

Year 2 Pure Mathematics Examination Revision

Health Check N° 8

Answer 1

$$2^{2x+1} - 17 \times 2^x + 2^3 = 0$$

$$2(2^x)^2 - 17(2^x) + 8 = 0$$

Let $y = 2^x$ in which case,

$$2y^2 - 17y + 8 = 0$$

$$(2y - 1)(y - 8) = 0$$

$$\text{Either } y = \frac{1}{2} \text{ or } y = 8$$

$$\text{That is, either } 2^x = \frac{1}{2} \text{ or } 2^x = 8$$

$$\therefore x = -1, 3$$

[4 marks]

Answer 2

$$3x^2 - y^3 - 5xy = 1$$

Differentiating implicitly gives,

$$6x - 3y^2 \frac{dy}{dx} - 5x \frac{dy}{dx} - 5y = 0$$

$$6x - 5y = \frac{dy}{dx} (3y^2 + 5x)$$

$$\frac{dy}{dx} = \frac{6x - 5y}{3y^2 + 5x}$$

$$\begin{aligned} \text{At the point } (2, 1), \quad \frac{dy}{dx} &= \frac{6 \times 2 - 5 \times 1}{3 \times 1^2 + 5 \times 2} \\ &= \frac{7}{13} \end{aligned}$$

[5 marks]

Answer 3

$$\begin{aligned} \text{(i)} \quad h &= 12 + \frac{9}{2} \sin\left(\frac{2\pi}{365} \times 25\right) \\ &= 13.877 \\ &= 13 \text{ hours, } 53 \text{ minutes} \end{aligned}$$

[2 marks]

$$\text{(ii)} \quad 16 \text{ hours, } 30 \text{ minutes}$$

[1 mark]

$$\begin{aligned}
 \text{(iii)} \quad \sin\left(\frac{2\pi}{365} \times d\right) &= 1 \\
 \left(\frac{2\pi}{365} \times d\right) &= \frac{\pi}{2} \\
 d &= \frac{\pi}{2} \times \frac{365}{2\pi} \\
 &= 91 \text{ days} \quad \text{That is, one season later}
 \end{aligned}$$

[1 mark]

$$\begin{aligned}
 \text{(iv)} \quad 12 + \frac{9}{2} \sin\left(\frac{2\pi}{365} \times d\right) &= 15 \\
 \frac{9}{2} \sin\left(\frac{2\pi}{365} \times d\right) &= 3 \\
 \sin\left(\frac{2\pi}{365} \times d\right) &= \frac{2}{3} \\
 \left(\frac{2\pi}{365} \times d\right) &= 0.729, 2.412 \\
 d &= 42.35, 140.11
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Days exceeding 15 hours of daylight} &= 140.11 - 42.35 \\
 &= 97.7 \\
 &= 97 \text{ days}
 \end{aligned}$$

[4 marks]

Answer 4

$$\begin{aligned}
 \text{(i)} \quad \text{LHS} &= x^2 + 18x + y^2 - 2y + 29 \\
 &= (-7)^2 + 18(-7) + (-6)^2 - 2(-6) + 29 \\
 &= 49 - 126 + 36 + 12 + 29 \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

$\therefore$ The point $(-7, -6)$ lies on C $\square$

[2 marks]

(ii) First, find the coordinates of the centre of the circle, A

$$\begin{aligned}
 x^2 + 18x + y^2 - 2y + 29 &= 0 \\
 [x^2 + 18x] + [y^2 - 2y] + 29 &= 0 \\
 [(x + 9)^2 - 81] + [(y - 1)^2 - 1] + 29 &= 0 \\
 (x + 9)^2 + (y - 1)^2 &= 53
 \end{aligned}$$

$\therefore$ The circle has centre $A(-9, 1)$ and radius $\sqrt{53}$

Secondly, get the gradient at P , either by using the fact that the tangent at P is perpendicular to the radius to P , or by implicitly differentiating the equation of the circle.

$$x^2 + 18x + y^2 - 2y + 29 = 0$$

$$2x + 18 + 2y \frac{dy}{dx} - 2 \frac{dy}{dx} = 0$$

Divide through by 2

$$x + 9 + y \frac{dy}{dx} - \frac{dy}{dx} = 0$$

$$x + 9 = \frac{dy}{dx} (1 - y)$$

$$\frac{dy}{dx} = \frac{x + 9}{1 - y}$$

$$= \frac{-7 + 9}{1 - (-6)} \text{ at } P(-7, -6)$$

$$= \frac{2}{7}$$

$$\therefore y = mx + c \Rightarrow -6 = \frac{2}{7}(-7) + c \Rightarrow c = -4$$

$$\therefore \text{Tangent to } C \text{ at } P \text{ is } y = \frac{2}{7}x - 4$$

(iii) We know that $A(-9, 1)$, $P(-7, -6)$ and $R(0, -4)$.

Also, there is a right angle at P in triangle APR where tangent meets radius.

Also, the length of AP is the radius of the circle, $\sqrt{53}$.

If the length of PR is found, the triangles area will be "half base times height".

$$\begin{aligned} |PR| &= \sqrt{(\Delta x)^2 + (\Delta y)^2} \\ &= \sqrt{(0 - 7)^2 + (-4 - (-6))^2} \\ &= \sqrt{49 + 4} \\ &= \sqrt{53} \end{aligned}$$

$$\begin{aligned} \therefore \text{Area } \triangle APR &= \frac{1}{2} \times \sqrt{53} \times \sqrt{53} \\ &= \frac{53}{2} \quad (\text{i.e. } 26.5) \end{aligned}$$

[4 marks]

Answer 5

(i) The question is telling us that,

$$ar = 4 \text{ and } ar^{15} = 9$$

As there is a $\frac{4}{9}$ in the result being aimed for, and no a ,

$$\frac{ar}{ar^{15}} = \frac{4}{9}$$

$$\frac{1}{r^{14}} = \frac{4}{9}$$

$$r^{-14} = \frac{4}{9}$$

$$\ln(r^{-14}) = \ln\left(\frac{4}{9}\right)$$

$$-14 \ln r - \ln\left(\frac{4}{9}\right) = 0$$

$$14 \ln r + \ln\left(\frac{4}{9}\right) = 0 \quad \square$$

[3 marks]

(ii) $14 \ln r + \ln\left(\frac{4}{9}\right) = 0$

$$\ln r = -\frac{1}{14} \ln\left(\frac{4}{9}\right)$$

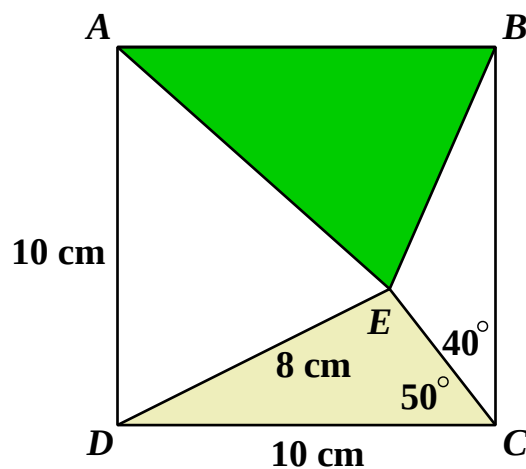
$$= 0.0579236$$

$$\therefore r = e^{0.0579236}$$

$$= 1.06 \text{ (3 sig fig asked for)}$$

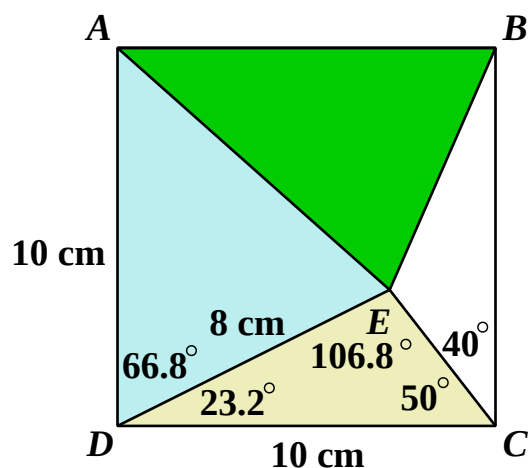
[3 marks]

Answer 6

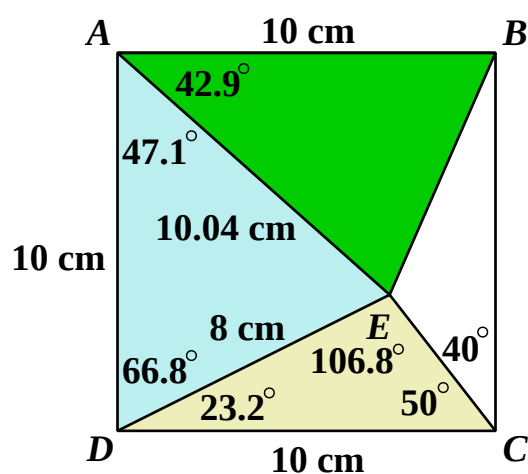


$$\text{In } \triangle CDE \quad \frac{\sin E}{10} = \frac{\sin 50}{8} \Rightarrow \sin E = 0.9576$$

$$\therefore E = 106.753^\circ \quad (\text{Reject } 73.247^\circ \text{ as told it's obtuse})$$



$$\begin{aligned}
 \text{In } \triangle AED \quad d^2 &= 10^2 + 8^2 - 2 \times 10 \times 8 \cos 66.753^\circ \\
 &= 100.85 \\
 d &= 10.04 \text{ cm} \\
 \cos A &= \frac{10^2 + 10.04^2 - 8^2}{2 \times 10 \times 10.04} \\
 &= 0.6813 \\
 A &= 47.1^\circ
 \end{aligned}$$



$$\begin{aligned}
 \text{In } \triangle ABE \quad \text{Area} &= \frac{1}{2} \times 10 \times 10.04 \times \sin 42.9^\circ \\
 &= 34.2 \text{ cm}^2
 \end{aligned}$$

[7 marks]

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