Year FM
Further Pure Mathematics Examination Revision
Health Check N° 4

Answer 1

$$2\,tanh\,x - 1 = 0$$

$$\frac{2\,e^{2x} - 2}{e^{2x} + 1} - 1 = 0$$

Multiply through by $e^{2x} + 1$

$$2\,e^{2x} - 2 - e^{2x} - 1 = 0$$

$$e^{2x} = 3$$

$$2x = ln\,3$$

$$x = \frac{1}{2}\,ln\,3$$

[4 marks]

Answer 2

(i) $y = sin\,x\,cosh\,x$

$$\frac{dy}{dx} = sin\,x\,sinh\,x + cos\,x\,cosh\,x$$

$$\frac{d^2y}{dx^2} = sin\,x\,cosh\,x + cos\,x\,sinh\,x + cos\,x\,sinh\,x - sin\,x\,cosh\,x$$

$$= 2\,cos\,x\,sinh\,x$$

$$\frac{d^3y}{dx^3} = 2\,cos\,x\,cosh\,x - 2\,sin\,x\,sinh\,x$$

$$\frac{d^4y}{dx^4} = 2\,cos\,x\,sinh\,x - 2\,sin\,x\,cosh\,x - 2\,sin\,x\,cosh\,x - 2\,cos\,x\,sinh\,x$$

$$= -4\,sin\,x\,cosh\,x$$

$$= -4\,y \qquad \square$$

[4 marks]

(ii) Using $f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}\,x^2 + \ldots + \frac{f^{(r)}(0)}{r!}\,x^r + \ldots$

$$y = 0 + x + 0 + \frac{2}{3!}\,x^3 + 0 - \frac{4}{5!}\,x^5$$

$$= x + \frac{1}{3}\,x^3 - \frac{1}{30}\,x^5$$

[4 marks]

Answer 3

(i) To be on the initial line, $r = a(3 + 2\cos(0))$

$$= 5a \qquad \Rightarrow \quad A(5a,\ 0)$$

$$r = a(5 - 2\cos(0))$$

$$= 3a \qquad \Rightarrow \quad B(3a,\ 0)$$

[2 marks]

(ii) The curves intersect when $a(3 + 2\cos\theta) = a(5 - 2\cos\theta)$

$$4\cos\theta = 2$$

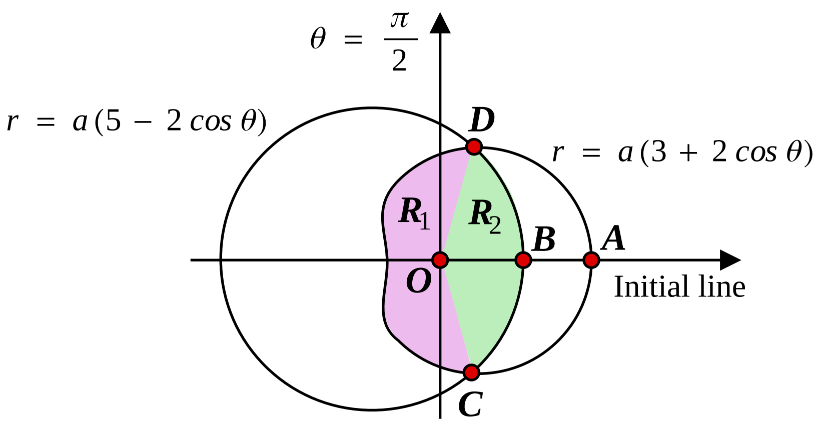
$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3},\ \frac{5\pi}{3}$$

$$\therefore\ C\left(4a,\ \frac{5\pi}{3}\right) \text{ and } D\left(4a,\ \frac{\pi}{3}\right)$$

[4 marks]

(iii)



$$R_1 = 2 \times \frac{1}{2}\int_{\frac{\pi}{3}}^{\pi} r^2\, d\theta = \int_{\frac{\pi}{3}}^{\pi} a^2\, (3 + 2\cos\theta)^2\, d\theta$$

$$= a^2\int_{\frac{\pi}{3}}^{\pi}\left(9 + 12\cos\theta + 4\cos^2\theta\right) d\theta$$

$$= a^2\int_{\frac{\pi}{3}}^{\pi}\left(9 + 12\cos\theta + 4\left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right)\right) d\theta$$

$$= a^2\int_{\frac{\pi}{3}}^{\pi}\left(11 + 12\cos\theta + 2\cos 2\theta\right) d\theta$$

$$= a^2\left[\,11\theta + 12\sin\theta + \sin 2\theta\,\right]_{\frac{\pi}{3}}^{\pi}$$

$$= a^2\left(\frac{22\pi}{3} - \frac{13\sqrt{3}}{2}\right)$$

$$R_2 = 2 \times \frac{1}{2} \int_0^{\frac{\pi}{3}} r^2 \, d\theta = \int_0^{\frac{\pi}{3}} a^2 \left(5 - 2 \cos \theta \right)^2 \, d\theta$$

$$= a^2 \int_0^{\frac{\pi}{3}} \left(25 - 20 \cos \theta + 4 \cos^2 \theta \right) d\theta$$

$$= a^2 \int_0^{\frac{\pi}{3}} \left(25 - 20 \cos \theta + 4 \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta \right) \right) d\theta$$

$$= a^2 \int_0^{\frac{\pi}{3}} \left(27 - 20 \cos \theta + 2 \cos 2\theta \right) d\theta$$

$$= a^2 \left[27\theta - 20 \sin \theta + \sin 2\theta \right]_0^{\frac{\pi}{3}}$$

$$= a^2 \left(\frac{27\pi}{3} - \frac{19\sqrt{3}}{2} \right)$$

$$R_1 + R_2 = a^2 \left(\frac{22\pi}{3} - \frac{13\sqrt{3}}{2} \right) + a^2 \left(\frac{27\pi}{3} - \frac{19\sqrt{3}}{2} \right)$$

$$= \frac{a^2}{3} \left(49\pi - 48\sqrt{3} \right)$$

[6 marks]

Answer 4

(i)

$$\begin{vmatrix} 1 & 1 & -1 \\ 3 & -1 & 2 \\ 5 & 1 & 0 \end{vmatrix} = 1\begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 3 & 2 \\ 5 & 0 \end{vmatrix} - 1\begin{vmatrix} 3 & -1 \\ 5 & 1 \end{vmatrix}$$

$$= 1(-1 \times 0 - 1 \times 2) - (3 \times 0 - 5 \times 2) - 1(3 \times 1 - 5(-1))$$

$$= -2 + 10 - 8 = 0 \qquad \square$$

[2 marks]

(ii)

$$2x + 2y - 2z = 4 \qquad (2 \times \text{Equation 1})$$

Add $\quad 3x - y + 2z = 5 \qquad$ (Equation 2)

$$5x + y = 9 \qquad \text{Which is Equation 3} \qquad \square$$

[2 marks]

(iii) For the point (0, 9, 7),

LHS $= x + y - z = 0 + 9 - 7 = 2 =$ RHS $\quad \therefore$ Eq 1 satisfied

LHS $= 3x - y + 2z = 0 - 9 + 14 = 5 =$ RHS $\quad \therefore$ Eq 2 satisfied

LHS $= 5x + y = 0 + 9 = 9 =$ RHS $\quad \therefore$ Eq 3 satisfied

Thus (0, 9, 7) is on all three planes $\quad \square$

For the point (1, 4, 3),

LHS $= x + y - z = 1 + 4 - 3 = 2 =$ RHS $\quad \therefore$ Eq 1 satisfied

LHS $= 3x - y + 2z = 3 - 4 + 6 = 5 =$ RHS $\quad \therefore$ Eq 2 satisfied

LHS $= 5x + y = 5 + 4 = 9 =$ RHS $\quad \therefore$ Eq 3 satisfied

Thus (1, 4, 3) is on all three planes $\quad \square$

$$r = \begin{pmatrix} 0 \\ 9 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 0 - 1 \\ 9 - 4 \\ 7 - 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 9 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix}$$ (Or an equivalent version of this)

[4 marks]