Answers to Grade Grabber 9

Answer 1

(a) **(i)** $E' = \{ 1, 3, 5, 7, 9, 11, ... \}$
$= \{$ the odd numbers $\}$

[1 mark]

(ii) $P \cap E = \{ 2 \}$

[1 mark]

(iii) $n(P \cap E) = 1$

[1 mark]

(iv) $E \cup E' = \varepsilon$
$= \{$ the positive integers $\}$

[1 mark]

(v) $P \cap P' = \varnothing$

[1 mark]

(b) Is $28577463968538 \in E$? Yes

[1 mark]

(c) Is $9 \in P$? No

[1 mark]

(d) Is $E' \subset P$? No

[1 mark]

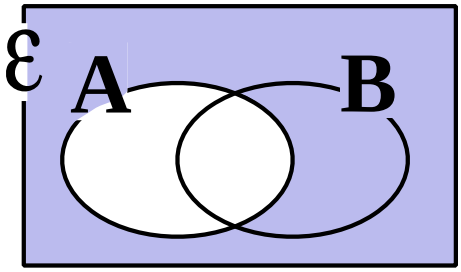
(e) Is $27 \notin P$? Yes

[1 mark]

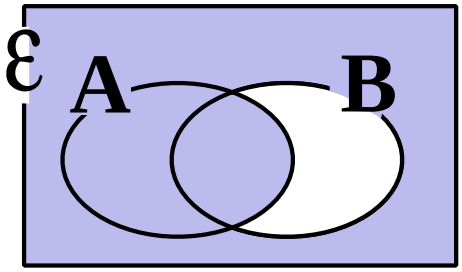
(f) Is it true that the set {factors of 15 } $\not\subset E$? Yes

[1 mark]

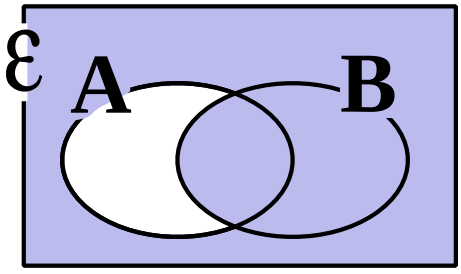
Answer 2



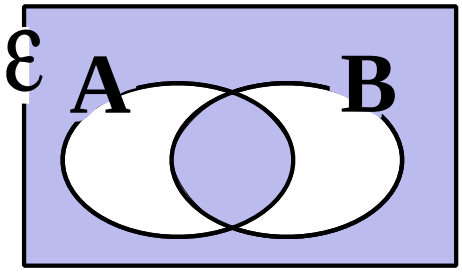
(i) A'



(ii) $A \cup B'$



(iii) $A' \cup B$



(iv) $(A \cup B') \cap (A' \cup B)$

[4 marks]

Answer 3

(i) $(2x - 3)(7x - 5) = 14x^2 - 31x + 15$

[2 marks]

(ii) $(3 + \sqrt{2})(3 - \sqrt{2}) = 7$ using a difference of two squares

[2 marks]

(iii) $(2x^2)^3 = 8x^6$

[2 marks]

Question 4

$$y = 5x^3 + \frac{1}{x^3} + 2x + \frac{1}{4}$$

$$= 5x^3 + x^{-3} + 2x + \frac{1}{4}$$

$$\frac{dy}{dx} = 15x^2 - 3x^{-4} + 2$$

$$= 15x^2 - \frac{3}{x^4} + 2$$

[4 marks]

Answer 5

(i)

$$m = \frac{\Delta Y}{\Delta X}$$

$$= \frac{11 - 6}{12 - 2}$$

$$= \frac{5}{10}$$

$$= \frac{1}{2}$$

[2 marks]

(ii)

$$y = mx + c$$

$$6 = \frac{1}{2} \times 2 + c$$

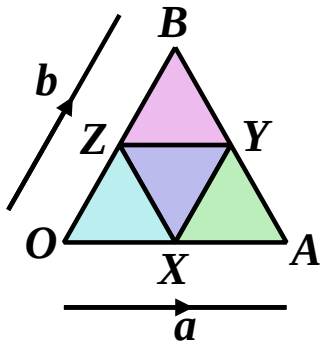
$$6 = 1 + c$$

$$c = 5$$

$$\therefore y = \frac{1}{2}x + 5$$

[2 marks]

Answer 6



(i) $\overrightarrow{OX} = \frac{1}{2}\boldsymbol{a}$ **(ii)** $\overrightarrow{AX} = -\frac{1}{2}\boldsymbol{a}$ **(iii)** $\overrightarrow{ZB} = \frac{1}{2}\boldsymbol{b}$

(iv) $\overrightarrow{YZ} = -\frac{1}{2}\boldsymbol{a}$ **(v)** $\overrightarrow{ZO} = -\frac{1}{2}\boldsymbol{b}$ **(vi)** $\overrightarrow{BA} = \boldsymbol{a} - \boldsymbol{b}$

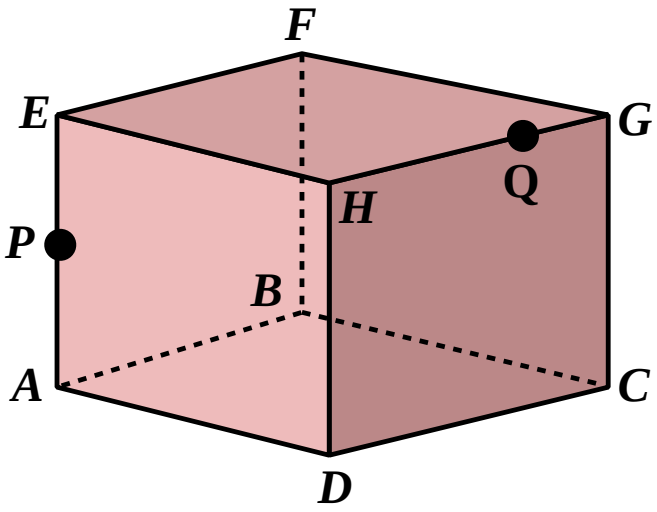
(vii) $\overrightarrow{ZX} = \frac{1}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$

(viii) $\overrightarrow{OY} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

(ix) $\overrightarrow{AZ} = -\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

[9 marks]

Answer 7



(i) $|EQ| = \sqrt{4^2 + 3^2}$ $|PD| = \sqrt{2^2 + 4^2}$

$= 5$ cm $= 2\sqrt{5}$ cm (About 4.472 cm)

[2 marks]

(ii) $|PQ| = \sqrt{2^2 + 5^2}$

$= \sqrt{29}$ cm (About 5.385 cm)

[1 mark]

(iii) the angle between PQ and the plane $EFGH$ is $\angle PQE$

$$\angle PQE = \arctan\left(\frac{2}{\sqrt{29}}\right)$$

$$= 20.4°$$

[2 marks]

(iv) For angle QPD first use Pythagoras' Theorem, then the Cosine Rule backwards on triangle QPD

$$|QD| = \sqrt{4^2 + 3^2}$$

$$= 5 \text{ cm}$$

$$\cos P = \frac{q^2 + d^2 - p^2}{2qd}$$

$$= \frac{(2\sqrt{5})^2 + (\sqrt{29})^2 - 5^2}{2 \times 2\sqrt{5} \times \sqrt{29}}$$

$$= \frac{20 + 29 - 25}{4\sqrt{145}}$$

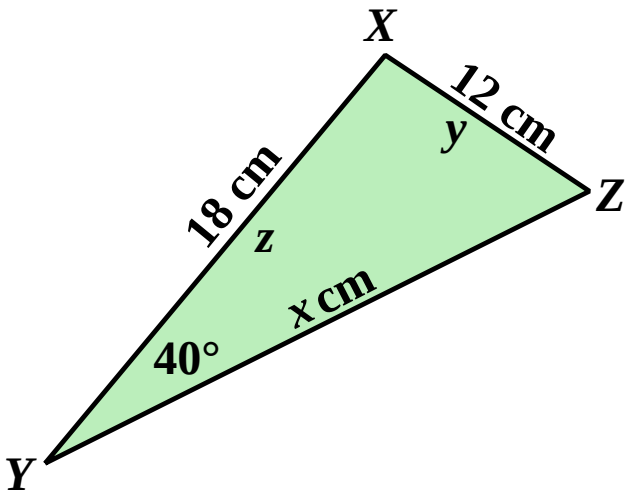
$$= 0.4983$$

$$P = 60.1°$$

[3 marks]

Answer 8

Label the triangle *XYZ*...



(i) By the inverted Sine Rule,

$$\frac{\sin Z}{18} = \frac{\sin 40}{12}$$

$$\sin Z = \frac{18 \sin 40}{12}$$

$$= 0.96418$$

$$Z = 74.62°$$

As the angles in a triangle sum to 180°,

$$\therefore X = 180 - 40 - 74.62$$

$$= 65.38°$$

Now to use the Sine Rule,

$$\frac{x}{\sin 65.38} = \frac{12}{\sin 40}$$

$$x = \frac{12 \sin 65.38}{\sin 40}$$

$$= 16.97 \text{ cm}$$

[3 marks]

(ii) Taking care to use the included angle,

$$Area \triangle = \frac{1}{2} x z \sin Y$$

$$= \frac{1}{2} \times 16.97 \times 18 \times \sin 40°$$

$$= 98.2 \text{ cm}^2$$

[2 marks]