Year 2 Pure Mathematics Examination Revision
Health Check N° 11

Answer 1

$$y = x(2x+1)^4$$
$$\frac{dy}{dx} = x \times 4(2x+1)^3 \times 2 + (2x+1)^4$$
$$= (2x+1)^3(8x+2x+1)$$
$$= (2x+1)^3(10x+1)$$
$$\therefore\ n = 3,\ A = 10 \text{ and } B = 1$$

[4 marks]

Answer 2

(i) The question is telling us that, $a\,r^3 = 16$ and $a\,r^6 = 250$

$$\therefore\ \frac{a\,r^6}{a\,r^3} = \frac{250}{16}$$
$$r^3 = \frac{125}{8}$$
$$r = \frac{5}{2}$$
$$\text{As } a\,r^3 = 16 \Rightarrow a\left(\frac{5}{2}\right)^3 = 16$$
$$a = \frac{16 \times 8}{125}$$
$$= \frac{128}{125}$$

[4 marks]

(ii) Equating the two ways that the common difference can be found,

$$(6k-9)-(4k+3) = (4k+3)-(3k)$$
$$2k-12 = k+3$$
$$k = 15$$

So the arithmetic progression is, 9, 27, 45, 63, 81, 99, 117, ...

That is, a is 9 and d is 18

$$Sum = \frac{n}{2}\{2a+(n-1)d\}$$
$$= \frac{n}{2}\{18+(n-1)18\}$$
$$= 9n^2$$
$$= (3n)^2 \quad \text{which is a square number} \quad \square$$

[5 marks]

(iii) The question is telling us that $a = 4$ and $\dfrac{a}{1 - r} = 12$

$$\therefore \frac{4}{1 - r} = 12$$

$$4 = 12(1 - r)$$

$$4 = 12 - 12r$$

$$12r = 8$$

$$r = \frac{2}{3}$$

[3 marks]

Question 3

(a) By the quotient rule,

$$f'(x) = \frac{(4 + x^2) \times 0 - (2x) \times 16}{(4 + x^2)^2}$$

$$= \frac{(-32x)}{(4 + x^2)^2}$$

$$f''(x) = \frac{(4 + x^2)^2 \ (-32) - 2(4 + x^2)^1 \ 2x(-32x)}{(4 + x^2)^4}$$

$$= \frac{(4 + x^2)(-32) - 4x(-32x)}{(4 + x^2)^3}$$

$$= \frac{-128 - 32x^2 + 128x^2}{(4 + x^2)^3}$$

$$= \frac{96x^2 - 128}{(4 + x^2)^3}$$

At the points of inflection, $f''(x) = 0$

$$\frac{96x^2 - 128}{(4 + x^2)^3} = 0$$

$$\therefore 96x^2 - 128 = 0$$

$$x^2 = \frac{128}{96}$$

$$x = \pm \frac{2\sqrt{3}}{3}$$

$$\Rightarrow\ y = 16 \div \left(4 + \left(\frac{2\sqrt{3}}{3}\right)^2\right)$$

$$= 16 \div \left(\frac{36 + 12}{9}\right)$$

$$= 16 \times \frac{9}{48}$$

$$= 3$$

So the two points of inflection are, $\left(\pm\frac{2\sqrt{3}}{3},\ 3\right)$

[6 marks]

(b) **(i)** The range of the original function is $f(x) \in \mathbb{R},\ 0 < f(x) \leqslant 4$

For $m(x)$ the range will be $m(x) \in \mathbb{R},\ 0 < m(x) \leqslant 1$

[1 mark]

(ii) In order to get $n(x)$, x first has to go into $f(x)$ but what can come out of $f(x)$ is now restricted by what is allowed in.

The range of $f(x)$ this time is $f(x) \in \mathbb{R},\ 0 < f(x) \leqslant 3$

For $n(x)$ the range will be $n(x) \in \mathbb{R},\ 4 < n(x) \leqslant 5$

[2 marks]

Answer 4

(a) $R\cos(x + \alpha) = R\cos x\cos\alpha - R\sin x\sin\alpha$

Compare this with the expression given of $5\cos x - 8\sin x$

$$\Rightarrow \frac{R\sin\alpha}{R\cos\alpha} = \frac{8}{5}$$

$$\alpha = \arctan\left(\frac{8}{5}\right)$$

$$= 1.0122 \text{ radians}$$

$$R = \sqrt{5^2 + 8^2}$$

$$= \sqrt{89}$$

$$\therefore\ 5\cos x - 8\sin x = \sqrt{89}\cos(x + 1.0122)$$

[4 marks]

(b)

$$T = 1100 + 5\cos\left(\frac{x}{3}\right) - 8\sin\left(\frac{x}{3}\right), \qquad 0 \leqslant x \leqslant 72$$

$$= 1100 + \sqrt{89}\cos\left(\frac{x}{3} + 1.0122\right) \qquad \text{using part (a)}$$

T will have a maximum value when the *cosine* function

has it's maximum value of 1

$$\therefore\ T_{MAX} = 1100 + \sqrt{89} \qquad (\text{About } 1109.4\ °\text{C})$$

In more detail, this will occur when,

$$\cos\left(\frac{x}{3} + 1.0122\right) = 1$$

$$\frac{x}{3} + 1.0122 = 0,\ 2\pi,\ 4\pi,\ \ldots$$

$$\frac{x}{3} = 5.271 \qquad (\text{As only after when first occurs})$$

$$x = 15.81 \text{ hours} \qquad (15 \text{ hours } 49 \text{ minutes})$$

[4 marks]

(c)

$$T = 1097$$

$$1100 + 5\cos\left(\frac{x}{3}\right) - 8\sin\left(\frac{x}{3}\right) = 1097$$

$$1100 + \sqrt{89}\cos\left(\frac{x}{3} + 1.0122\right) = 1097$$

$$\sqrt{89}\cos\left(\frac{x}{3} + 1.0122\right) = -3$$

$$\cos\left(\frac{x}{3} + 1.0122\right) = -\frac{3}{\sqrt{89}} \qquad (WA \text{ is } 1.2471)$$

$$\frac{x}{3} + 1.0122 = 1.8944,\ 4.3887,\ 8.1776,\ 10.6719,\ \ldots$$

$$\frac{x}{3} = 0.8822,\ 3.3765,\ 7.1654,\ 9.6597,\ \ldots$$

$$x = 2.6466,\ 10.1295,\ 21.4962,\ 28.9791,\ \ldots$$

x = 2 hours 39 minutes,

10 hours 8 minutes,

21 hours 30 minutes

[4 marks]

Answer 5

(a) As 1st January 2005 is four years after 1st January 2001,

$$32000 = Ap^4 \Rightarrow A = \frac{32000}{p^4}$$

Similarly, as 1st January 2012 is eleven years after 1st January 2001,

$$50000 = Ap^{11} \Rightarrow A = \frac{50000}{p^{11}}$$

$$\therefore \frac{32000}{p^4} = \frac{50000}{p^{11}}$$

$$\frac{p^{11}}{p^4} = \frac{50000}{32000}$$

$$p^7 = \frac{25}{16}$$

$$p = 1.0658$$

$$A = \frac{32000}{p^4}$$

$$= \frac{32000}{1.0658^4}$$

$$= 24799.7$$

Which is approximately 24 800 ☐

[4 marks]

(b) **(i)** A is the value of the car on 1st January 2001

(ii) p is a measure of the growth rate of the value.
1.0658 corresponds to an annual increase of 6.58%

[2 marks]

(c) To find the critical value, $v = 100000$

$$24799.7 \times 1.0658^n = 100000$$

$$1.0658^n = 4.0323$$

$$ln\, 1.0658^n = ln\, 4.0323$$

$$n = \frac{ln\, 4.0323}{ln\, 1.0658}$$

$$= 21.88$$

$\therefore$ The value first exceeds £100 000 in 2022

[4 marks]