Year FM

Further Pure Mathematics Examination Revision

Health Check N° 2

Answer 1

(i)

$$\mathbf{A} = \begin{pmatrix} 3 & k & 0 \\ -2 & 1 & 2 \\ 5 & 0 & k+3 \end{pmatrix}$$

$$\begin{aligned} det\,\mathbf{A} &= 3\begin{vmatrix} 1 & 2 \\ 0 & k+3 \end{vmatrix} - k\begin{vmatrix} -2 & 2 \\ 5 & k+3 \end{vmatrix} + 0\begin{vmatrix} -2 & 1 \\ 5 & 0 \end{vmatrix} \\ &= 3\left(k+3\right) - k(-2k - 6 - 10) + 0 \\ &= 3k + 9 + 2k^2 + 16k \\ &= 2k^2 + 19k + 9 \end{aligned}$$

[2 marks]

(ii) If **A** is singular then its determinant will equal zero.

$$2k^2 + 19k + 9 = 0$$

$$(2k+1)(k+9) = 0$$

$$\text{Either } k = -\frac{1}{2} \text{ or } k = -9$$

[2 marks]

Answer 2

(a) As $f(z) = z^3 + 3z^2 + kz + 48$ and $f(4\mathrm{i}) = 0$ it follows that,

$$(4\mathrm{i})^3 + 3(4\mathrm{i})^2 + k(4\mathrm{i}) + 48 = 0$$

$$-64\mathrm{i} - 48 + 4k\mathrm{i} + 48 = 0$$

$$0 + 4\mathrm{i}(k - 16) = 0 + 0\mathrm{i}$$

$$\therefore\ k = 16$$

[2 marks]

(b) As the coefficients of the polynomial are all real, the two complex roots occur as a conjugate pair.

$\therefore\ z = 4\mathrm{i}$ is a root, along with $z = -4\mathrm{i}$

The product of the two known factors $(z - 4\mathrm{i})$ with $(z + 4\mathrm{i})$ is $z^2 + 16$

and it's now obvious that the third factor is $(z + 3)$

Thus the three roots are, 4i, – 4i, – 3

[3 marks]

Answer 3

Method 1 : Core 1 Method

$$\frac{\cos 2x + \mathrm{i}\sin 2x}{\cos 9x - \mathrm{i}\sin 9x}$$

$$= \frac{\cos 2x + \mathrm{i}\sin 2x}{\cos 9x - \mathrm{i}\sin 9x} \times \frac{\cos 9x + \mathrm{i}\sin 9x}{\cos 9x + \mathrm{i}\sin 9x}$$

$$= \frac{\cos 2x\cos 9x - \sin 2x\sin 9x + \mathrm{i}(\sin 2x\cos 9x + \cos 2x\sin 9x)}{\cos^2 9x + \sin^2 9x}$$

$$= \cos(2x + 9x) + \mathrm{i}(\sin(2x + 9x))$$

$$= \cos 11x + \mathrm{i}\sin 11x$$

Method 2 : Core 2 Method

$$\frac{\cos 2x + \mathrm{i}\sin 2x}{\cos 9x - \mathrm{i}\sin 9x} = \frac{e^{\mathrm{i}2x}}{e^{-\mathrm{i}9x}} \text{ using Euler's Relation}$$

$$= e^{\mathrm{i}11x}$$

$$= \cos 11x + \mathrm{i}\sin 11x$$

[4 marks]

Answer 4

(a)

$$\sum_{r=1}^{n} r^2(r - 1) = \sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r^2$$

$$= \frac{1}{4}n^2(n + 1)^2 - \frac{1}{6}n(n + 1)(2n + 1)$$

$$= \frac{1}{12}n(n + 1)(3n(n + 1) - 2(2n + 1))$$

$$= \frac{1}{12}n(n + 1)(3n^2 + 3n - 4n - 2)$$

$$= \frac{1}{12}n(n + 1)(3n^2 - n - 2)$$

$$\therefore\ p = 3,\ q = -1 \text{ and } r = -2$$

[4 marks]

(b)

$$\sum_{r=50}^{100} r^2(r - 1) = \sum_{r=1}^{100} r^2(r - 1) - \sum_{r=1}^{49} r^2(r - 1)$$

$$= \frac{1}{12} \times 100 \times 101 \times 29898 - \frac{1}{12} \times 49 \times 50 \times 7152$$

$$= 25164150 - 1460200$$

$$= 23703950$$

[2 marks]

Answer 5

(i) 24 mm

[1 mark]

(ii) Get the points of intersection of the two curves,

$$12 - x^2 = 8 - 0.2\,x^2$$
$$4 = 0.8\,x^2$$
$$x = \pm\sqrt{5} \qquad \text{(About 2.236 mm)}$$

$$Volume = 2 \times \pi \int_0^{2.236} \left(12 - x^2\right)^2 - \left(8 - 0.2\,x^2\right)^2 \, dx$$
$$= 2\pi \int_0^{2.236} 144 - 24\,x^2 + x^4 - 64 + 3.2\,x^2 - 0.04\,x^4 \, dx$$
$$= 2\pi \int_0^{2.236} 80 - 20.8\,x^2 + 0.96\,x^4 \, dx$$
$$= 2\pi \left[80x - 6.93\,x^3 + 0.192\,x^5 \right]_0^{2.236}$$
$$= 2\pi \left[178.9 - 77.5 + 10.7 \right]$$
$$= 704.3 \text{ mm}^3$$

$$mass = density \times volume$$
$$= 19.3 \times \left(\frac{704.3}{1000}\right)$$
$$= 13.6\,g$$

[9 marks]

(iii) Air bubble within the metal
Not “pure” gold
An engraving may have removed some gold

[1 mark]