Year 2 Pure Mathematics Examination Revision
Health Check N° 6

Answer 1

$$y = \frac{x^4}{\cos 3x}$$

Using the Quotient Rule...

$$\frac{dy}{dx} = \frac{(\cos 3x)\ 4x^3 - (-\sin 3x)(3)\ x^4}{(\cos 3x)^2}$$

$$= \frac{x^3(4\cos 3x + 3x\sin 3x)}{\cos^2 3x}$$

[4 marks]

Answer 2

(a)

$$Arc\ Length = r\theta$$

$$Arc\ DEA = 7 \times 2.1$$

$$= 14.7 \text{ cm}$$

[2 marks]

(b)

$$Angle\ CBD = \pi - 2.1$$

$$= 1.042 \text{ radians}$$

$$Perimeter\ ABCDEA = 7 + 7\cos 1.042 + 7\sin 1.042 + 14.7$$

$$= 31.3 \text{ cm} \quad \text{(to 1 decimal place asked for)}$$

[4 marks]

Answer 3

(a)

$$R\sin(x + \alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$$

Compare this with the expression given of $12\sin x + 5\cos x$

$$\Rightarrow \frac{R\sin\alpha}{R\cos\alpha} = \frac{5}{12}$$

$$\alpha = \arctan\left(\frac{5}{12}\right)$$

$$= 22.6°$$

$$R = \sqrt{5^2 + 12^2}$$

$$= 13$$

$$\therefore\ 12\sin x + 5\cos x = 13\sin(x + 22.6)$$

[4 marks]

(b) Notice that v is speed and not velocity (i.e. Can't have a negative speed)

$$v(x) = \frac{50}{12\,sin\left(\frac{2x}{5}\right)^{\circ} + 5\,cos\left(\frac{2x}{5}\right)^{\circ}}$$

$$= \frac{50}{13\,sin\left(\frac{2x}{5} + 22.6\right)} \qquad \text{using part (a)}$$

v will have a minimum value when the *sine* function, on the denominator, has it's maximum value of 1

$$\therefore\ v_{MIN} = \frac{50}{13} \qquad (\text{About } 3.85\text{ m s}^{-1})$$

[2 marks]

(c)

$$sin\left(\frac{2x}{5} + 22.6\right) = 1 \qquad 0 \leqslant x \leqslant 300$$

$$\frac{2x}{5} + 22.6 = 90$$

$$\frac{2x}{5} = 67.4$$

$$x = 168.5\text{ minutes}$$

[1 mark]

Answer 4

$$f(x) = \frac{32}{(3x+1)^2} + \frac{x}{9}$$

$$= 32\,(3x+1)^{-2} + \frac{x}{9}$$

(to avoid the unnecessarily complicated Quotient Rule method)

$$f'(x) = -64\,(3x+1)^{-3} \times 3 + \frac{1}{9}$$

$$= -\frac{192}{(3x+1)^3} + \frac{1}{9}$$

For the minimum, $f'(x) = 0$

$$\therefore \frac{1}{9} = \frac{192}{(3x+1)^3}$$

$$(3x+1)^3 = 1728$$

$$3x + 1 = 12$$

$$x = \frac{11}{3}$$

[4 marks]

Answer 5

(a)

$$\begin{aligned}
\text{LHS} &= \sin 2x - \tan x \\
&= 2\sin x\cos x - \frac{\sin x}{\cos x} \\
&= \frac{2\sin x\cos^2 x}{\cos x} - \frac{\sin x}{\cos x} \\
&= \frac{\sin x\left(2\cos^2 x - (1)\right)}{\cos x} \\
&= \frac{\sin x\left(2\cos^2 x - \left(\cos^2 x + \sin^2 x\right)\right)}{\cos x} \\
&= \frac{\sin x\left(\cos^2 x - \sin^2 x\right)}{\cos x} \\
&= \tan x\cos 2x \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[4 marks]

(b)

$$\begin{aligned}
\sin 2x - \tan x &= 3\tan x\sin x \\
\tan x\cos 2x &= 3\tan x\sin x
\end{aligned}$$

do not divide by $\tan x$ as this would be a division by zero

$$\begin{aligned}
\tan x\cos 2x - 3\tan x\sin x &= 0 \\
\tan x\left(\cos 2x - 3\sin x\right) &= 0
\end{aligned}$$

Either $\tan x = 0$ in which case $x = 0°,\ 180°$

$$\begin{aligned}
\text{Or}\ \cos 2x - 3\sin x &= 0 \\
\cos^2 x - \sin^2 x - 3\sin x &= 0 \\
1 - 2\sin^2 x - 3\sin x &= 0 \\
2\sin^2 x + 3\sin x - 1 &= 0
\end{aligned}$$

$$\begin{aligned}
\sin x &= \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-1)}}{2 \times 2} \\
&= \frac{-3 \pm \sqrt{17}}{4}
\end{aligned}$$

Either $x = 16.3°$ or $x = 163.7°$

$\therefore$ Solution Set is $x \in \left\{ 0°,\ 16.3°,\ 163.7°,\ 180° \right\}$

[5 marks]