Year 2 Pure Mathematics Examination Revision
Health Check N° 3

Answer 1

$$\frac{1}{1 + 2x} = (1 + 2x)^{-1}$$

(i) The Binomial Theorem

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$$

... with $n = -1$ and x replaced with $2x$ yields ...

$$(1 + 2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(2x)^3 + \ldots$$

$$= 1 - 2x + 4x^2 - 8x^3 + \ldots$$

[4 marks]

(ii) The series could be written as,

$$2^0 x^0 - 2^1 x^1 + 2^2 x^2 - 2^3 x^3 + \ldots$$

So the coefficient of x^8 will be $+\ 2^8 = 256$

[1 mark]

Answer 2

At a point of inflection $\dfrac{d^2y}{dx^2} = 0$

$$y = 5x^3 + ax^2 + 3x + 2$$

$$\frac{dy}{dx} = 15x^2 + 2ax + 3$$

$$\frac{d^2y}{dx^2} = 30x + 2a$$

We require $30x + 2a = 0$

$$2a = -30x$$

$$a = -15x$$

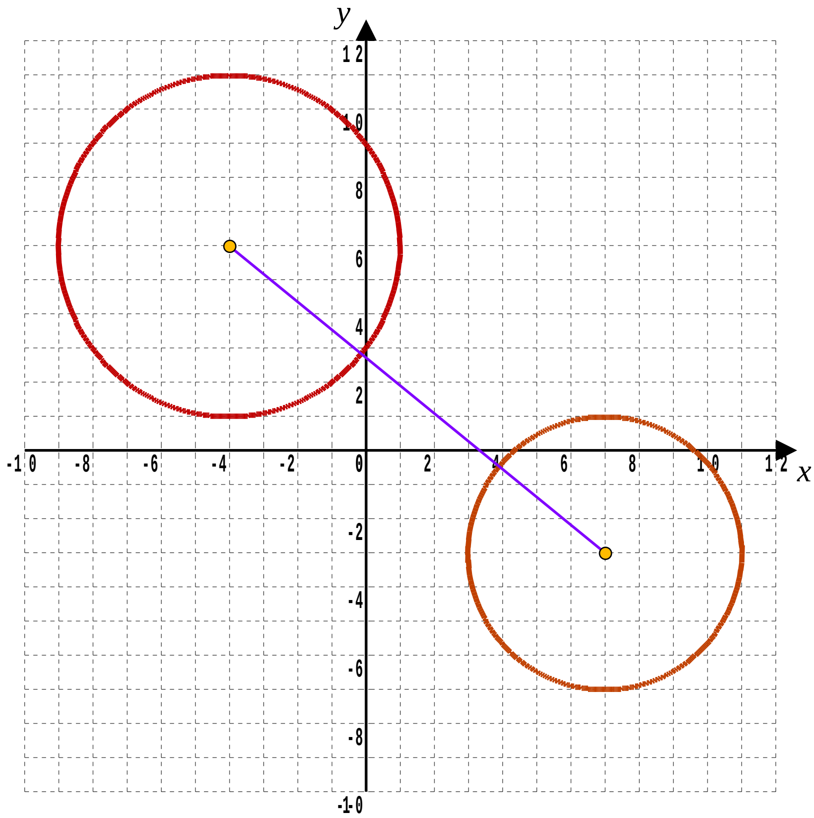
The point of inflection is at $x = \dfrac{1}{5}$ and so $a = -3$

[4 marks]

Answer 3

$$(x+4)^2 + (y-6)^2 = 25 \text{ has centre } (-4,\ 6),\ \text{radius } 5$$

$$(x-7)^2 + (y+3)^2 = 16 \text{ has centre } (7, -3),\ \text{radius } 4$$



The distance, D, between the two centres is,

$$\begin{aligned} D &= \sqrt{(\Delta x)^2 + (\Delta y)^2} \\ &= \sqrt{(-4-7)^2 + (6-(-3))^2} \\ &= \sqrt{(-11)^2 + 9^2} \\ &= \sqrt{121 + 81} \\ &= \sqrt{202} \end{aligned}$$

The minimum distance between the two circles is thus,

$$\begin{aligned} \sqrt{202} - 5 - 4 &= \sqrt{202} - 9 \\ &= 5.213 \ \text{(3 decimal places asked for)} \end{aligned}$$

[4 marks]

Answer 4

(i) (0, 0.4) and (12, 2.4)

$$m = \frac{\Delta Y}{\Delta X} = \frac{2.4 - 0.4}{12 - 0} = \frac{1}{6}$$

$$log_{10} N = \frac{1}{6} \times T + 0.4$$

[2 marks]

(ii) You may use a remembered result or work it out from scratch. Here's the 'from scratch' method;

$$log_{10} N = \frac{1}{6} \times T + 0.4$$

$$10^{log_{10} N} = 10^{\frac{1}{6} T + 0.4}$$

$$N = 10^{\frac{1}{6} T} \, 10^{0.4}$$

$$N = 2.512 \times 1.468^{T}$$

That is, $a = 2.512$ and $b = 1.468$

[4 marks]

(iii) a is the (theoretical) initial number of sick people
That is, the value of N when $T = 0$

[1 mark]

(iv) When $T = 21$,

$$N = 2.512 \times 1.468^{21}$$

$$= 7966 \text{ people}$$

Once an outbreak is detected, mitigation measures are likely to be implemented, slowing the rate of transmission. (for example)

[2 marks]

Answer 5

$$\boldsymbol{a} + \boldsymbol{b} = k (\boldsymbol{b} - \boldsymbol{c})$$

$$\begin{pmatrix} 8 \\ 23 \end{pmatrix} + \begin{pmatrix} -15 \\ x \end{pmatrix} = k \left(\begin{pmatrix} -15 \\ x \end{pmatrix} - \begin{pmatrix} -13 \\ 2 \end{pmatrix} \right)$$

$$\begin{pmatrix} -7 \\ 23 + x \end{pmatrix} = k \begin{pmatrix} -2 \\ x - 2 \end{pmatrix}$$

$$\therefore k = 3.5$$

$$23 + x = 3.5x - 7$$

$$2.5x = 30$$

$$x = 12$$

[4 marks]

Answer 6

(i)

$$p(x) = x^2, \qquad q(x) = 5 - 2x$$

$$pq(x) = qp(x)$$

$$p\left(5 - 2x\right) = q\left(x^2\right)$$

$$(5 - 2x)^2 = 5 - 2x^2$$

$$25 - 20x + 4x^2 = 5 - 2x^2$$

$$6x^2 - 20x + 20 = 0$$

divide through by 2

$$3x^2 - 10x + 10 = 0 \qquad \square$$

[3 marks]

(ii) Looking at the discriminant,

$$(-10)^2 - 4 \times 3 \times 10 = 100 - 120$$

$$= -20$$

As the discriminant is less than zero the quadratic equation

$$3x^2 - 10x + 10 = 0$$

has no real solutions ☐

[1 mark]