

Answers to Grade Grabber 9

Answer 1

(a) (i) $E' = \{ 1, 3, 5, 7, 9, 11, \dots \}$
 $= \{ \text{the odd numbers} \}$

[1 mark]

(ii) $P \cap E = \{ 2 \}$

[1 mark]

(iii) $n(P \cap E) = 1$

[1 mark]

(iv) $E \cup E' = \varepsilon$
 $= \{ \text{the positive integers} \}$

[1 mark]

(v) $P \cap P' = \emptyset$

[1 mark]

(b) Is $28577463968538 \in E$? Yes

[1 mark]

(c) Is $9 \in P$? No

[1 mark]

(d) Is $E' \subset P$? No

[1 mark]

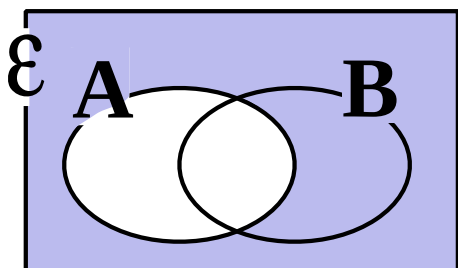
(e) Is $27 \notin P$? Yes

[1 mark]

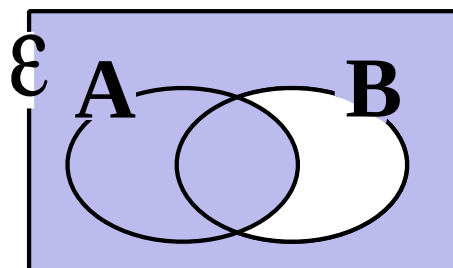
(f) Is it true that the set $\{ \text{factors of } 15 \} \not\subset E$? Yes

[1 mark]

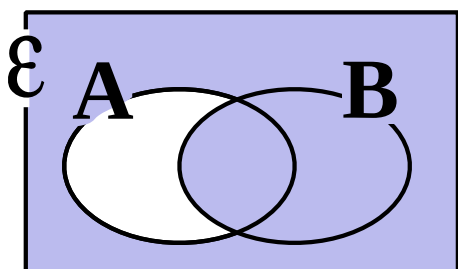
Answer 2



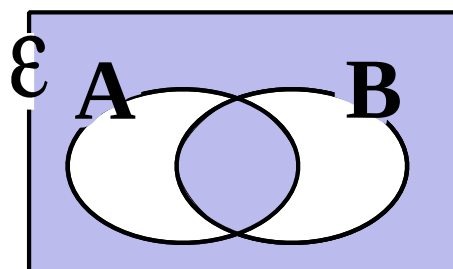
(i) A'



(ii) $A \cup B'$



(iii) $A' \cup B$



(iv) $(A \cup B') \cap (A' \cup B)$

[4 marks]

Answer 3

(i) $(2x - 3)(7x - 5) = 14x^2 - 31x + 15$

[2 marks]

(ii) $(3 + \sqrt{2})(3 - \sqrt{2}) = 7$ using a difference of two squares

[2 marks]

(iii) $(2x^2)^3 = 8x^6$

[2 marks]

Question 4

$$\begin{aligned} y &= 5x^3 + \frac{1}{x^3} + 2x + \frac{1}{4} \\ &= 5x^3 + x^{-3} + 2x + \frac{1}{4} \\ \frac{dy}{dx} &= 15x^2 - 3x^{-4} + 2 \\ &= 15x^2 - \frac{3}{x^4} + 2 \end{aligned}$$

[4 marks]

Answer 5

(i)

$$\begin{aligned} m &= \frac{\Delta Y}{\Delta X} \\ &= \frac{11 - 6}{12 - 2} \\ &= \frac{5}{10} \\ &= \frac{1}{2} \end{aligned}$$

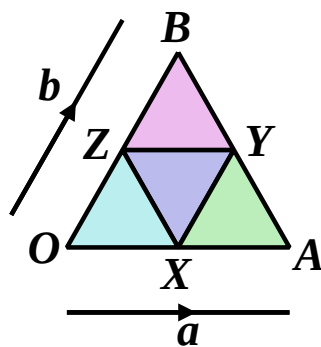
[2 marks]

(ii)

$$\begin{aligned} y &= mx + c \\ 6 &= \frac{1}{2} \times 2 + c \\ 6 &= 1 + c \\ c &= 5 \\ \therefore y &= \frac{1}{2}x + 5 \end{aligned}$$

[2 marks]

Answer 6



(i) $\overrightarrow{OX} = \frac{1}{2} \mathbf{a}$

(ii) $\overrightarrow{AX} = -\frac{1}{2} \mathbf{a}$

(iii) $\overrightarrow{ZB} = \frac{1}{2} \mathbf{b}$

(iv) $\overrightarrow{YZ} = -\frac{1}{2} \mathbf{a}$

(v) $\overrightarrow{ZO} = -\frac{1}{2} \mathbf{b}$

(vi) $\overrightarrow{BA} = \mathbf{a} - \mathbf{b}$

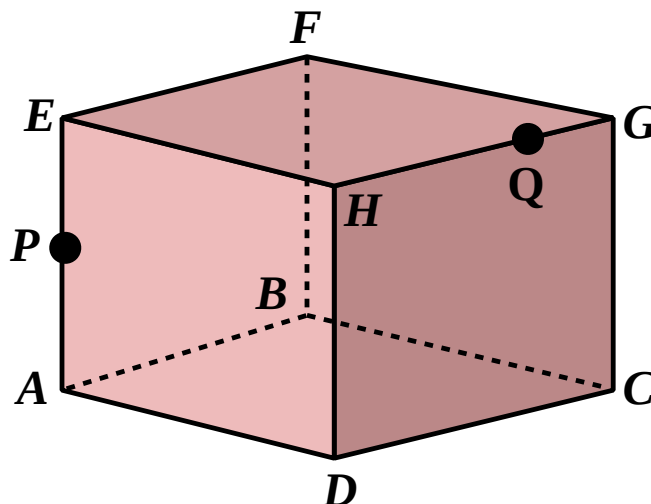
(vii) $\overrightarrow{ZX} = \frac{1}{2} \mathbf{a} - \frac{1}{2} \mathbf{b}$

(viii) $\overrightarrow{OY} = \frac{1}{2} \mathbf{a} + \frac{1}{2} \mathbf{b}$

(ix) $\overrightarrow{AZ} = -\mathbf{a} + \frac{1}{2} \mathbf{b}$

[9 marks]

Answer 7



(i) $|EQ| = \sqrt{4^2 + 3^2} = 5 \text{ cm}$ $|PD| = \sqrt{2^2 + 4^2} = 2\sqrt{5} \text{ cm}$ (About 4.472 cm) [2 marks]

(ii) $|PQ| = \sqrt{2^2 + 5^2} = \sqrt{29} \text{ cm}$ (About 5.385 cm) [1 mark]

(iii) the angle between PQ and the plane $EFGH$ is $\angle PQE$
 $\angle PQE = \arctan\left(\frac{2}{\sqrt{29}}\right) = 20.4^\circ$ [2 marks]

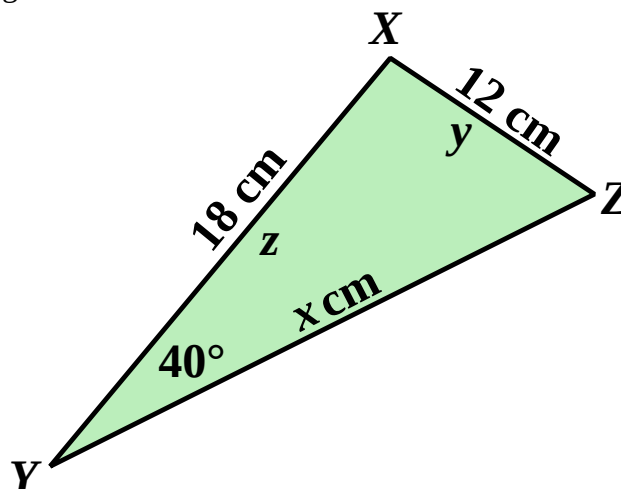
(iv) For angle QPD first use Pythagoras' Theorem, then the Cosine Rule backwards on triangle QPD

$$\begin{aligned} |QD| &= \sqrt{4^2 + 3^2} = 5 \text{ cm} \\ \cos P &= \frac{q^2 + d^2 - p^2}{2qd} \\ &= \frac{(2\sqrt{5})^2 + (\sqrt{29})^2 - 5^2}{2 \times 2\sqrt{5} \times \sqrt{29}} \\ &= \frac{20 + 29 - 25}{4\sqrt{145}} \\ &= 0.4983 \\ P &= 60.1^\circ \end{aligned}$$

[3 marks]

Answer 8

Label the triangle XYZ...



- (i) By the inverted Sine Rule,

$$\begin{aligned}\frac{\sin Z}{18} &= \frac{\sin 40}{12} \\ \sin Z &= \frac{18 \sin 40}{12} \\ &= 0.96418 \\ Z &= 74.62^\circ\end{aligned}$$

As the angles in a triangle sum to 180° ,

$$\begin{aligned}\therefore X &= 180 - 40 - 74.62 \\ &= 65.38^\circ\end{aligned}$$

Now to use the Sine Rule,

$$\begin{aligned}\frac{x}{\sin 65.38} &= \frac{12}{\sin 40} \\ x &= \frac{12 \sin 65.38}{\sin 40} \\ &= 16.97 \text{ cm}\end{aligned}$$

[3 marks]

- (ii) Taking care to use the included angle,

$$\begin{aligned}\text{Area } \Delta &= \frac{1}{2} x z \sin Y \\ &= \frac{1}{2} \times 16.97 \times 18 \times \sin 40^\circ \\ &= 98.2 \text{ cm}^2\end{aligned}$$

[2 marks]

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