

Year 2 Pure Mathematics Examination Revision
Health Check N° 4

Answer 1

To have distinct real roots, the discriminant must be positive.

$$D > 0$$

$$b^2 - 4ac > 0$$

$$8^2 - 4 \times k \times 5 > 0$$

$$64 - 20k > 0$$

$$-20k > -64$$

$$20k < 64$$

$$k < \frac{64}{20}$$

$$k < 3.2$$

[3 marks]

Answer 2

$$f(x) = px^3 - 3px^2 + x^2 - 4$$

$$f'(x) = 3px^2 - 6px + 2x$$

$$f''(x) = 6px - 6p + 2$$

We're told that, $f''(2) = -1$

$$6p(2) - 6p + 2 = -1$$

$$12p - 6p = -3$$

$$6p = -3$$

$$p = -\frac{1}{2}$$

[3 marks]

Answer 3**(a)** Use the (inverted) sine rule,

$$\frac{\sin DAO}{1.8} = \frac{\sin 0.84}{3.9}$$

$$\sin DAO = 0.3437$$

$$DAO = 0.351 \text{ radians}$$

[2 marks]**(b)** As the three angles in a triangle add up to π radians,

$$ODA = \pi - 0.351 - 0.84$$

$$= 1.951 \text{ radians}$$

Again, using the sine rule,

$$\frac{AO}{\sin 1.951} = \frac{3.9}{\sin 0.84}$$

$$= 4.86$$

$$= 4.9 \text{ to one decimal place} \quad \square$$

[3 marks]

$$\begin{aligned} \text{(c)} \quad \text{Area Sector} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \times 1.8^2 \times (2\pi - 0.84) \\ &= 8.82 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area triangle} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 1.8 \times 4.9 \times \sin 0.84 \\ &= 3.28 \text{ m}^2 \end{aligned}$$

$$\therefore \text{Area Shop Sign} = 8.82 + 3.28$$

$$= 12.1 \text{ m}^2$$

[3 marks]

$$\text{(d)} \quad \text{Perimeter} = \text{arc } BCD + DA + AB$$

$$= 1.8 \times (2\pi - 0.84) + 3.9 + (4.9 - 1.8)$$

$$= 9.8 + 3.9 + 3.1$$

$$= 16.8 \text{ m}$$

[3 marks]

Answer 4

- (i) This could be done using the Quotient Rule but that's using a sledgehammer to crack a nut.
Instead, use the Chain Rule like this...

$$\begin{aligned}
 y &= \frac{1}{4x + 1} \\
 &= (4x + 1)^{-1} \\
 \frac{dy}{dx} &= (-1)(4x + 1)^{-2} \times 4 \\
 &= -\frac{4}{(4x + 1)^2} \\
 \therefore \text{When } x &= \frac{1}{4}, \quad \frac{dy}{dx} = -1
 \end{aligned}$$

[3 marks]

- (ii) When $x = \frac{1}{4}$, $y = \frac{1}{2}$ and so the tangent will be the straight line through the point $\left(\frac{1}{4}, \frac{1}{2}\right)$ with gradient -1

$$\begin{aligned}
 y &= mx + c \\
 \frac{1}{2} &= (-1) \times \frac{1}{4} + c \\
 c &= \frac{3}{4}
 \end{aligned}$$

and the tangent's equation is thus $y = -x + \frac{3}{4}$

[2 marks]**Answer 5**

$$\begin{aligned}
 x &= 3 \sin y \\
 \frac{dx}{dy} &= 3 \cos y \\
 \text{But } \cos^2 y + \sin^2 y &= 1 \\
 \frac{dx}{dy} &= 3 \sqrt{1 - \sin^2 y} \\
 &= \sqrt{9} \sqrt{1 - \sin^2 y} \\
 &= \sqrt{9 - (3 \sin y)^2} \\
 &= \sqrt{9 - x^2} \\
 \therefore \frac{dy}{dx} &= \frac{1}{\sqrt{9 - x^2}} \quad \square
 \end{aligned}$$

[4 marks]

Answer 6

$$\begin{aligned}\int_1^2 11 - x - 2x^2 - 8x^{-3} dx &= \left[11x - \frac{x^2}{2} - \frac{2x^3}{3} - \frac{8x^{-2}}{(-2)} \right]_1^2 \\&= \left[11x - \frac{x^2}{2} - \frac{2x^3}{3} + \frac{4}{x^2} \right]_1^2 \\&= \left[22 - 2 - \frac{16}{3} + 1 \right] - \left[11 - \frac{1}{2} - \frac{2}{3} + 4 \right] \\&= \left[\frac{47}{3} \right] - \left[\frac{83}{6} \right] \\&= \frac{11}{6}\end{aligned}$$

[4 marks]

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