Year FM
Further Pure Mathematics Examination Revision
Health Check N° 1

Answer 1

(i) $\quad sinh\, 3x = 4\, sinh^3 x + 3\, sinh\, x$

[1 mark]

(ii) $\quad \text{RHS} = 4\, sinh^3 x + 3\, sinh\, x$

$$= 4\left(\frac{e^x - e^{-x}}{2}\right)^3 + 3\left(\frac{e^x - e^{-x}}{2}\right)$$

$$= \frac{1}{2}\left(e^x - e^{-x}\right)^3 + \frac{3}{2}\left(e^x - e^{-x}\right)$$

$$= \frac{1}{2}\left(e^{3x} - 3\, e^{2x} e^{-x} + 3\, e^x e^{-2x} - e^{-3x}\right) + \frac{3}{2}\left(e^x - e^{-x}\right)$$

$$= \frac{1}{2}\left(e^{3x} - 3\, e^x + 3\, e^{-x} - e^{-3x} + 3\, e^x - 3\, e^{-x}\right)$$

$$= \frac{e^{3x} - e^{-3x}}{2}$$

$$= sinh\, 3x$$

$$= \text{LHS} \qquad \square$$

[3 marks]

Answer 2

$$4x^3 - 12x^2 - x + 3 = 0$$

Divide through by 4

$$x^3 - 3x^2 - \frac{1}{4}x + \frac{3}{4} = 0$$

(i) $\quad (\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

(ii) $\quad \alpha + \beta + \gamma = 3$

(iii) $\quad \alpha\beta + \beta\gamma + \gamma\alpha = -\frac{1}{4}$

(iv) $\quad \alpha\beta\gamma = -\frac{3}{4}$

(v) $\quad \alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$$= (3)^2 - 2 \times \left(-\frac{1}{4}\right) = \frac{19}{2} \text{ or } 9.5$$

[1, 1, 1, 1, 3 marks]

Answer 3

$$x = \sin^4\theta\sqrt{\cos\theta}, \qquad y = \cos\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

$$V = \pi\int_{\theta=0}^{\theta=\frac{\pi}{2}} x^2\,\frac{dy}{d\theta}\,d\theta$$

$$= \pi\int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin^8\theta\cos\theta\,(-\sin\theta)\,d\theta$$

$$= (-1)\pi\int_{\theta=0}^{\theta=\frac{\pi}{2}} \cos\theta\,(\sin\theta)^9\,d\theta$$

$f'(x)\;[f(x)]^n$ "Chain Rule Backwards"

$$= (-1)\pi\left[\frac{(\sin\theta)^{10}}{10}\right]_0^{\frac{\pi}{2}}$$

$$= (-1)\pi\left[\frac{1}{10} - 0\right]$$

$$= \frac{\pi}{10}$$ as volume cannot be negative

[4 marks]

Answer 4

(a)
$$\sum_{r=1}^{n}\left(r^2 - r - 1\right) = \sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1$$

$$= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - n$$

$$= \frac{1}{6}n\left\{(n+1)(2n+1) - 3(n+1) - 6\right\}$$

$$= \frac{1}{6}n\left\{2n^2 + 3n + 1 - 3n - 3 - 6\right\}$$

$$= \frac{1}{6}n\left\{2n^2 - 8\right\}$$

$$= \frac{1}{3}n\left\{n^2 - 4\right\}$$ "Difference of two squares"

$$= \frac{1}{3}n(n-2)(n+2) \qquad \square$$

[4 marks]

$$\sum_{r=10}^{40}\left(r^2 - r - 1\right) = \sum_{r=1}^{40}\left(r^2 - r - 1\right) - \sum_{r=1}^{9}\left(r^2 - r - 1\right)$$

$$= \frac{1}{3}\times 40\times 38\times 42 - \frac{1}{3}\times 9\times 7\times 11$$

$$= 21049$$

[3 marks]

Answer 5

$$\begin{aligned}
\mathbf{AB} &= \mathbf{I} + 2\mathbf{A} \\
\mathbf{AB} - 2\mathbf{A} &= \mathbf{I} \\
\mathbf{A}\left(\mathbf{B} - 2\mathbf{I}\right) &= \mathbf{I} \\
\mathbf{A} &= \left(\mathbf{B} - 2\mathbf{I}\right)^{-1} \\
&= \left(\begin{pmatrix} 3 & -2 \\ -4 & 8 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \right)^{-1} \\
&= \begin{pmatrix} 1 & -2 \\ -4 & 6 \end{pmatrix}^{-1} \\
&= \frac{1}{1 \times 6 - (-4)(-2)} \begin{pmatrix} 6 & 2 \\ 4 & 1 \end{pmatrix} \\
&= \frac{1}{(-2)} \begin{pmatrix} 6 & 2 \\ 4 & 1 \end{pmatrix} \\
&= \begin{pmatrix} -3 & -1 \\ -2 & -0.5 \end{pmatrix}
\end{aligned}$$

[3 marks]