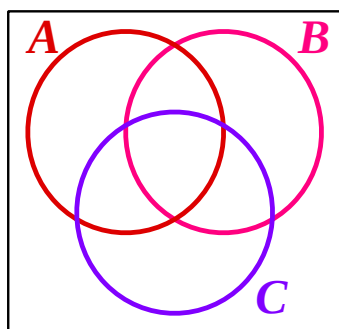
Shade : B'



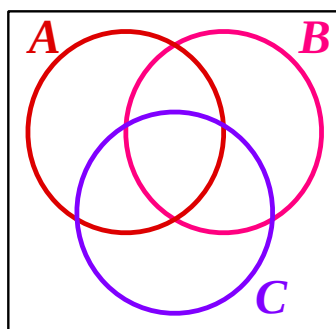
Shade : $A \cap B'$



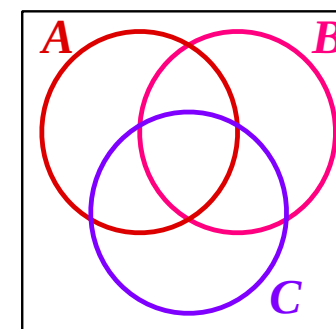
Shade : $A \cup B'$



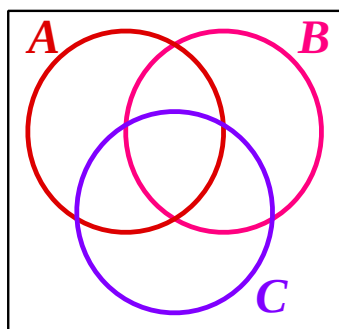
Shade : $A \cup C$



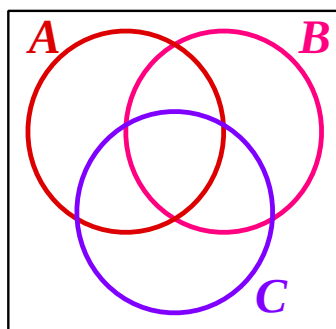
Shade : $(A \cup C)'$



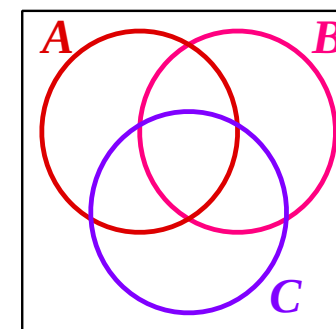
Shade : $A \cap B \cap C'$



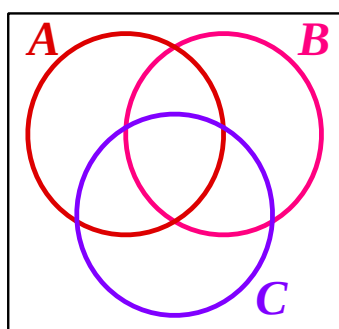
Shade : $A \cap C$



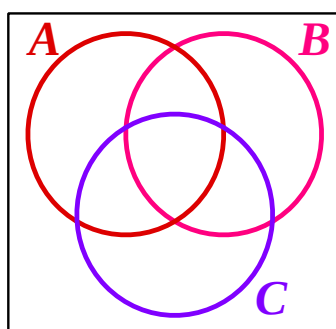
Shade : $(A \cap C)'$



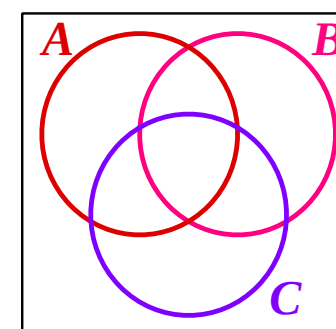
Shade : $B \cup B'$



Shade : $A' \cup C'$



Shade : $C \cap C$



Shade : $C \cup A \cup B'$

[12 marks]

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3.4 Answers

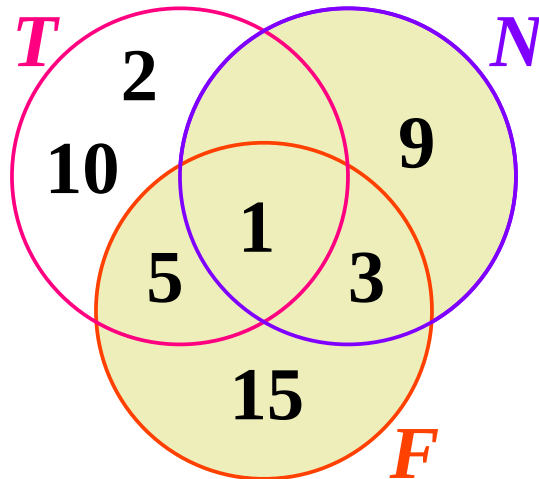
3.4.1 Solution to 3.2 Example

$$T = \{\text{Factors of } 10\} = \{1, 2, 5, 10\}$$

$$N = \{\text{Factors of } 9\} = \{1, 3, 9\}$$

$$F = \{\text{Factors of } 15\} = \{1, 3, 5, 15\}$$

(i), (ii)



[3, 1 marks]

(iii) $N \cup F = \{1, 3, 5, 9, 15\}$

[1 mark]

(iv) $n(N \cup F) = 5$

[1 mark]

3.4.2 Solutions to 3.3 Exercise

Answer 1

(i) $T = \{1, 2, 11, 22\}$

[1 mark]

(ii) $n\{\text{factors of } 22\} = 4$

[1 mark]

Answer 2

(a) The 35 in the Venn diagram is the number of people at the nightclub who like pop music but do not like techno or drum & bass music.

[1 mark]

(b) (i) $n\{\text{people who like **Pop** music}\} = 35 + 5 + 2 + 1$
 $= 43$

[1 mark]

(ii) $n\{\text{people who like **Techo** music}\} = 11 + 4 + 5 + 2$
 $= 22$

[1 mark]

(iii) $n(D) = 5 + 11 + 2 + 1$
 $= 19$

[1 mark]

(iv) $n(P \cap D) = 2 + 1$
 $= 3$

[1 mark]

(v) $n(P \cup D) = 35 + 5 + 2 + 1 + 5 + 11$
 $= 59$

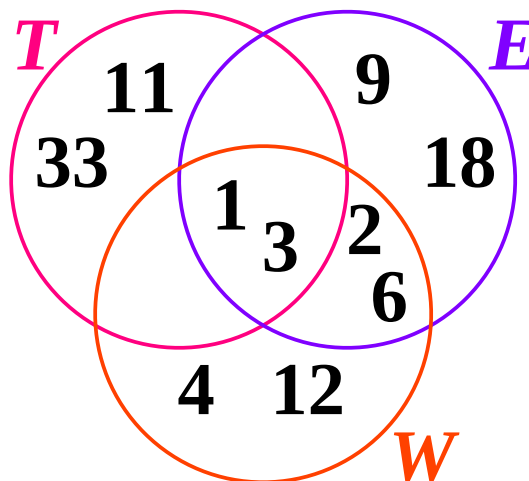
[1 mark]

Answer 3

- (a) $T = \{1, 3, 11, 33\}$
 $E = \{1, 2, 3, 6, 9, 18\}$
 $W = \{1, 2, 3, 4, 6, 12\}$

[3 marks]

(b)



[3 marks]

- (c) (i) $n(T) = 4$ (ii) $n(E) = 6$
 (iii) $n(T \cap E) = 2$ (iv) $n(T \cup E) = 8$
 (v) $n(E \cap W) = 4$ (vi) $n(E \cup W) = 8$
 (vii) $n(T \cap E \cap W) = 2$ (viii) $n(T \cup E \cup W) = 10$

[8 marks]

- (d) Does $n(T) + n(E) = n(T \cup E)$? No !

[1 mark]

- (e) $hcf\{33, 18, 12\} = 3$

[1 mark]

Answer 4

- (i) A (ii) $A \cup B$
 (iii) $(A \cup B)'$ (iv) B'
 (v) $A \cap B$ (vi) $A \cap B'$

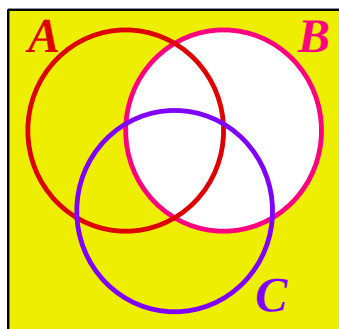
[6 marks]

Answer 5

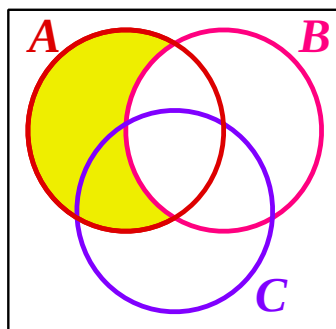
- (i) T (ii) F (iii) T (iv) F (v) F

[5 marks]

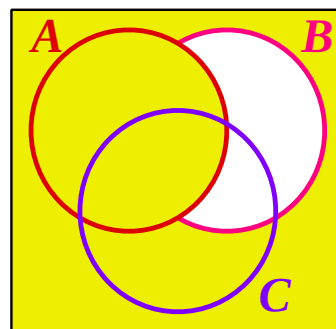
Answer 6



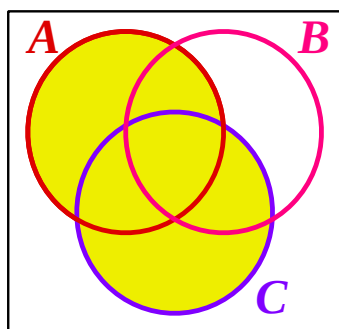
Shade : B'



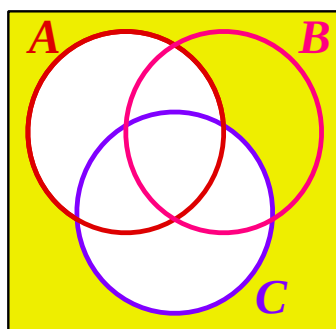
Shade : $A \cap B$



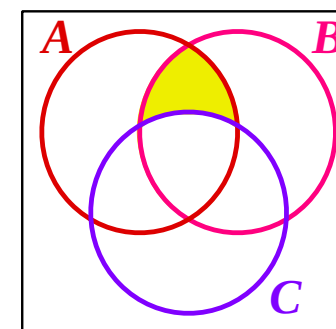
Shade : C'



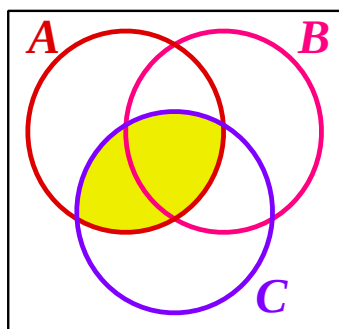
Shade : A'



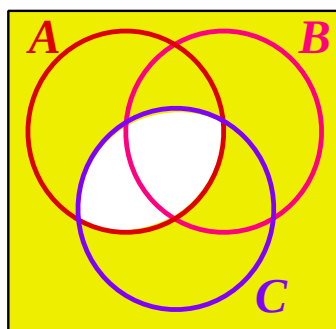
Shade : $A \cap C$



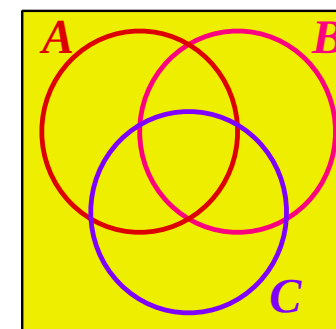
Shade : $A \cap B$



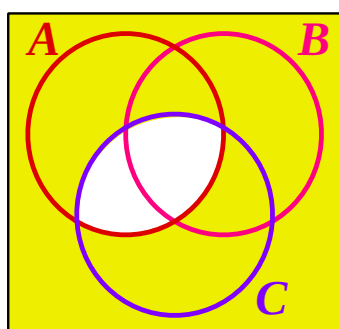
Shade : $A \cap C$



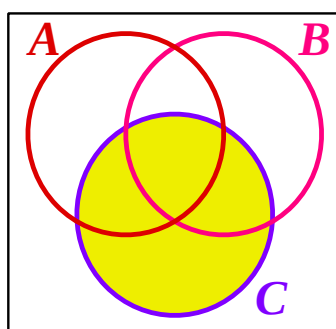
Shade : $A \cap C$



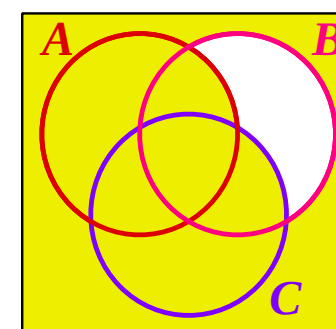
Shade : $A \cap B$



Shade : $A \cap C$



Shade : $A \cap C$



Shade : $A \cap B$

[12 marks]

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In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

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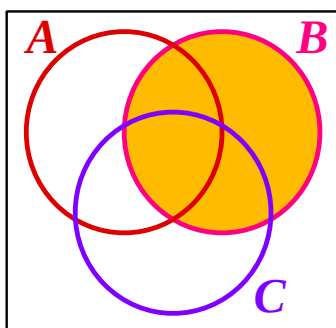
Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk

4.1 Venn Diagram Backwards

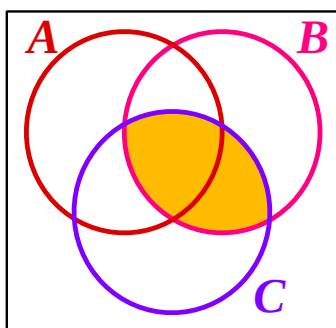
Previously, the questions involving shading on a Venn diagram have given a piece of Set Theory algebra, and asked that the region specified be shaded. However, such questions can be asked the other way round; the shading is given and the algebra describing the region asked for.

4.2 Example

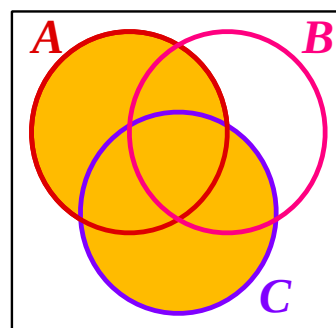
On each of the following diagrams, describe the region that is shaded by using Set Theory symbols.



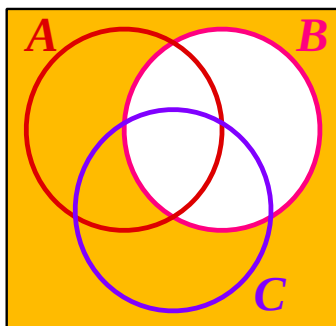
Shade :



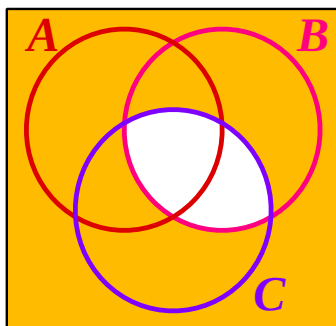
Shade :



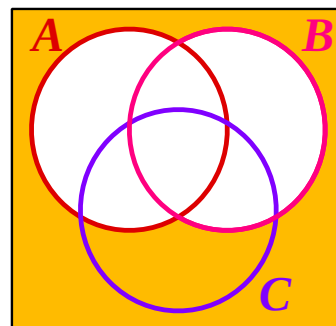
Shade :



Shade :



Shade :



Shade :

[6 marks]

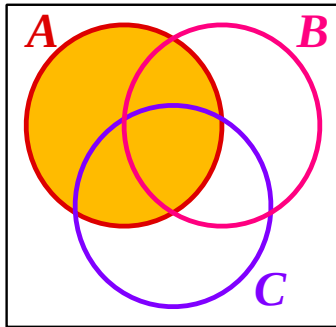
Often, more than one answer is possible, although there is usually an elegant, simplest answer, which is to be striven for.

4.3 Exercise

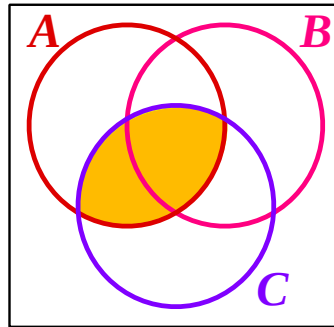
Marks Available : 40

Question 1

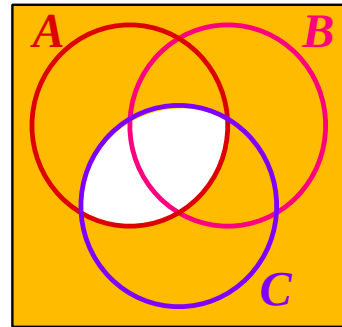
On each of the following diagrams, describe the region that is shaded by using Set Theory symbols.



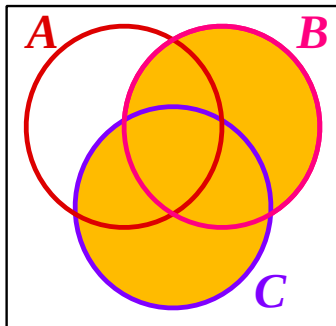
Shade :



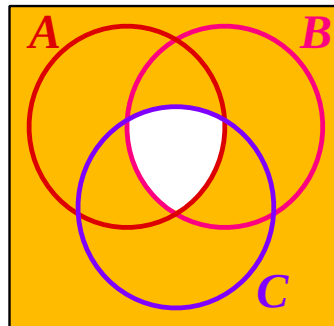
Shade :



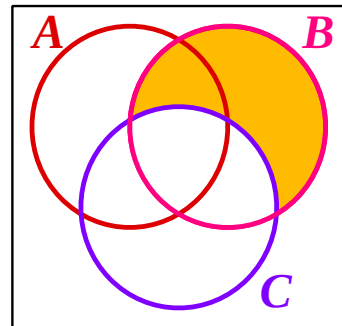
Shade :



Shade :



Shade :



Shade :

[6 marks]

Question 2

Given that $D = \{\text{Names of days in a week}\}$

What is $n\{\text{Names of days in a week}\}$

[1 mark]

Question 3

Let $S = \{\text{Square numbers less than } 40\}$
 $F = \{\text{Factors of } 32\}$
 $T = \{\text{Triangular numbers less than } 40\}$

(i) List the elements of set S

[1 mark]

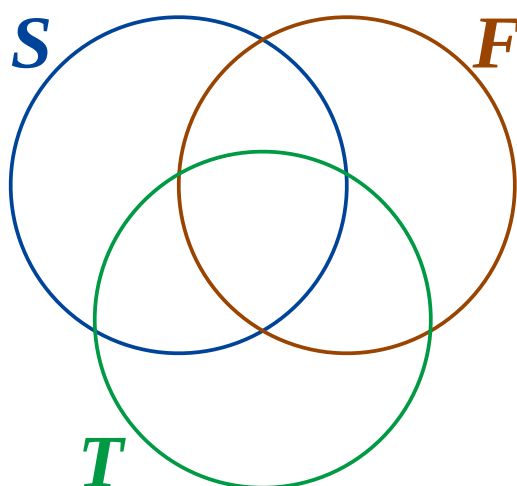
(ii) List the elements of set F

[1 mark]

(iii) List the elements of set T

[1 mark]

(iv) Complete the Venn diagram to show the relationship between S , F and T .



[3 marks]

(v) By counting elements, if any, determine :

(a) $n(F)$

(d) $n(S \cup F \cup T)$

(b) $n(S \cup F)$

(e) $n(S \cap F \cap T')$

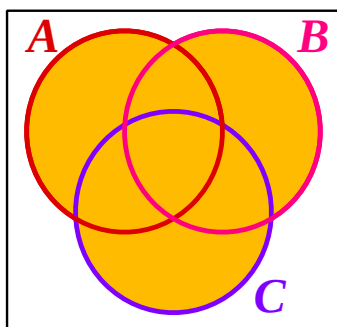
(c) $n(S \cap F)$

(f) $n((S \cap F) \cup T)$

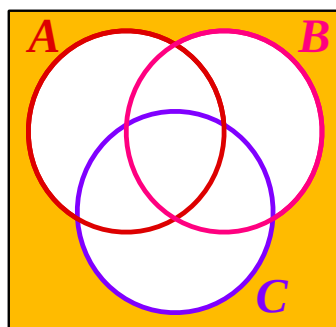
[6 marks]

Question 4

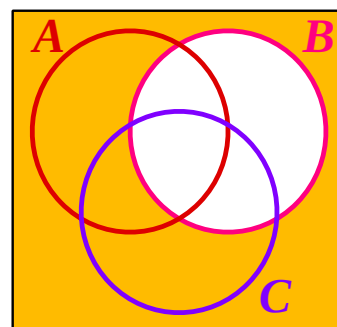
On each of the following diagrams, describe the region that is shaded by using Set Theory symbols.



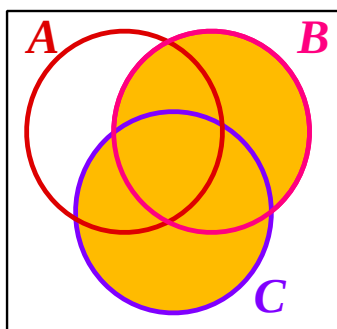
Shade :



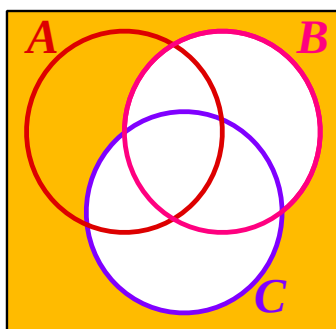
Shade :



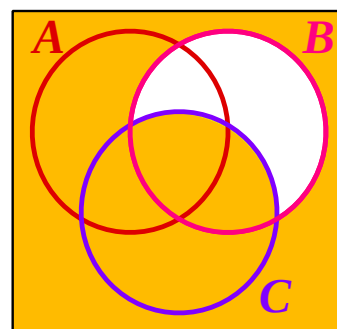
Shade :



Shade :



Shade :



Shade :

[6 marks]

Question 5

In this question,

$V = \{\text{vowels}\}$

$C = \{\text{consonants}\}$

Describe in words the set $V \cup C$

[1 mark]

Question 6

Let ε = All positive integers

Let $A = \{\text{The factors of } 100\}$

Let $B = \{\text{The factors of } 60\}$

(i) List the elements of set A

[2 marks]

(ii) List the elements of set B

[2 marks]

(iii) For each of the following, decide if the given statement is TRUE or FALSE

(a) $n(A) = 9$

(f) $n(A \cap B) > 5$

(b) $n(B) \neq 9$

(g) $7 \notin (A \cup B)$

(c) $3 \in (A \cup B)$

(h) $101 \in A'$

(d) $3 \in (A \cap B)$

(i) $101 \in (A' \cup B')$

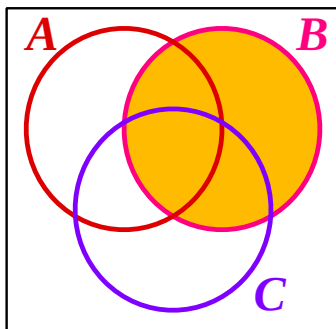
(e) $50 \notin B'$

(j) $12 \in (A \cup B)'$

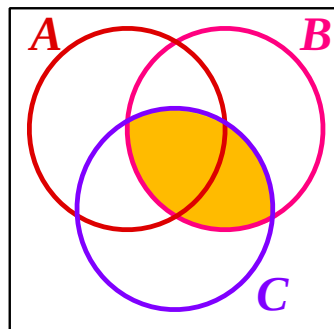
[10 marks]

4.4 Answers

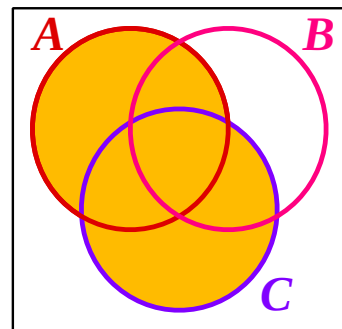
4.4.1 Solutions to 4.2 Example



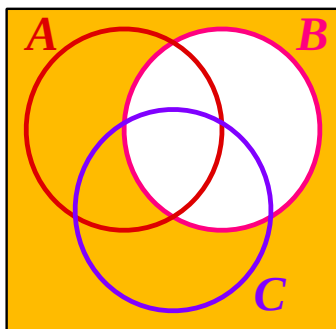
Shade : B



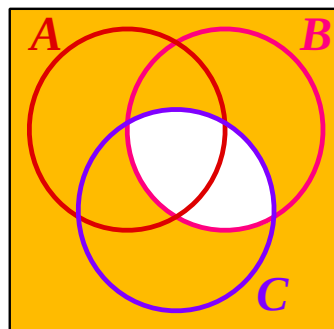
Shade : $B \cap C$



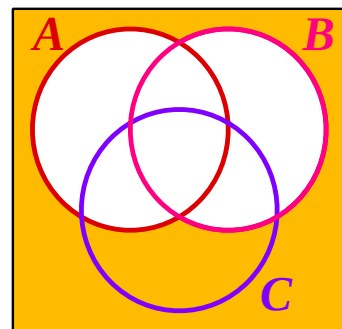
Shade : $A \cup C$



Shade : B'



Shade : $(B \cap C)'$

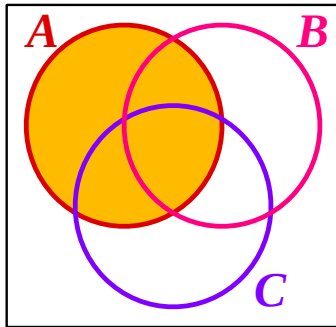


Shade : $(A \cup B)'$

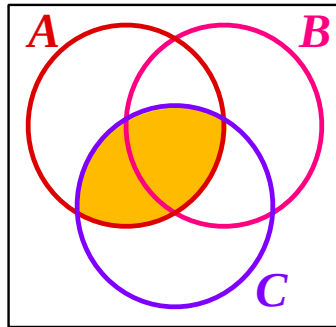
[6 marks]

4.4.2 Solutions to 4.3 Exercise

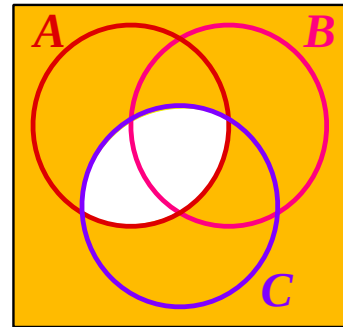
Answer 1



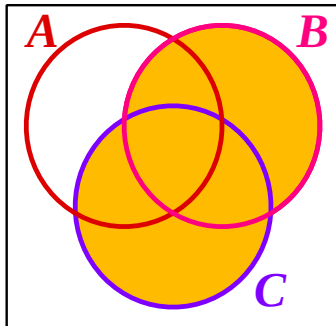
Shade : A



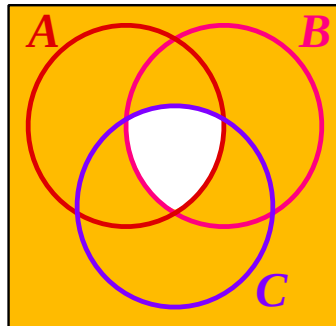
Shade : $A \cap C$



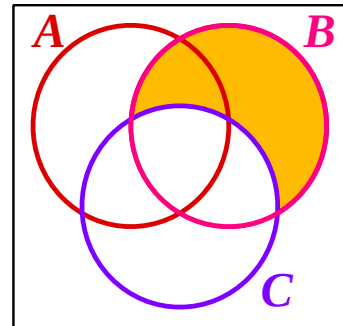
Shade : $(A \cap C)'$



Shade : $B \cup C$



Shade : $(A \cap B \cap C)'$



Shade : $B \cap C'$

[6 marks]

Answer 2

$n\{\text{Names of days in a week}\} = 7$

[1 mark]

Answer 3

(i) $S = \{\text{Square numbers less than 40}\}$
 $= \{1, 4, 9, 16, 25, 36\}$

[1 mark]

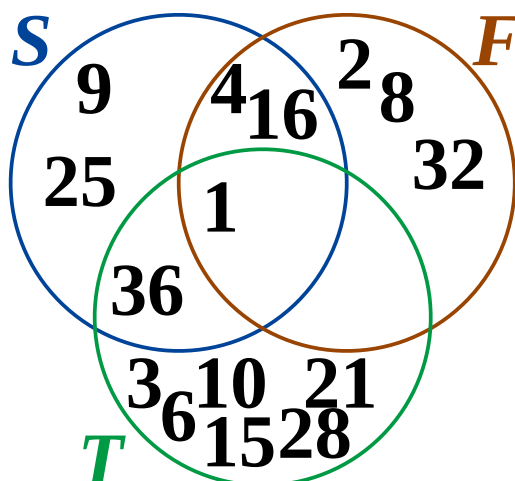
(ii) $F = \{\text{Factors of 32}\}$
 $= \{1, 2, 4, 8, 16, 32\}$

[1 mark]

(iii) $T = \{\text{Triangular numbers less than 40}\}$
 $= \{1, 3, 6, 10, 15, 21, 28, 36\}$

[1 mark]

(iv)



[3 marks]

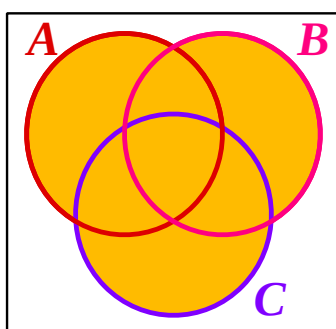
(v) (a) $n(F) = 6$ (d) $n(S \cup F \cup T) = 15$

(b) $n(S \cup F) = 9$ (e) $n(S \cap F \cap T') = 2$

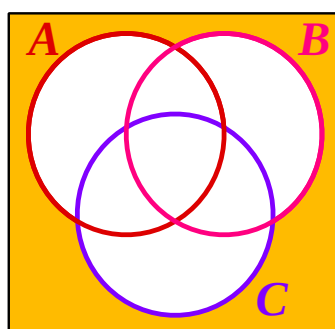
(c) $n(S \cap F) = 3$ (f) $n((S \cap F) \cup T) = 10$

[6 marks]

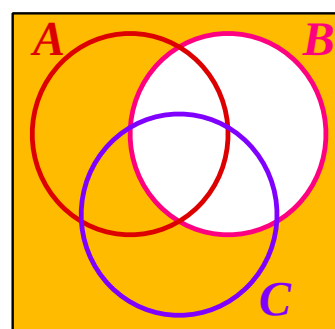
Answer 4



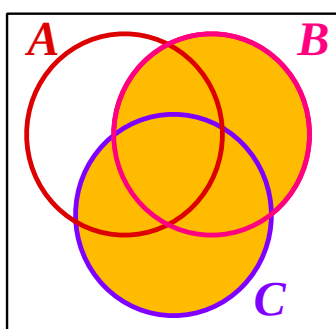
Shade : $A \cup B \cup C$



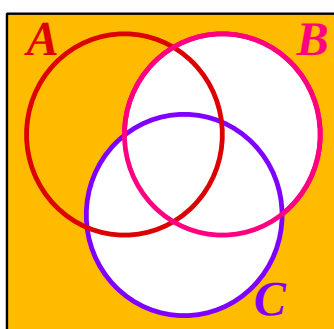
Shade : $(A \cup B \cup C)'$



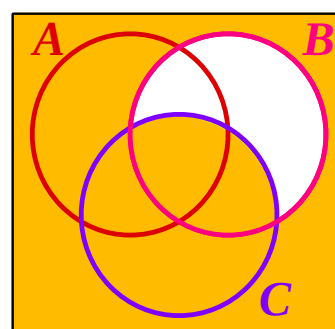
Shade : B'



Shade : $B \cup C$



Shade : $(B \cup C)'$



Shade : $(B \cap C')'$

[6 marks]

Answer 5

$$V \cup C = \{\text{The Alphabet}\}$$

[1 mark]**Answer 6**

(i) $A = \{\text{The factors of } 100\}$
 $= \{ 1, 2, 4, 5, 10, 20, 25, 50, 100 \}$

[2 marks]

(ii) $B = \{\text{The factors of } 60\}$
 $= \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$

[2 marks]**(iii)**

(a) $n(A) = 9$	T	(f) $n(A \cap B) > 5$	T
(b) $n(B) \neq 9$	T	(g) $7 \notin (A \cup B)$	T
(c) $3 \in (A \cup B)$	T	(h) $101 \in A'$	T
(d) $3 \in (A \cap B)$	F	(i) $101 \in (A' \cup B')$	T
(e) $50 \notin B'$	F	(j) $12 \in (A \cup B)'$	F

[10 marks]

5.1 Revision

Marks Available : 100

Question 1

List the elements of the following sets;

(i) $Q = \{\text{The factors of 18}\}$

$Q = \{ \quad, \quad, \quad, \quad, \quad, \quad \}$

(ii) $R = \{\text{The factors of 12}\}$

$R = \{ \quad, \quad, \quad, \quad, \quad, \quad \}$

(iii) $Q \cap R$

$Q \cap R = \{ \quad, \quad, \quad, \quad \}$

[6 marks]

Question 2

Explain each of the following symbols;

(i) $\emptyset$

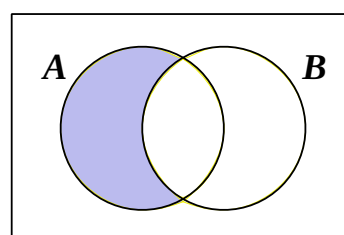
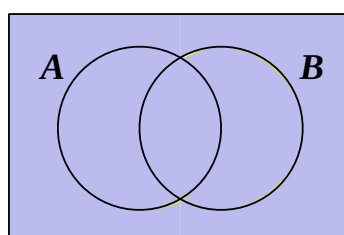
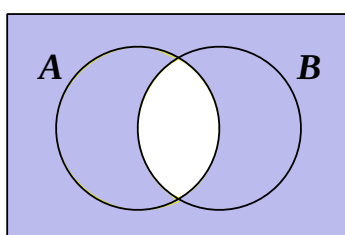
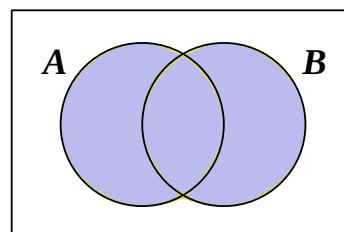
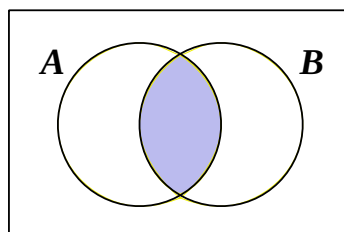
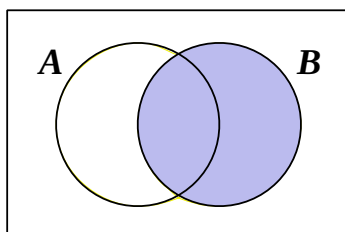
(ii) $\notin$

(iii) $\cup$

[6 marks]

Question 3

For each Venn Diagrams use set notation to describe the shading.



[12 marks]

Question 4

Let $S = \{\text{The first six multiples of 6}\}$
 $F = \{\text{Factors of 36}\}$
 $T = \{\text{The first eight triangular numbers}\}$

(i) List the elements of set S

$S = \{ \quad, \quad, \quad, \quad, \quad, \quad \}$

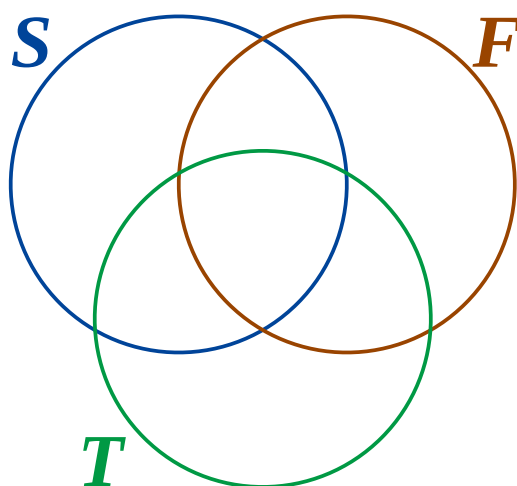
(ii) List the elements of set F

$F = \{ \quad, \quad, \quad, \quad, \quad, \quad, \quad, \quad, \quad \}$

(iii) List the elements of set T

$T = \{ \quad, \quad, \quad, \quad, \quad, \quad, \quad, \quad \}$

(iv) Complete the Venn diagram to show the relationship between S , F and T



List all elements, if any, that are in the following intersections.

(v) $S \cap F = \{ \quad, \quad, \quad, \quad \}$

(vi) $S \cap F \cap T = \{ \quad, \quad \}$

(vii) $S \cap F \cap T' = \{ \quad, \quad \}$

(viii) $F \cap T' \cap S' = \{ \quad, \quad, \quad \}$

(ix) What is $n(S \cup F)$? $\quad$

[18 marks]

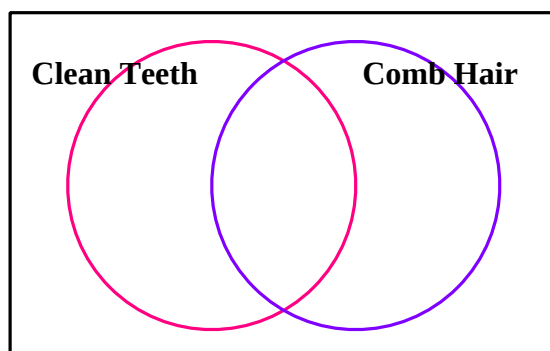
Question 5

A group of 45 teenagers were asked if they cleaned their teeth that morning. They were also asked if they combed their hair.

Here is a summary of their answers :

- ◇ 38 said they cleaned their teeth.
- ◇ 29 said they combed their hair.
- ◇ 23 said they had had both cleaned their teeth and combed their hair.

- (i) Complete the Venn Diagram to clarify the teenagers' replies :



- (ii) How many teenagers neither cleaned their teeth nor combed their hair ?

[6 marks]

Question 6

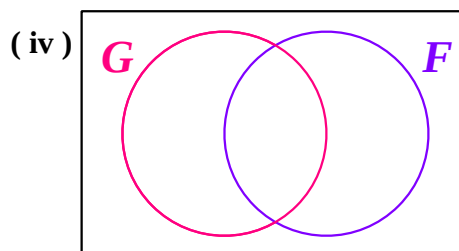
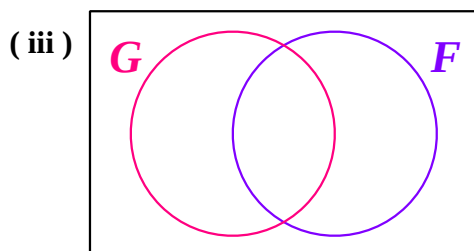
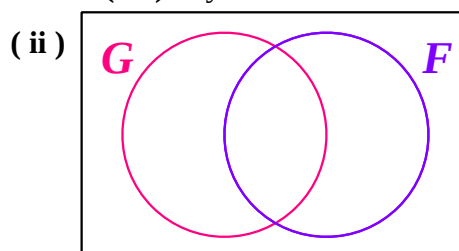
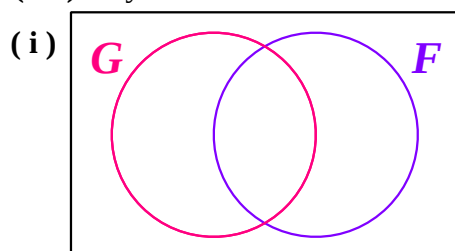
In a class of children, G is the set of girls and F is the set of those who like football. Shade the part that represents:

On (i) girls who like football.

On (ii) girls who dislike football.

On (iii) boys who like football.

On (iv) boys who do not like football.



[8 marks]

Question 7

Set $M = \{\text{Names of seasons in a year}\}$

What is $n\{\text{Names of seasons in a year}\}$?

[2 marks]

Question 8

TRUE or FALSE ?

- (i) $n\{\text{factors } 21\} = 4$
- (ii) $n\{\text{factors of } 17\} = 2$
- (iii) $n\{\text{factors of an even number}\} = 3$
- (iv) $n\{\text{factors of square number}\} = 3$
- (v) $n\{\text{common factors of } 15 \text{ and } 24\} = 3$

[10 marks]

Question 9

In this question,

$L = \{\text{Objects made from the metal lead}\}$

$B = \{\text{Types of balloon}\}$

- (i) Describe the set $L \cap B$ in words.
- (ii) If $L \cap B = \emptyset$, describe what this means.

[4 marks]

Question 10

In this question, consider all the positive integers.

Within this consideration, $P = \{\text{Prime numbers}\}$

$C = \{\text{Composite numbers}\}$

Describe in words the set $P \cup C$

[2 marks]

Question 11

Let : A be the set of numbers in the infinite sequence 4, 8, 12, 16, 20, ...
 B be the set of numbers in the infinite sequence 2, 6, 10, 14, 18, ...

- (i) List the a few members of the set $A \cup B$
- (ii) What is the special name given to the set $A \cup B$?
- (iii) Describe $A \cap B$

[6 marks]

Question 12

In a class of 30 pupils,

- ◇ 18 say they like pancakes.
- ◇ 13 say they like maple syrup.
- ◇ 6 say they do not like either pancakes nor maple syrup.

- (i) Draw a Venn Diagram to clarify what the pupils say.

- (ii) How many of the pupils like both pancakes and maple syrup ?

[6 marks]

Question 13

Let $S = \{\text{Square numbers}\}$
 $F = \{1, 4, 16, 64, 256, 1024\}$

(a) (i) List some elements of set S

(ii) Describe set F in words

(b) For each of the following, decide if the given statement is TRUE or FALSE.

(i) $25 \in S$

(vi) $S \cup F = S$

(ii) $1024 \notin S$

(vii) $\pi \in S'$

(iii) $64 \in S \cap F$

(viii) $F \cap \{\text{Prime numbers}\} = \emptyset$

(iv) $n(F) = 8$

(ix) $36 \in S \cap F'$

(v) $n(S \cap F) = 6$

(x) $3 \notin S \cup F$

[14 marks]

5.2 Answers to 5.1 Revision

Answer 1

(i) $Q = \{ 1, 2, 3, 6, 9, 18 \}$

(ii) $R = \{ 1, 2, 3, 4, 6, 12 \}$

(iii) $Q \cap R = \{ 1, 2, 3, 6 \}$

[6 marks]

Answer 2

(i) $\emptyset$: “empty set”

(ii) $\notin$: “is not an element of”

(iii) $\cup$: “union”

[6 marks]

Answer 3

(i) B

(ii) $A \cap B$

(iii) $A \cup B$

(iv) $(A \cap B)'$

(v) $A \cup A'$

(vi) $A \cap B'$

Other equivalent answers are possible

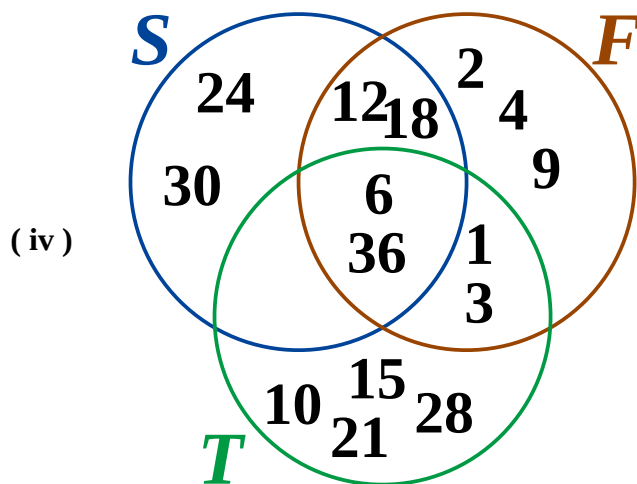
[12 marks]

Answer 4

(i) $S = \{ 6, 12, 18, 24, 30, 36 \}$

(ii) $F = \{ 1, 2, 3, 4, 6, 9, 12, 18, 36 \}$

(iii) $T = \{ 1, 3, 6, 10, 15, 21, 28, 36 \}$



(v) $S \cap F = \{ 6, 12, 18, 36 \}$

(vi) $S \cap F \cap T = \{ 6, 36 \}$

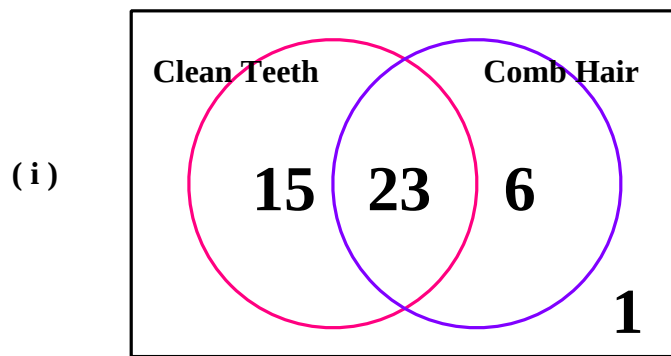
(vii) $S \cap F \cap T' = \{ 12, 18 \}$

(viii) $F \cap T' \cap S' = \{ 2, 4, 9 \}$

(ix) $n(S \cup F) = 11$

[18 marks]

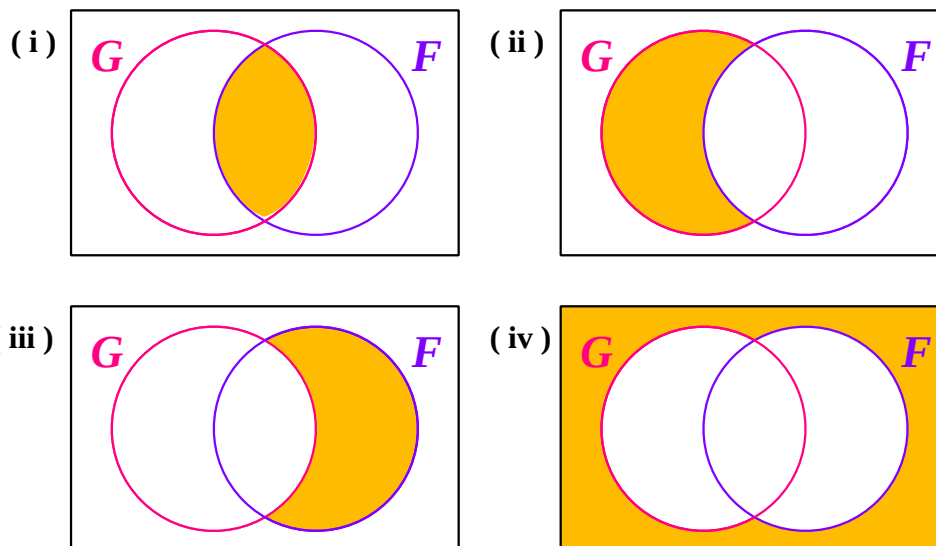
Answer 5



(ii) 1

[6 marks]

Answer 6



[8 marks]

Answer 7

$n\{\text{Names of seasons in a year}\} = 4$

[2 marks]

Answer 8

- (i) T
- (ii) T
- (iii) F
- (iv) F
- (v) F

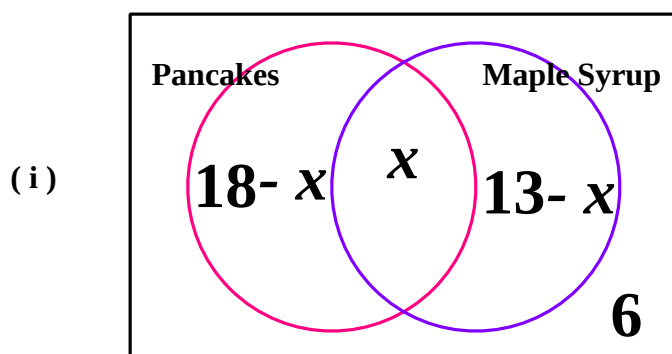
[10 marks]

Answer 9

- (i) Lead balloons
- (ii) There are no lead balloons

[4 marks]

Answer 10

$$P \cup C \text{ is all integers } \geq 2$$
[2 marks]**Answer 11****(i)** $A \cup B = \{ 2, 4, 6, 8, 10, 12, 14, \dots \}$ **(ii)** The even numbers**(iii)** $A \cap B = \emptyset$ **[6 marks]****Answer 12****(ii)** Let x be the number of pupils who like both pancakes and maple syrup.
In which case,

$$(18 - x) + x + (13 - x) + 6 = 30$$

$$37 - x = 30$$

$$x = 7$$

[6 marks]**Answer 13****(a) (i)** $S = \{ 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, \dots \}$ **(ii)** $F =$ Powers of 4. i.e. 4^x for $x = 0, 1, 2, 3, 4$

(b) (i)	T	(vi)	T
(ii)	F	(vii)	T
(iii)	T	(viii)	T
(iv)	F	(ix)	T
(v)	T	(x)	T

[14 marks]

Lesson 6

GCSE Mathematics Set Theory II

6.1 Test

Question 1

List the elements of the following sets;

(i) $M = \{\text{The factors of } 50\}$

$$M = \{ \quad , \quad , \quad , \quad , \quad , \quad \}$$

(ii) $R = \{\text{The factors of } 45\}$

$$R = \{ \quad , \quad , \quad , \quad , \quad , \quad \}$$

(iii) $M \cap R$

$$M \cap R = \{ \quad , \quad \}$$

[6 marks]

Question 2

Explain each of the following symbols;

(i) $\cap$

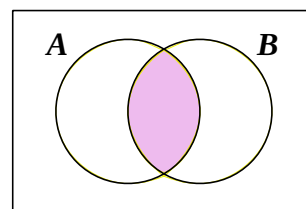
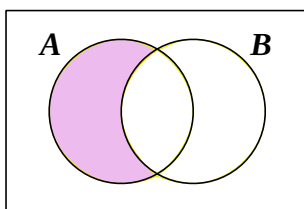
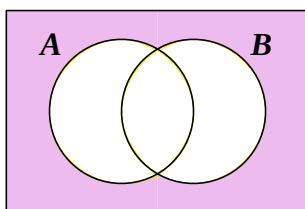
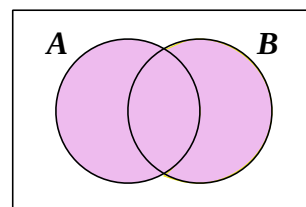
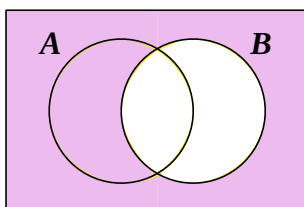
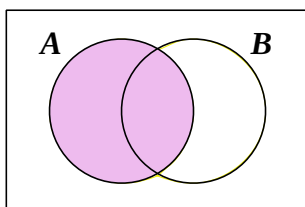
(ii) $\emptyset$

(iii) $\in$

[6 marks]

Question 3

For each of the Venn diagrams below use set notation to describe the shading.



[12 marks]

Question 4

Let $S = \{\text{The first five square numbers}\}$

Let $M = \{\text{The first six multiples of 4}\}$

Let $F = \{\text{The factors of 16}\}$

- (i) List the elements of set S

$S = \{ \quad , \quad , \quad , \quad , \quad \}$

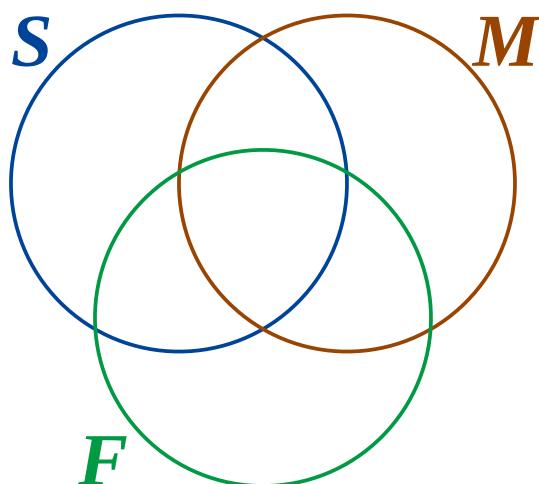
- (ii) List the elements of set M

$M = \{ \quad , \quad , \quad , \quad , \quad , \quad \}$

- (iii) List the elements of set F

$F = \{ \quad , \quad , \quad , \quad , \quad \}$

- (iv) Complete the Venn diagram to show the relationship between S , M and F



List all elements, if any, that are in the following intersections,

- (v) $S \cap F = \{ \quad , \quad , \quad \}$

- (vii) $S \cap M \cap F = \{ \quad , \quad \}$

- (vi) $F \cap M = \{ \quad , \quad , \quad \}$

- (viii) $F \cap M \cap S' = \{ \quad \}$

- (ix) What is $n(S \cup M)$? $\quad$

[18 marks]

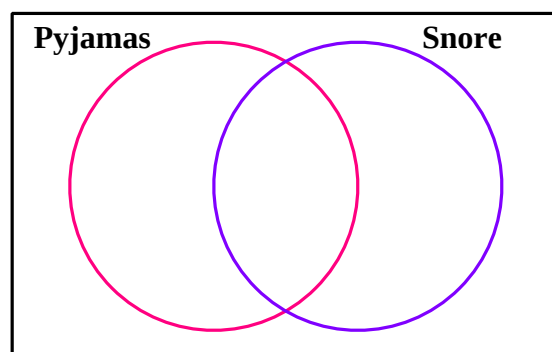
Question 5

A group of 58 wives were asked if their husbands wore pyjamas in bed. They were also asked if their husbands snored.

Here is a summary of their answers :

- ◇ 33 said their husbands wore pyjamas in bed.
- ◇ 26 said their husbands snored.
- ◇ 18 said their husbands both wore pyjamas and snored.

(i) Complete the Venn Diagram to clarify the wives' replies :



(ii) How many husbands neither wore pyjamas nor snored ?

[6 marks]

Question 6

In a class of children, B is the set of boys and M is the set of those who like maths.

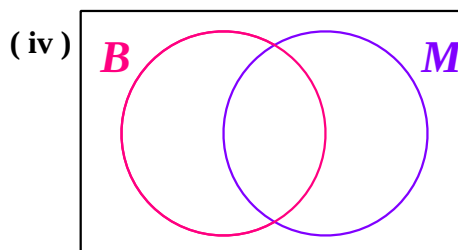
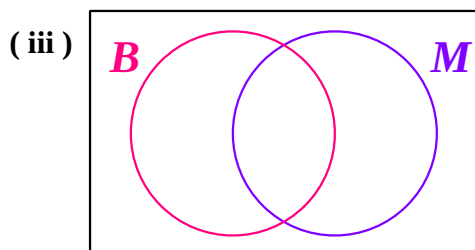
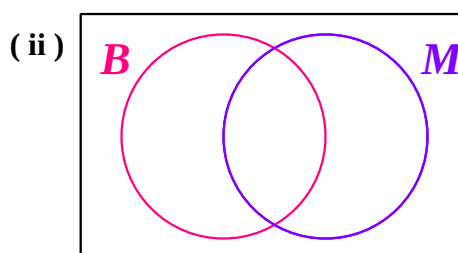
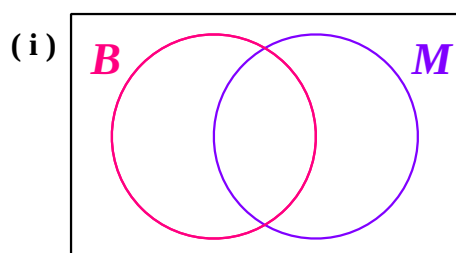
Shade the part that represents:

On (i) boys who like maths.

On (ii) boys who dislike maths.

On (iii) girls who like maths.

On (iv) girls who do not like maths.



[8 marks]

Question 7

Set $M = \{\text{Names of months in a year}\}$

What is $n\{\text{Names of months in a year}\}$?

[2 marks]

Question 8

TRUE or FALSE ?

- (i) $n\{\text{factors } 9\} = 3$
- (ii) $n\{\text{factors of } 23\} = 3$
- (iii) $n\{\text{factors of a prime number}\} = 1$
- (iv) $n\{\text{factors of square number}\} = 3$
- (v) $n\{\text{common factors of } 15 \text{ and } 20\} = 3$

[10 marks]

Question 9

In this question,

$F = \{\text{Flying objects}\}$

$P = \{\text{Types of pig}\}$

- (i) Describe the set $F \cap P$ in words.
- (ii) If $F \cap P = \emptyset$, describe what this means.

[4 marks]

Question 10

In this question, consider all the positive integers.

Within this consideration, $E = \{\text{Even numbers}\}$

$O = \{\text{Odd numbers}\}$

Describe in words the set $E \cup O$

[2 marks]

Question 11

Let : A be the set of numbers in the infinite sequence 1, 9, 25, 49, 81, ...
 B be the set of numbers in the infinite sequence 4, 16, 36, 64, 100, ...

- (i) List the some members of the set $A \cup B$
- (ii) What is the special name given to the set $A \cup B$?
- (iii) Describe $A \cap B$

[6 marks]

Question 12

In a class of 30 pupils,

- ◇ 19 say they like haggis.
- ◇ 11 say they like neeps.
- ◇ 5 say they do not like either haggis nor neeps.

- (i) Draw a two hoop Venn Diagram to clarify what the pupils say.

- (ii) How many of the pupils like both haggis and neeps ?

[6 marks]

Question 13

Let $T = \{\text{Triangular numbers}\}$

Let $M = \{\text{Multiples of 3}\}$

(a) List the elements of;

(i) set T

(ii) set M

(b) For each of the following, decide if the given statement is TRUE or FALSE.

(i) $10 \in T$

(vi) $9 \notin T \cup M$

(ii) $3693 \notin M$

(vii) $2 \in M'$

(iii) $6 \in T \cap M$

(viii) $T \cap M' = \emptyset$

(iv) $n(T) = 8$

(ix) $n(T \cap M) > 36$

(v) $2 \notin M \cup T$

(x) $T \cap \{\text{Prime numbers}\}' = \emptyset$

[14 marks]

6.2 Answers to 6.1 Test

Answer 1

(i) $M = \{ 1, 2, 5, 10, 25, 50 \}$

(ii) $R = \{ 1, 3, 5, 9, 15, 45 \}$

(iii) $M \cap R = \{ 1, 5 \}$

[6 marks]

Answer 2

(i) $\cap$: “intersection”

(ii) $\emptyset$: “empty set”

(iii) $\in$: “is an element of”

[6 marks]

Answer 3

(i) A

(ii) B'

(iii) $A \cup B$

(iv) $(A \cup B)'$

(v) $A \cap B'$

(vi) $A \cap B$

Other equivalent answers are possible

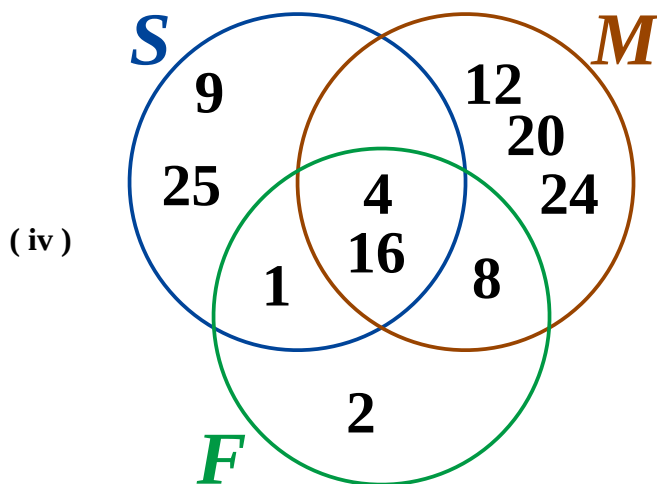
[12 marks]

Answer 4

(i) $S = \{ 1, 4, 9, 16, 25 \}$

(ii) $M = \{ 4, 8, 12, 16, 20, 24 \}$

(iii) $F = \{ 1, 2, 4, 8, 16 \}$



(v) $S \cap F = \{ 1, 4, 16 \}$

(vi) $S \cap M \cap F = \{ 4, 16 \}$

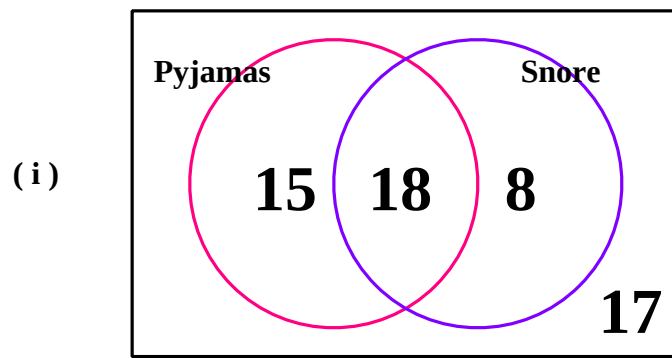
(vii) $F \cap M = \{ 4, 8, 16 \}$

(viii) $F \cap M \cap S' = \{ 8 \}$

(ix) $n(S \cup M) = 9$

[18 marks]

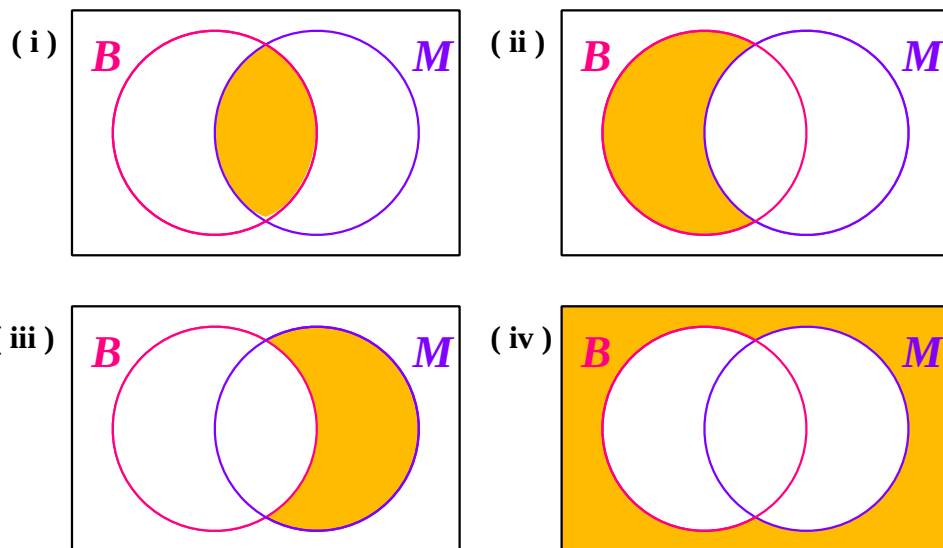
Answer 5



(ii) 17

[6 marks]

Answer 6



[8 marks]

Answer 7

$n\{\text{Names of months in a year}\} = 12$

[2 marks]

Answer 8

- (i) T
- (ii) F
- (iii) F
- (iv) F
- (v) F

[10 marks]

Answer 9

- (i) Flying pigs
- (ii) There are no flying pigs

[4 marks]

Answer 10

$E \cup O$ is all the positive integers

[2 marks]

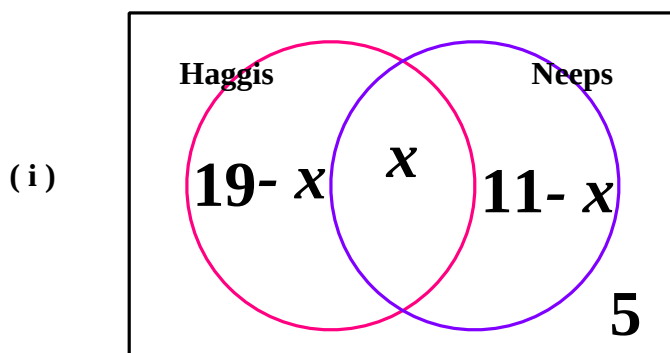
Answer 11

(i) $A \cup B = \{ 1, 4, 9, 16, 25, 36, 49, \dots \}$

(ii) Square numbers

(iii) $A \cap B = \emptyset$

[6 marks]

Answer 12

(ii) Let x be the number of pupils who like both haggis and neeps.
In which case,

$$(19 - x) + x + (11 - x) + 5 = 30$$

$$35 - x = 30$$

$$x = 5$$

[6 marks]

Answer 13

(a) (i) $T = \{ 1, 3, 6, 10, 15, 21, 28, 36, \dots \}$

(ii) $M = \{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, \dots \}$

(b) (i) T (vi) F

(ii) F (vii) T

(iii) T (viii) F

(iv) F (ix) T

(v) T (x) F, most obviously because of the 3

[14 marks]