Year 2 Pure Mathematics Examination Revision
Health Check N° 4

Answer 1
To have distinct real roots, the discriminant must be positive.

$$\begin{aligned} D &> 0 \\ b^2 - 4ac &> 0 \\ 8^2 - 4 \times k \times 5 &> 0 \\ 64 - 20k &> 0 \\ -20k &> -64 \\ 20k &< 64 \\ k &< \frac{64}{20} \\ k &< 3.2 \end{aligned}$$

[3 marks]

Answer 2

$$\begin{aligned} f(x) &= p x^3 - 3p x^2 + x^2 - 4 \\ f'(x) &= 3p x^2 - 6p x + 2x \\ f''(x) &= 6px - 6p + 2 \\ \text{We're told that, } f''(2) &= -1 \\ 6p(2) - 6p + 2 &= -1 \\ 12p - 6p &= -3 \\ 6p &= -3 \\ p &= -\frac{1}{2} \end{aligned}$$

[3 marks]

Answer 3

(a) Use the (inverted) sine rule,

$$\frac{\sin DAO}{1.8} = \frac{\sin 0.84}{3.9}$$

$$\sin DAO = 0.3437$$

$$DAO = 0.351 \text{ radians}$$

[2 marks]

(b) As the three angles in a triangle add up to π radians,

$$ODA = \pi - 0.351 - 0.84$$

$$= 1.951 \text{ radians}$$

Again, using the sine rule,

$$\frac{AO}{\sin 1.951} = \frac{3.9}{\sin 0.84}$$

$$= 4.86$$

$$= 4.9 \text{ to one decimal place} \qquad \square$$

[3 marks]

(c)

$$Area\ Sector = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \times 1.8^2 \times (2\pi - 0.84)$$

$$= 8.82 \text{ m}^2$$

$$Area\ triangle = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 1.8 \times 4.9 \times \sin 0.84$$

$$= 3.28 \text{ m}^2$$

$$\therefore Area\ Shop\ Sign = 8.82 + 3.28$$

$$= 12.1 \text{ m}^2$$

[3 marks]

(d)

$$Perimeter = arc\ BCD + DA + AB$$

$$= 1.8 \times (2\pi - 0.84) + 3.9 + (4.9 - 1.8)$$

$$= 9.8 + 3.9 + 3.1$$

$$= 16.8 \text{ m}$$

[3 marks]

Answer 4

(i) This could be done using the Quotient Rule but that's using a sledgehammer to crack a nut.
Instead, use the Chain Rule like this...

$$y = \frac{1}{4x + 1}$$

$$= (4x + 1)^{-1}$$

$$\frac{dy}{dx} = (-1)(4x + 1)^{-2} \times 4$$

$$= -\frac{4}{(4x + 1)^2}$$

$$\therefore \text{ When } x = \frac{1}{4}, \quad \frac{dy}{dx} = -1$$

[3 marks]

(ii) When $x = \frac{1}{4}, y = \frac{1}{2}$ and so the tangent will be the straight line through the point $\left(\frac{1}{4}, \frac{1}{2} \right)$ with gradient -1

$$y = mx + c$$

$$\frac{1}{2} = (-1) \times \frac{1}{4} + c$$

$$c = \frac{3}{4}$$

and the tangent's equation is thus $y = -x + \frac{3}{4}$

[2 marks]

Answer 5

$$x = 3\, sin\, y$$

$$\frac{dx}{dy} = 3\, cos\, y$$

$$\text{But } cos^2 y + sin^2 y = 1$$

$$\frac{dx}{dy} = 3\sqrt{1 - sin^2 y}$$

$$= \sqrt{9}\sqrt{1 - sin^2 y}$$

$$= \sqrt{9 - (3\, sin\, y)^2}$$

$$= \sqrt{9 - x^2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{9 - x^2}} \qquad \square$$

[4 marks]

Answer 6

$$\int_1^2 11 - x - 2x^2 - 8x^{-3}\ dx = \left[\ 11x - \frac{x^2}{2} - \frac{2x^3}{3} - \frac{8x^{-2}}{(-2)}\ \right]_1^2$$

$$= \left[\ 11x - \frac{x^2}{2} - \frac{2x^3}{3} + \frac{4}{x^2}\right]_1^2$$

$$= \left[\ 22 - 2 - \frac{16}{3} + 1\ \right] - \left[\ 11 - \frac{1}{2} - \frac{2}{3} + 4\ \right]$$

$$= \left[\ \frac{47}{3}\right] - \left[\ \frac{83}{6}\ \right]$$

$$= \frac{11}{6}$$

[4 marks]