Year 2 Pure Mathematics Examination Revision
Health Check N° 8

Answer 1

$$2^{2x+1} - 17 \times 2^x + 2^3 = 0$$

$$2(2^x)^2 - 17(2^x) + 8 = 0$$

Let $y = 2^x$ in which case,

$$2y^2 - 17y + 8 = 0$$

$$(2y - 1)(y - 8) = 0$$

Either $y = \frac{1}{2}$ or $y = 8$

That is, either $2^x = \frac{1}{2}$ or $2^x = 8$

$$\therefore x = -1,\ 3$$

[4 marks]

Answer 2

$$3x^2 - y^3 - 5xy = 1$$

Differentiating implicitly gives,

$$6x - 3y^2\frac{dy}{dx} - 5x\frac{dy}{dx} - 5y = 0$$

$$6x - 5y = \frac{dy}{dx}\left(3y^2 + 5x\right)$$

$$\frac{dy}{dx} = \frac{6x - 5y}{3y^2 + 5x}$$

At the point $(2,\ 1)$, $\dfrac{dy}{dx} = \dfrac{6 \times 2 - 5 \times 1}{3 \times 1^2 + 5 \times 2}$

$$= \frac{7}{13}$$

[5 marks]

Answer 3

(i) $h = 12 + \frac{9}{2}\sin\left(\frac{2\pi}{365} \times 25\right)$

$= 13.877$

$= 13$ hours, 53 minutes

[2 marks]

(ii) 16 hours, 30 minutes

[1 mark]

(iii)

$$\sin\left(\frac{2\pi}{365} \times d\right) = 1$$

$$\left(\frac{2\pi}{365} \times d\right) = \frac{\pi}{2}$$

$$d = \frac{\pi}{2} \times \frac{365}{2\pi}$$

$$= 91 \text{ days} \quad \text{That is, one season later}$$

[1 mark]

(iv)

$$12 + \frac{9}{2}\sin\left(\frac{2\pi}{365} \times d\right) = 15$$

$$\frac{9}{2}\sin\left(\frac{2\pi}{365} \times d\right) = 3$$

$$\sin\left(\frac{2\pi}{365} \times d\right) = \frac{2}{3}$$

$$\left(\frac{2\pi}{365} \times d\right) = 0.729,\ \ 2.412$$

$$d = 42.35,\ \ 140.11$$

$$\therefore \text{ Days exceeding 15 hours of daylight } = 140.11 - 42.35$$

$$= 97.7$$

$$= 97 \text{ days}$$

[4 marks]

Answer 4

(i)

$$\text{LHS} = x^2 + 18x + y^2 - 2y + 29$$

$$= (-7)^2 + 18(-7) + (-6)^2 - 2(-6) + 29$$

$$= 49 - 126 + 36 + 12 + 29$$

$$= 0$$

$$= \text{RHS}$$

$\therefore$ The point $(-7, -6)$ lies on C ☐

[2 marks]

(ii) First, find the coordinates of the centre of the circle, A

$$x^2 + 18x + y^2 - 2y + 29 = 0$$

$$\left[x^2 + 18x\right] + \left[y^2 - 2y\right] + 29 = 0$$

$$\left[(x + 9)^2 - 81\right] + \left[(y - 1)^2 - 1\right] + 29 = 0$$

$$(x + 9)^2 + (y - 1)^2 = 53$$

$\therefore$ The circle has centre $A(-9,\ 1)$ and radius $\sqrt{53}$

Secondly, get the gradient at P, either by using the fact that the tangent at P is perpendicular to the radius to P, or by implicitly differentiating the equation of the circle.

$$x^2 + 18x + y^2 - 2y + 29 = 0$$

$$2x + 18 + 2y\frac{dy}{dx} - 2\frac{dy}{dx} = 0$$

Divide through by 2

$$x + 9 + y\frac{dy}{dx} - \frac{dy}{dx} = 0$$

$$x + 9 = \frac{dy}{dx}(1 - y)$$

$$\frac{dy}{dx} = \frac{x + 9}{1 - y}$$

$$= \frac{-7 + 9}{1 - (-6)} \quad \text{at } P(-7, -6)$$

$$= \frac{2}{7}$$

$$\therefore\ y = mx + c \Rightarrow -6 = \frac{2}{7}(-7) + c \Rightarrow c = -4$$

$$\therefore\ \text{Tangent to } C \text{ at } P \text{ is } y = \frac{2}{7}x - 4$$

(iii) We know that $A(-9,\ 1)$, $P(-7, -6)$ and $R(0, -4)$.

Also, there is a right angle at P in triangle APR where tangent meets radius.

Also, the length of AP is the radius of the circle, $\sqrt{53}$.

If the length of PR is found, the triangles area will be “half base times height”.

$$|PR| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$= \sqrt{(0 - 7)^2 + (-4 - (-6))^2}$$

$$= \sqrt{49 + 4}$$

$$= \sqrt{53}$$

$$\therefore\ Area\ \Delta APR = \frac{1}{2} \times \sqrt{53} \times \sqrt{53}$$

$$= \frac{53}{2} \qquad \text{(i.e. 26.5)}$$

[4 marks]

Answer 5

(i) The question is telling us that,

$$ar = 4 \text{ and } a\,r^{15} = 9$$

As there is a $\frac{4}{9}$ in the result being aimed for, and no a,

$$\frac{ar}{a\,r^{15}} = \frac{4}{9}$$

$$\frac{1}{r^{14}} = \frac{4}{9}$$

$$r^{-14} = \frac{4}{9}$$

$$ln\left(r^{-14}\right) = ln\left(\frac{4}{9}\right)$$

$$-14\,ln\,r - ln\left(\frac{4}{9}\right) = 0$$

$$14\,ln\,r + ln\left(\frac{4}{9}\right) = 0 \qquad \square$$

[3 marks]

(ii)

$$14\,ln\,r + ln\left(\frac{4}{9}\right) = 0$$

$$ln\,r = -\frac{1}{14}ln\left(\frac{4}{9}\right)$$

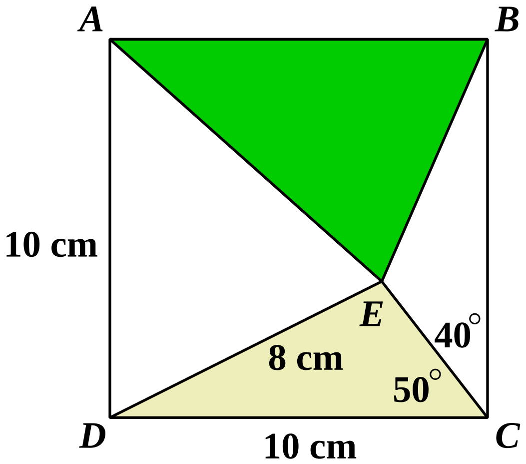
$$= 0.0579236$$

$$\therefore\ r = e^{0.0579236}$$

$$= 1.06 \ \text{(3 sig fig asked for)}$$

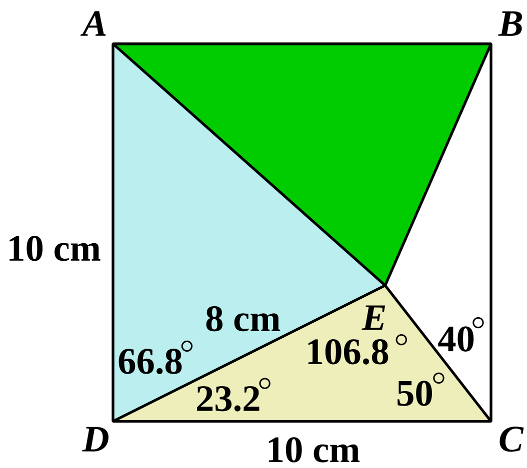
[3 marks]

Answer 6



In $\Delta CDE \qquad \frac{sin\,E}{10} = \frac{sin\,50}{8} \Rightarrow sin\,E = 0.9576$

$\therefore\ E = 106.753°$ (Reject 73.247° as told it's obtuse)



In $\triangle AED$ $\quad d^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \cos 66.753°$

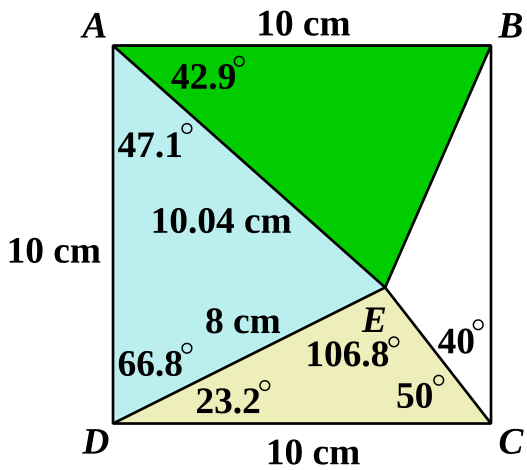
$$= 100.85$$

$$d = 10.04 \text{ cm}$$

$$\cos A = \frac{10^2 + 10.04^2 - 8^2}{2 \times 10 \times 10.04}$$

$$= 0.6813$$

$$A = 47.1°$$



In $\triangle ABE$ $\quad Area = \frac{1}{2} \times 10 \times 10.04 \times \sin 42.9°$

$$= 34.2 \text{ cm}^2$$

[7 marks]