Year 2 Pure Mathematics Examination Revision
Health Check N° 1

Answer 1

$$\sin(x + 30) = 2\cos x$$
$$\sin x \cos 30 + \cos x \sin 30 = 2\cos x$$
$$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = 2\cos x$$
multiply through by 2
$$\sqrt{3}\sin x + \cos x = 4\cos x$$
$$\sqrt{3}\sin x = 3\cos x$$
$$\frac{\sin x}{\cos x} = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$\tan x = \sqrt{3}$$
$$x = 60°,\ 240°$$

[5 marks]

Answer 2

As $(4x + 1)$ is a factor $f\left(-\frac{1}{4}\right) = 0$ by the factor theorem

$$\therefore 4\left(-\frac{1}{4}\right)^3 + a\left(-\frac{1}{4}\right)^2 + 20\left(-\frac{1}{4}\right) + 4 = 0$$
$$-\frac{1}{16} + \frac{a}{16} - 5 + 4 = 0$$
$$\frac{a}{16} = \frac{1}{16} + 1$$
multiply through by 16
$$a = 1 + 16$$
$$a = 17$$

[5 marks]

Answer 3

(i)

$$g(x) = x^3 - 12x^2 + 50x - 7$$

$$g'(x) = 3x^2 - 24x + 50$$

[2 marks]

(ii)

$$\begin{aligned} g'(x) &= 3x^2 - 24x + 50 \\ &= \left[3x^2 - 24x \right] + 50 \\ &= 3\left[x^2 - 8x \right] + 50 \\ &= 3\left[(x - 4)^2 - 16 \right] + 50 \\ &= 3(x - 4)^2 - 48 + 50 \\ &= 3(x - 4)^2 + 2 \end{aligned}$$

- The smallest that $(x - 4)^2$ can be is zero.
- Even then, 2 is added on.
- Therefore, the gradient $g'(x)$ is always positive
- Therefore, $g(x)$ is an increasing function ☐

[5 marks]

Answer 4

(i) When $T = 0$,

$$S = \frac{390}{1 + e^{4 - 1.4 \times 0}}$$

$$= \frac{390}{1 + e^{4}}$$

$$= 7 \text{ squirrels}$$

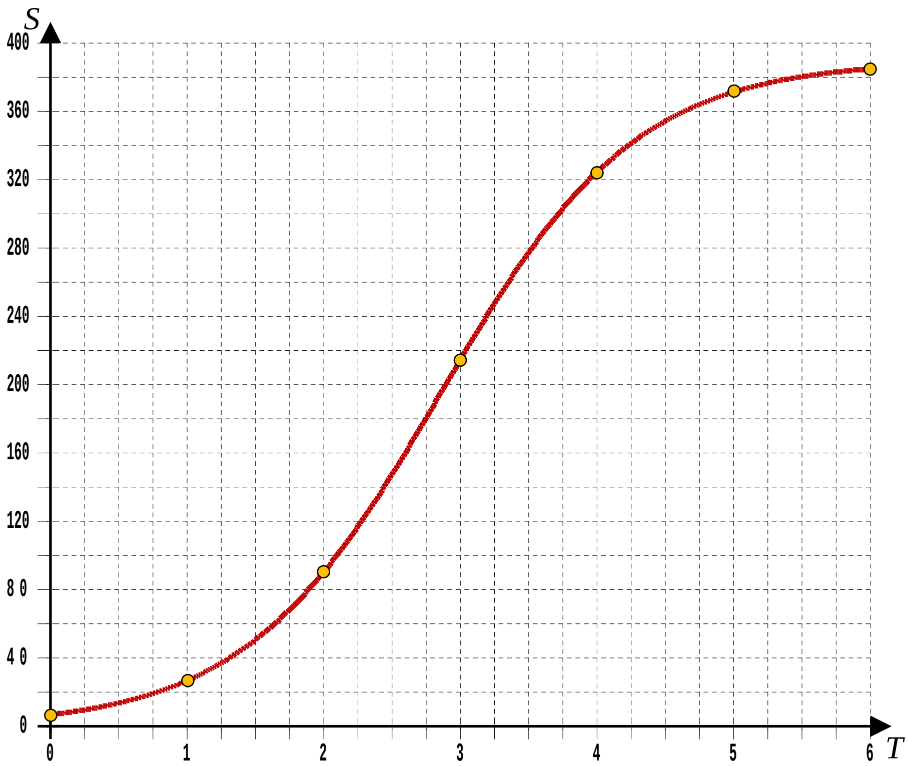
[1 mark]

(ii)

T (years)	0	1	2	3	4	5	6
S (N° of Squirrels)	7	27	90	214	324	372	385

[2 marks]

(iii)



[2 marks]

(iv) It is true that the equation contains an exponential but as T becomes large the power of $e^{4 - 1.4T}$ will be negative; it has exponential decay, not growth.

As $T \to \infty$, $e^{4 - 1.4T} \to 0$, and so $\dfrac{390}{1 + e^{4 - 1.4 \times T}} \to 390$

Thus the model predicts a stable, unchanging long term population in the forest of 390 squirrels.

[3 marks]

Answer 5

$$2^{2x} + 3\left(2^{x-1}\right) - 1 = 0$$

$$\left(2^x\right)^2 + \frac{3\left(2^x\right)}{2} - 1 = 0$$

multiply through by 2

$$2\left(2^x\right)^2 + 3\left(2^x\right) - 2 = 0$$

Let $z = 2^x$ in which case,

$$2z^2 + 3z - 2 = 0$$

$$\left(2z - 1\right)\left(z + 2\right) = 0$$

Either $z = \frac{1}{2}$ or $z = -2$

But $2^x = -2$ has no solutions

so the only solution comes from $2^x = \frac{1}{2}$

That is, $x = -1$

[5 marks]