Year 2 Pure Mathematics Examination Revision
Health Check N° 9

Answer 1

$$e^{x} + 12e^{-x} = 7$$

Multiply through by e^{x}

$$(e^{x})^{2} + 12 = 7(e^{x})$$

$$(e^{x})^{2} - 7(e^{x}) + 12 = 0$$

$$(e^{x} - 3)(e^{x} - 4) = 0$$

Either $e^{x} = 3$ or $e^{x} = 4$

$x = ln\,3$ or $x = ln\,4$

[4 marks]

Answer 2

The Binomial Theorem states that,

$$(1 + x)^{n} = 1 + nx + \frac{n(n-1)}{2!}x^{2} + \ldots$$

... with $n = \frac{1}{2}$ and x replaced with $-6x$ yields ...

$$(1 - 6x)^{\frac{1}{2}} =$$

$$1 + \frac{1}{2}(-6x) + \frac{1}{2!} \times \frac{1}{2} \times \left(-\frac{1}{2}\right)(-6x)^{2} + \frac{1}{3!} \times \frac{1}{2} \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)(-6x)^{3}$$

$$= 1 - 3x - \frac{9}{2}x^{2} - \frac{27}{2}x^{3}$$

[4 marks]

Answer 3

$$\begin{aligned}
\text{LHS} &= csc\,\theta\, sec^{2}\,\theta \\
&= \left(\frac{1}{\sin\theta}\right)(1 + tan^{2}\,\theta) \\
&= \left(\frac{1}{\sin\theta}\right)\left(1 + \frac{sin^{2}\,\theta}{cos^{2}\,\theta}\right) \\
&= \left(\frac{1}{\sin\theta}\right) + \left(\frac{\sin\theta}{cos^{2}\,\theta}\right) \\
&= csc\,\theta + \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta} \\
&= csc\,\theta + tan\,\theta\, sec\,\theta \\
&= \text{RHS} \quad \square
\end{aligned}$$

[4 marks]

Answer 4

(i)

$$f(x) = \left(x^2 - 4x\right)\left(e^{-x}\right)$$

$$f'(x) = \left(x^2 - 4x\right)\left(-e^{-x}\right) + \left(2x - 4\right)\left(e^{-x}\right)$$

$$f'(1) = \left(-3\right)\left(-e^{-1}\right) + \left(-2\right)\left(e^{-1}\right)$$

$$= \frac{3}{e} - \frac{2}{e}$$

$$= \frac{1}{e}$$

For the tangent, $y = mx + c$

$$-\frac{3}{e} = \frac{1}{e} \times 1 + c \Rightarrow c = -\frac{4}{e}$$

$$\therefore\ y = \frac{x}{e} - \frac{4}{e}$$

[5 marks]

(ii) The tangent will intersect the x-axis when $y = 0$,

$$x - 4 = 0 \Rightarrow x = 4 \Rightarrow A(4,\ 0)$$

For the normal, $y = mx + c$

$$-\frac{3}{e} = -e \times 1 + c \Rightarrow c = e - \frac{3}{e}$$

$$\therefore y = -ex + e - \frac{3}{e}$$

The normal will intersect the x-axis when $y = 0$,

$$ex = e - \frac{3}{e} \Rightarrow x = 1 - \frac{3}{e^2} \Rightarrow B\left(1 - \frac{3}{e^2},\ 0\right)$$

For $\triangle APB$,

$$Area = \frac{1}{2}\left(4 - 1 + \frac{3}{e^2}\right)\left(\frac{3}{e}\right)$$

$$= \frac{1}{2}\left(\frac{3e^2 + 3}{e^2}\right)\left(\frac{3}{e}\right)$$

$$= \frac{9\left(e^2 + 1\right)}{2e^3} \qquad \square$$

[8 marks]

Answer 5

(i) $a, \quad ar, \; a\,r^2, \; a\,r^3, \; a\,r^4, \; \ldots a\,r^{17}$

$1^{st}, \; 2^{nd}, \; 3^{rd}, \; 4^{th}, \; 5^{th}, \; \ldots 18^{th}$

The given sequence is in Geometric Progression with $a = 10, \; r = 2$

$$u_{18} = 10 \times 2^{17}$$

$$= £\,1\,310\,720$$

[2 marks]

(ii)

$$SUM_{GP} = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{10(2^{18} - 1)}{2 - 1}$$

$$= £\,2\,621\,430$$

[3 marks]