Year 2 Pure Mathematics Examination Revision
Health Check N° 2

Answer 1

$$f(x) = \left(x + \frac{1}{x} \right)^2$$

$$= x^2 + 2 + \frac{1}{x^2}$$

$$\int_1^3 f(x)\, dx = \int_1^3 x^2 + 2 + x^{-2}\, dx$$

$$= \left[\frac{x^3}{3} + 2x + \frac{x^{-1}}{(-1)} \right]_1^3$$

$$= \left[\frac{x^3}{3} + 2x - \frac{1}{x} \right]_1^3$$

$$= \left[9 + 6 - \frac{1}{3} \right] - \left[\frac{1}{3} + 2 - 1 \right]$$

$$= 14\frac{2}{3} - 1\frac{1}{3}$$

$$= 13\frac{1}{3}$$

[5 marks]

Answer 2

(i) When $Y = 2014$, $U = 1$, gives $1 = a\, e^{b\,(2014 - 2014)} \Rightarrow a = 1$

When $Y = 2020$, $U = 2$, gives $2 = 1\, e^{b\,(2020 - 2014)} \Rightarrow b = \frac{1}{6}\, ln\, 2$

(As an approximate decimal $b = 0.11552...$)

[2 marks]

(ii) When $y = 2050$, $U = e^{\frac{1}{6} ln\, 2\,(2050 - 2014)}$

$$= e^{6\, ln\, 2}$$

$$= e^{ln\, 2^6}$$

$$= 2^6$$

$= 64$ billion active users

[2 marks]

(iii) The human population of Earth is predicted to be around 10 billion in 2050, placing enormous strain on the planet's ability to support life. (It's currently about 7.9 billion)
$\therefore$ The prediction of 64 billion active users is not possible in reality !

[2 marks]

Answer 3

(i)

$$y = x^4 - x$$

$$\frac{dy}{dx} = 4x^3 - 1$$

$$\frac{d^2y}{dx^2} = 12x^2$$

At $(0,\ 0)$, $\dfrac{d^2y}{dx^2} = 0$ satisfying the first condition.

As any number, other than zero, when squared is positive,

the sign of $\dfrac{d^2y}{dx^2}$ either side of $(0,\ 0)$ must be positive, and so

the second condition is not satisfied.

$\therefore\ y = x^4 - x$ does not have a point of inflection at $(0,\ 0)$ □

[3 marks]

(ii)

$$y = x^3 + x$$

$$\frac{dy}{dx} = 3x^2 + 1$$

$$\frac{d^2y}{dx^2} = 6x$$

At $(0,\ 0)$, $\dfrac{d^2y}{dx^2} = 0$ satisfying the first condition.

When $x < 0$, $\dfrac{d^2y}{dx^2}$ will be negative

When $x > 0$, $\dfrac{d^2y}{dx^2}$ will be positive

The sign of $\dfrac{d^2y}{dx^2}$ on one side of $(0,\ 0)$ is different to that on the other

and so the second condition is satisfied.

$\therefore\ y = x^3 + x$ has a point of inflection at $(0,\ 0)$ □

[3 marks]

Answer 4

(a) From the cosine rule,

$$
\begin{aligned}
\cos AOB &= \frac{5^2 + 5^2 - 6^2}{2 \times 5 \times 5} \\
&= \frac{25 + 25 - 36}{50} \\
&= \frac{14}{50} \\
&= \frac{7}{25} \qquad \square
\end{aligned}
$$

[2 marks]

(b)

$$
\begin{aligned}
\therefore\ AOB &= \arccos\left(\frac{7}{25}\right) \\
&= 1.287 \text{ radians}
\end{aligned}
$$

[1 mark]

(c)

$$
\begin{aligned}
Area\ Sector &= \frac{1}{2} r^2 \theta \\
&= \frac{1}{2} \times 5^2 \times 1.287 \\
&= 16.1 \text{ m}^2
\end{aligned}
$$

[2 marks]

(d)

$$
\begin{aligned}
Area\ Shaded &= Area\ Sector \ - \ Area\ Triangle \\
&= 16.1 - \frac{1}{2} \times 5 \times 5 \times \sin 1.287 \\
&= 16.1 - 12.0 \\
&= 4.1 \text{ m}^2
\end{aligned}
$$

[3 marks]

Answer 5

$$\begin{aligned}
3\,log_8 2 + log_8(7 - x) &= 2 + log_8 x \\
log_8 2^3 + log_8(7 - x) - log_8 x &= 2 \\
log_8\left(\frac{2^3(7 - x)}{x}\right) &= 2 \\
\therefore \left(\frac{2^3(7 - x)}{x}\right) &= 8^2 \\
8(7 - x) &= 64x \\
56 - 8x &= 64x \\
64x + 8x &= 56 \\
72x &= 56 \\
x &= \frac{56}{72} \\
&= \frac{7}{9}
\end{aligned}$$

[5 marks]

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