Year 2 Pure Mathematics Examination Revision
Health Check N° 7

Answer 1

$$\frac{x^2 + 4x + 4}{y^2 - 6y + 9} \div \frac{x^2 - 4}{y^2 - 9} = \frac{(x+2)(x+2)}{(y-3)(y-3)} \times \frac{(y-3)(y+3)}{(x-2)(x+2)}$$
$$= \frac{(x+2)(y+3)}{(y-3)(x-2)}$$

[4 marks]

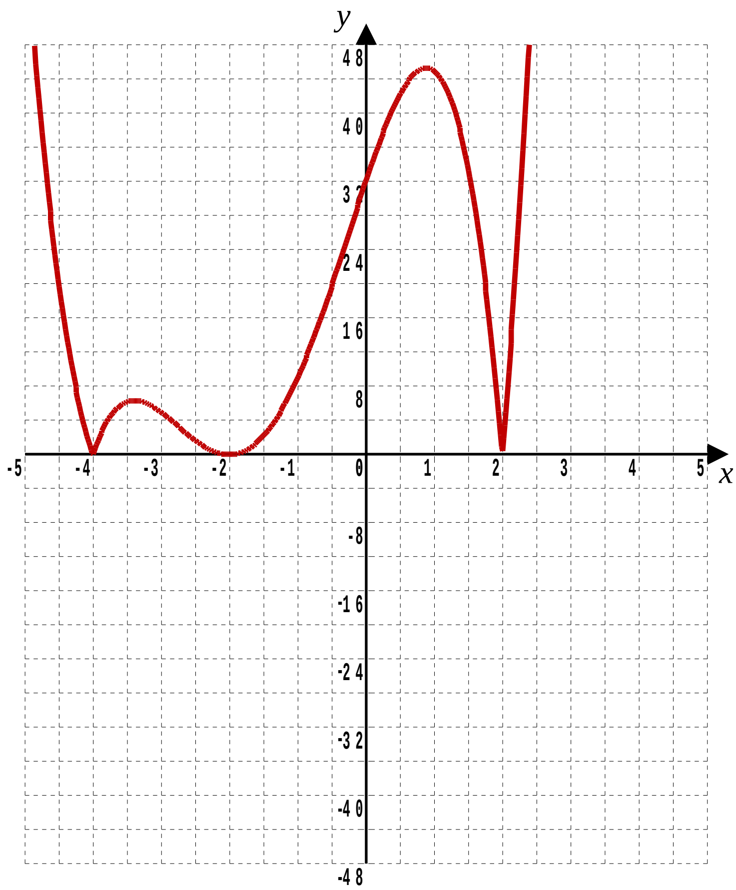
Answer 2

$$\sum_{r=3}^{7} 3\left(-\frac{1}{2}\right)^r = 3\sum_{r=3}^{7}\left(-\frac{1}{2}\right)^r$$
$$= 3\left(-\frac{1}{2^3} + \frac{1}{2^4} - \frac{1}{2^5} + \frac{1}{2^6} - \frac{1}{2^7}\right)$$
$$= 3\left(-\frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} - \frac{1}{128}\right)$$
$$= 3\left(-\frac{11}{128}\right) = -\frac{33}{128}$$

[4 marks]

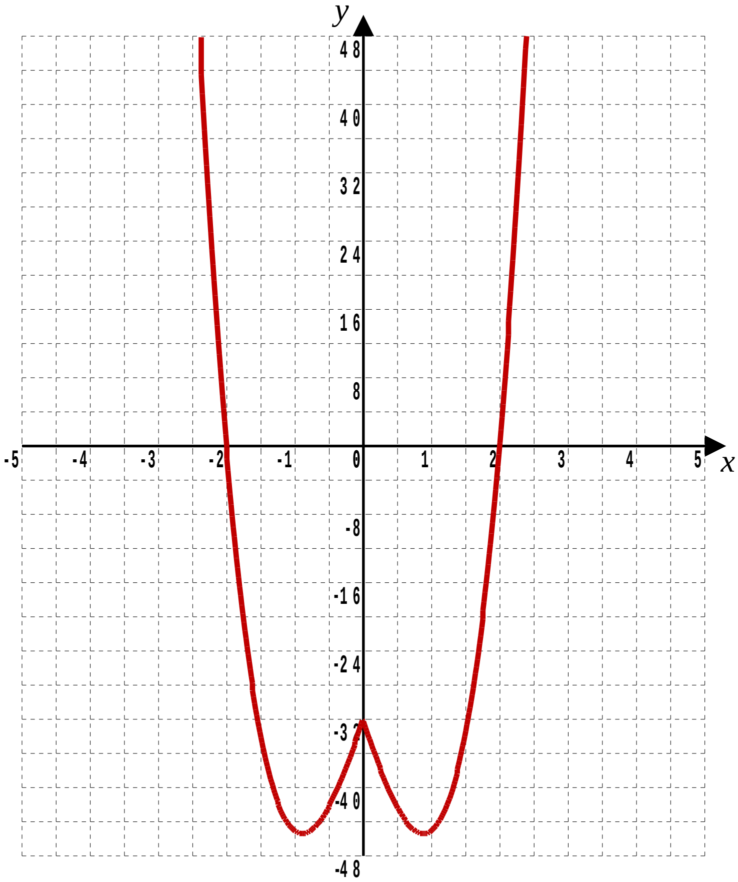
Answer 3

(i)



[3 marks]

(ii)



[3 marks]

Answer 4

The endpoints are likely to correspond to $t = \pm \dfrac{\pi}{2}$

(i) When $t = -\dfrac{\pi}{2}$, $x = 18 \times \left(-\dfrac{\pi}{2}\right) - 32\,sin\left(-\dfrac{\pi}{2}\right)$

$= -9\pi + 32$ (About 3.726)

and $y = 8 - 16\,cos\left(-\dfrac{\pi}{2}\right)$

$= 8$

When $t = \dfrac{\pi}{2}$, $x = 18 \times \left(\dfrac{\pi}{2}\right) - 32\,sin\left(\dfrac{\pi}{2}\right)$

$= 9\pi - 32$ (About – 3.726)

and $y = 8 - 16\,cos\left(-\dfrac{\pi}{2}\right)$

$= 8$

Thus, the width of the opening of the vase is given by,

$$-9\pi + 32 - 9\pi + 32 = 64 - 18\pi$$

$= 7.45$ cm (to 3 significant figures)

[4 marks]

(ii) $y = 0$ when $8 - 16\cos t = 0$

$$\cos t = \frac{8}{16}$$

$$t = \pm\frac{\pi}{3}$$

When $t = -\frac{\pi}{3}$, $x = 18 \times \left(-\frac{\pi}{3}\right) - 32\sin\left(-\frac{\pi}{3}\right)$

$= -6\pi + 16\sqrt{3}$ (About 8.863 cm)

and when $t = \frac{\pi}{3}$, $x = 18 \times \left(\frac{\pi}{3}\right) - 32\sin\left(\frac{\pi}{3}\right)$

$= 6\pi - 16\sqrt{3}$

Thus the radius of the vase at the surface of the water is 8.863 cm

[3 marks]

Answer 5

(i) The graph of $f(x)$ will cross the x-axis when $x^2 (2 - 5x)^5 = 0$

That is, at $(0, 0)$ and $\left(\frac{2}{5}, 0\right)$

[2 marks]

(ii) By the product rule, $f'(x) = x^2(5)(2 - 5x)^4(-5) + 2x(2 - 5x)^5$

$= -25x^2 (2 - 5x)^4 + 2x (2 - 5x)^5$

$= x(2 - 5x)^4(-25x + 4 - 10x)$

$= x(2 - 5x)^4(4 - 35x)$

[3 marks]

(iii) When $x = 0$, $f'(0) = 0 \times 2^4 \times 4$ $\Rightarrow f'(0) = 0$ ☐

When $x = \frac{2}{5}$, $f'\left(\frac{2}{5}\right) = \left(\frac{2}{5}\right) \times 0^4 \times (-10)$ $\Rightarrow f'\left(\frac{2}{5}\right) = 0$ ☐

The function is a polynomial of degree 7 with a negative coefficient for the x^7 term. So its graph must “enter” top left and “exit” bottom right.

Looking at $f(x) = x^2 (2 - 5x)^5$ the square indicates a tangent to the x-axis at the origin, so (0, 0) must be a minimum.

The power of 5 indicates a proper crossing of the x-axis but, as the point $\left(\frac{2}{5}, 0\right)$ has a gradient of zero that must be a point of inflection.

[4 marks]