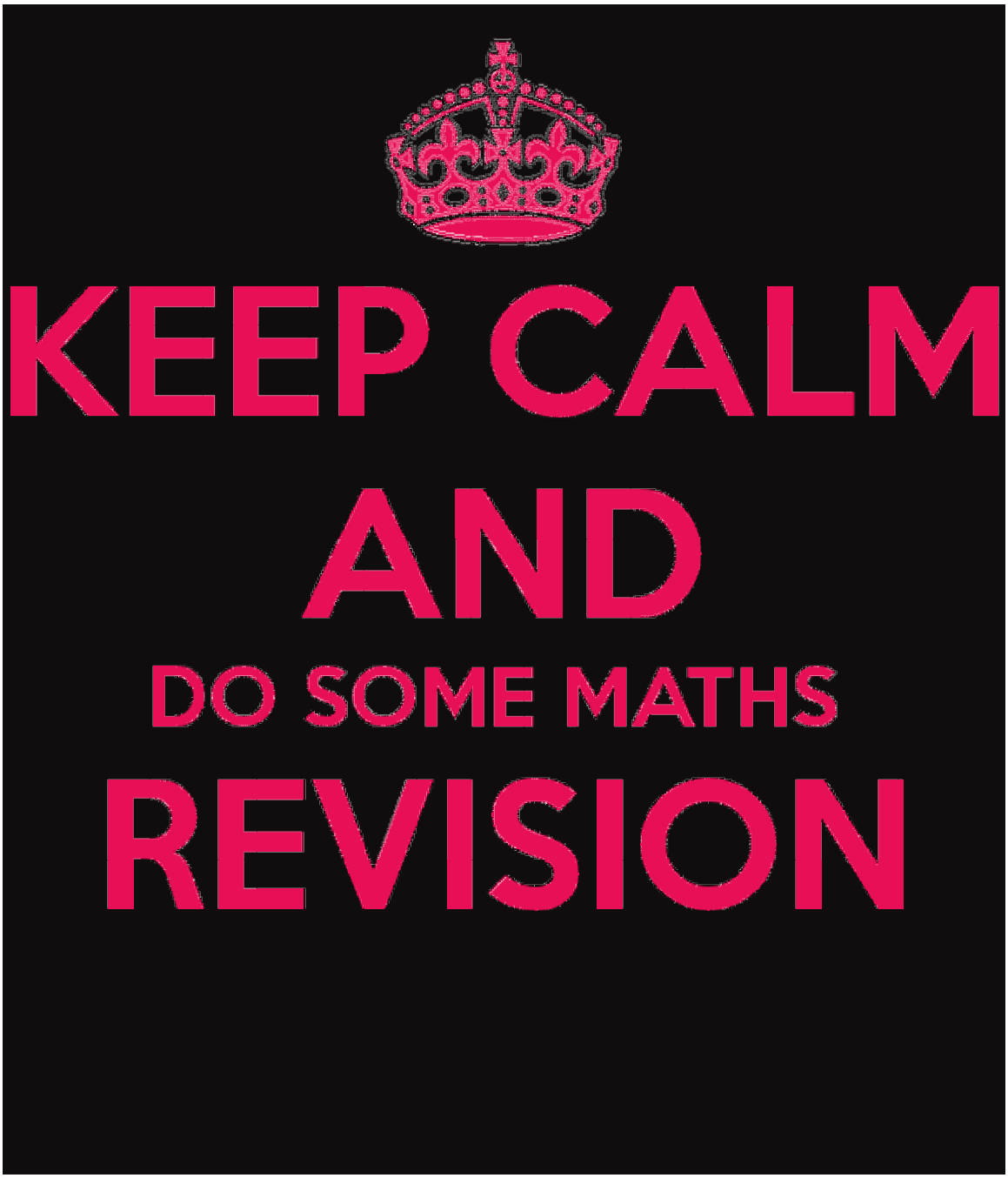
Grade Grabber

R E V I S I O N 2023



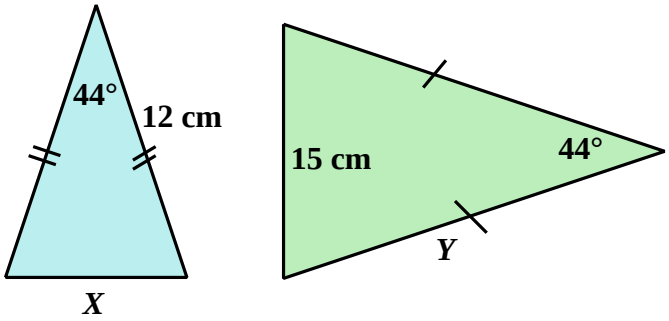
IGCSE Mathematics

Grade Grabber 7

40 Mark Paper

Question 1

Two isosceles triangles are sketched below;



(i) Explain why the two triangles are similar.

[1 mark]

(ii) The sine rule tells us that for any triangle with vertices A, B and C

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use the sine rule to calculate the integer length of the side marked X.

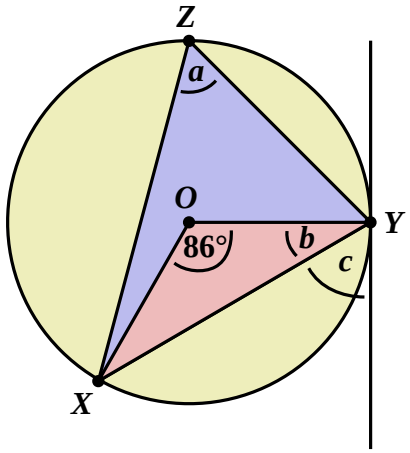
[2 marks]

(iii) Use the fact that the triangles are similar to calculate the side marked Y.

[2 marks]

Question 2

In the diagram below, *O* is the centre of a circle.
X, *Y* and *Z* are three points on the circumference of the circle.
The line touching at *Y* is a tangent.



Write down the size of the angles;

(i) *a* **(ii)** *b* **(iii)** *c*

[2, 2, 2 marks]

Question 3

The formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, can be used to find solutions to a quadratic equation written in the form $ax^2 + bx + c = 0$, where *a*, *b* and *c* are constants.

Show how this formula may be used to solve the equation, $x^2 + x = 1$
Write your solutions to an accuracy of three decimal places.

[4 marks]

Question 4

Two numbers can be described as "coprime" if they have only 1 as a common factor.
For example, 2078 and 4520 are NOT coprime because they both divide by 2.

(a) Which **one** of the following pairs of numbers are coprime ?

(i) 21, 24

(ii) 35, 50

(iii) 33, 77

(iv) 25, 27

For each pair that are not coprime state a factor they have in common.

[4 marks]

(b) 441 can be written as $3 \times 3 \times 7 \times 7$
We say 441 has been written as a product of primes.

(i) Write 550 as a product of primes.

[1 mark]

(ii) Are 441 and 550 coprime ?

[1 mark]

Question 5

Make x the subject of the formula $y = \dfrac{7 - 3x}{x + 5}$

[4 marks]

Question 6

This question is about solving the equation :

$$\frac{3}{(x+3)} + \frac{2}{(x+7)} = 1$$

(i) Show that this equation can be written in the form $x^2 + 5x - 6 = 0$

[4 marks]

(ii) Hence write down the two solutions to the original equation.

[2 marks]

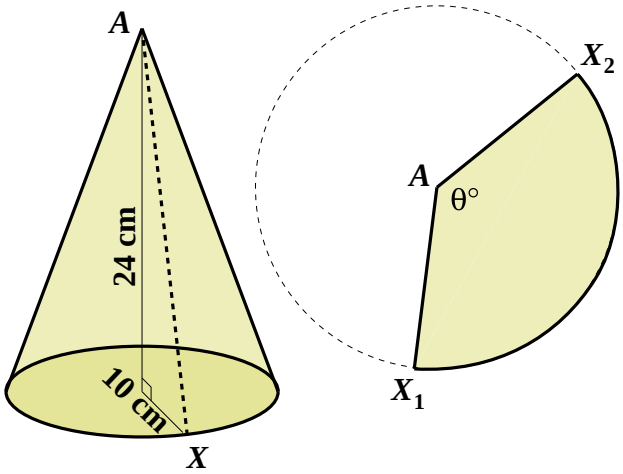
Question 7

Use algebra to show that $0.\dot{7}\dot{7}\dot{2} = \frac{17}{22}$

[2 marks]

Question 8

A wizard's hat is made out of a cone of paper.
It is 24 cm high and the base radius is 10 cm.



(i) Use the theorem of Pythagoras to calculate the slant height, AX.

[1 mark]

(ii) As a part of a magic spell the cone is to be cut along AX and the paper laid flat as the words "***Passus meus IGCSEus at grade Nineus***" are muttered. Find the length of the curved arc, $X_1 X_2$, of the sector.

[2 marks]

(iii) Find the angle, θ, of the sector.

[2 marks]

(iv) Find the area of the paper.

[2 marks]

Answers to Grade Grabber 7

Answer 1

(i) Each has angles of 44°, 68° and 68°

[1 mark]

(ii)

$$\frac{X}{\sin 44} = \frac{12}{\sin 68}$$

$$X = \frac{12 \sin 44}{\sin 68}$$

$$= 8.99 \text{ cm}$$

As we're told X is an integer, $X = 9$ cm

[2 marks]

(iii)

$$\frac{Y}{12} = \frac{15}{9}$$

$$Y = \frac{15 \times 12}{9}$$

$$= 20 \text{ cm}$$

[2 marks]

Answer 2

(i) $a = \frac{1}{2} \times 86 \Rightarrow a = 43°$

[2 marks]

(ii) $b = \frac{180 - 86}{2} \Rightarrow b = 47°$

[2 marks]

(iii) $c = 90 - 47 \Rightarrow c = 43°$

[2 marks]

Note: The equivalence of angles a and c is The Alternate Segment Theorem

Answer 3

$$x^2 + x = 1 \Rightarrow x^2 + x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-1)}}{2 \times 1} \quad * \text{ Must have this line of working } *$$

$$= 0.618 \text{ or } -1.618$$

[4 marks]

Answer 4

(a) **(i)** 21, 24 are not coprime as they both divide by 3

(ii) 35, 50 are not coprime as they both divide by 5

(iii) 33, 77 are not coprime as they both divide by 11

(iv) 25, 27 are coprime (as they both only divide by 1)

(b) **(i)** $550 = 2 \times 5 \times 5 \times 11$

(ii) 441, 550 are coprime

[4, 2 marks]

Answer 5

$$y = \frac{7 - 3x}{x + 5}$$

$$y(x + 5) = 7 - 3x$$

$$xy + 5y = 7 - 3x$$

$$xy + 3x = 7 - 5y$$

$$x(y + 3) = 7 - 5y$$

$$x = \frac{7 - 5y}{y + 3}$$

[4 marks]

Answer 6

(i)

$$\frac{3}{(x + 3)} + \frac{2}{(x + 7)} = 1$$

Multiply through by $(x + 3)(x + 7)$

$$3(x + 7) + 2(x + 3) = (x + 3)(x + 7)$$

$$3x + 21 + 2x + 6 = x^2 + 3x + 7x + 21$$

$$x^2 + 10x + 21 - 5x - 27 = 0$$

$$x^2 + 5x - 6 = 0 \qquad \square$$

[4 marks]

(ii)

$$(x + 6)(x - 1) = 0$$

$$\therefore \; x = -6 \quad \text{or} \quad x = 1$$

[2 marks]

Answer 7

Let $x = 0.\dot{7}\dot{7}2 \Rightarrow 10000x = 7727.2727272 \ldots$

Subtract $100x = 77.2727272 \ldots$

$$9900x = 7650$$

$$x = \frac{7650}{9900} \Rightarrow x = \frac{17}{22} \qquad \square$$

[2 marks]

Answer 8

(i) Using Pythagoras' Theorem, as instructed,

$$|AX| = \sqrt{24^2 + 10^2}$$
$$= 26 \text{ cm}$$

[1 mark]

(ii) The length of the arc $X_1 X_2$ is the circumference of the circular cone's base

$$C = 2\pi r$$
$$= 2 \times \pi \times 10$$
$$= 62.83 \text{ cm}$$

[2 marks]

(iii) Working out the circumference of the "opened up" circle of which the sector is a part,

$$C = 2\pi r$$
$$= 2 \times \pi \times 26 \qquad \text{from part (i)}$$
$$= 163.36 \text{ cm}$$
$$\theta = \frac{62.83}{163.36} \times 360$$
$$= 138.5°$$

[2 marks]

(iv)
$$Area = \frac{138.5}{360} \pi \times 26^2$$
$$= 817 \text{ cm}^2$$

[2 marks]

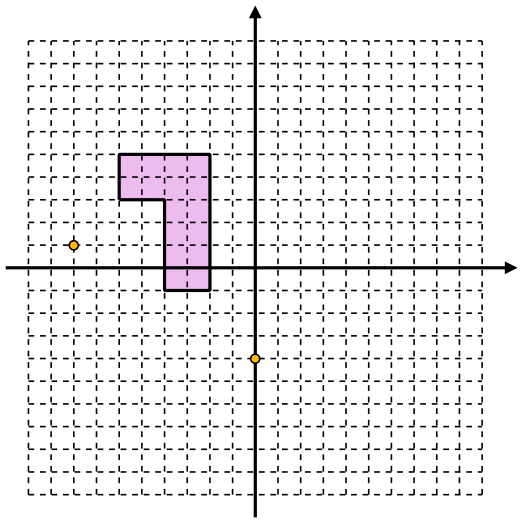
IGCSE Mathematics

Grade Grabber 8

40 Mark Paper

Question 1

The shape with vertices $(-2, -1)$, $(-2, 5)$, $(-6, 5)$, $(-4, 3)$ and $(-4, -1)$ has been plotted on the grid below.



On the grid mark where the shape would be if it were;

(i) Rotated 90° about the point $(0, -4)$

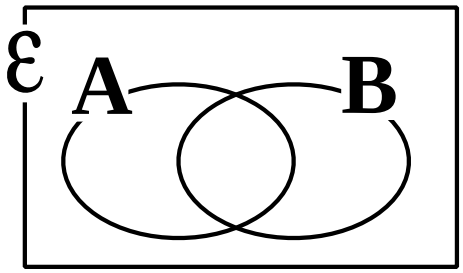
[2 marks]

(ii) Enlarged with scale factor 2, centre $(-8, 1)$

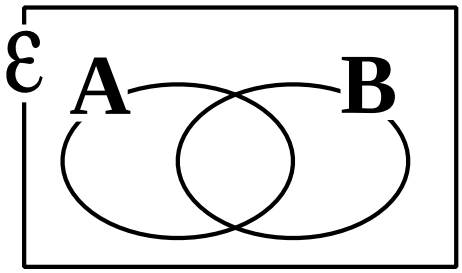
[2 marks]

Question 2

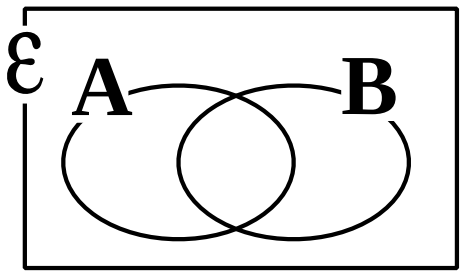
On each diagram, shade in the region specified,



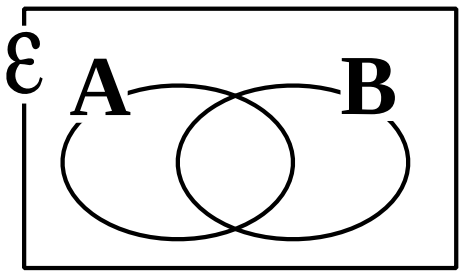
(i) $A \cap B$



(ii) $(A \cup B)'$



(iii) $A \cap B'$



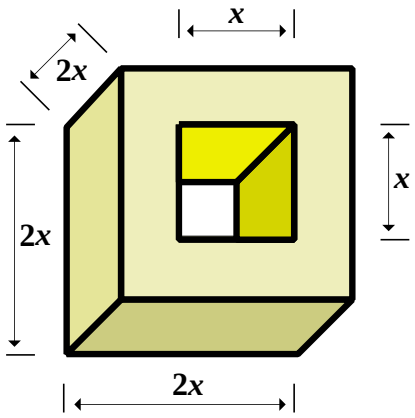
(iv) $A' \cup B'$

[4 marks]

Question 3

A *holed cube* is illustrated below.

It can be thought of as being a cube of side length $2x$ with an "x by x by $2x$" hole passing through.



If $x = 3.5$ cm, calculate;

(i) The volume of the *holed cube*,

[3 marks]

(ii) The surface area of the *holed cube*.

[3 marks]

Question 4

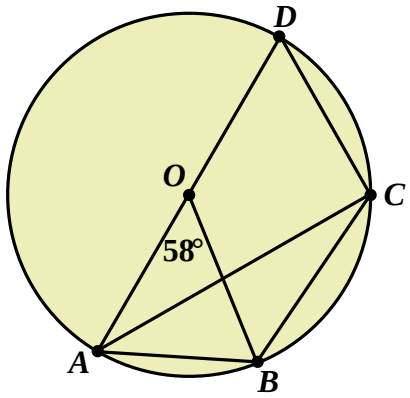
Write each of the following in the form, $k\sqrt{2}$, where k is an integer.

(i) $\sqrt{50}$ **(ii)** $\sqrt{6} \times \sqrt{48}$ **(iii)** $\dfrac{14}{\sqrt{2}}$

[1, 1, 1 marks]

Question 5

In the diagram below, O is the centre of a circle, with diameter AOD,
A, B, C and D are points on the circumference of the circle and $\angle AOB = 58°$



(a) **(i)** Write down the value of $\angle ACB$.

[1 mark]

(ii) Give a reason for your answer.

[1 mark]

(b) **(i)** Write down the value of $\angle BCD$.

[1 mark]

(ii) Give a reason for your answer.

[1 mark]

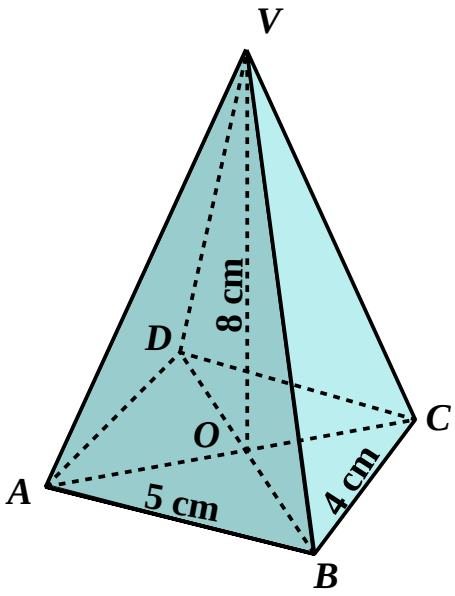
Question 6

Draco bought a wand from Ollivander's wand shop for a price of 3451 Knuts.
At the end of each year, the value of the wand has depreciated by 12%.
Work out the value of Draco's wand at the end of three years.
Give your answer to the nearest Knut.

[3 marks]

Question 7

A pyramid *ABCDV* has a rectangular, horizontal, base *ABCD* of sides 5 cm and 4 cm.
The vertex *V*, is vertically above the centre of the base *O*.
The pyramid has height 8 cm.



Find the angle *VAC*.

[4 marks]

Question 8

Given that $x = 4 \times 5^{28}$ and $y = 8 \times 5^{25}$

(i) Find the highest common factor (HCF) of *x* and *y*
Give your answer in index form

[2 marks]

(ii) Find the lowest common multiple (LCM) of *x* and *y*
Give your answer in index form

[2 marks]

Question 9

There are 15 beads in a box and n of the beads are red.
Jonty takes one bead at random from the box and does not replace it.
He takes a second bead at random from the box.

The probability that he takes 2 red beads is $\frac{1}{7}$

(i) Show that $n^2 - n - 30 = 0$

[4 marks]

(ii) Solve this equation.

[2 marks]

(iii) Explain why only one of the two answers is appropriate.

[1 mark]

(iv) How many of the original 15 beads were red ?

[1 mark]

Answers to Grade Grabber 8

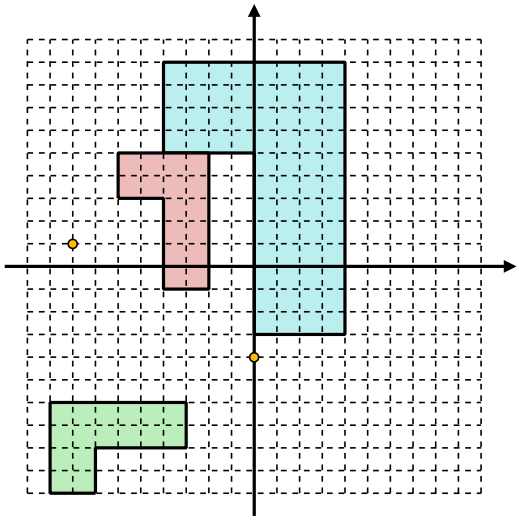
Answer 1

(i) The green shape below

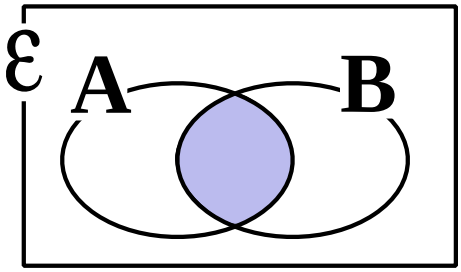
[2 marks]

(ii) The blue shape below

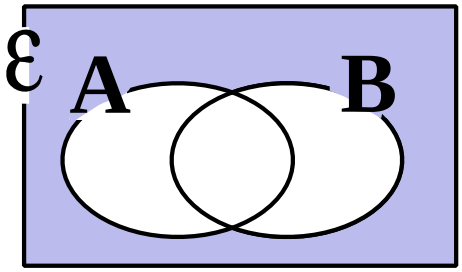
[2 marks]



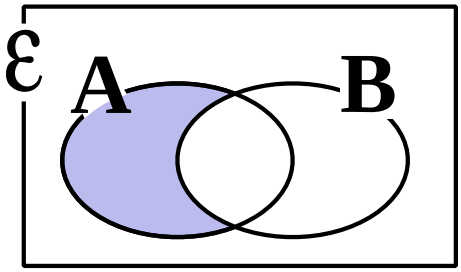
Answer 2



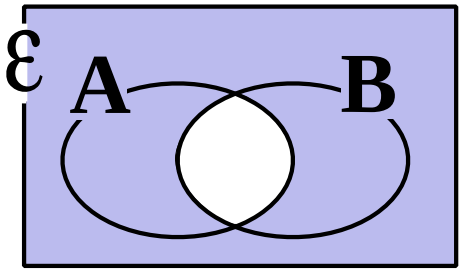
(i) $A \cap B$



(ii) $(A \cup B)'$



(iii) $A \cap B'$



(iv) $A' \cup B'$

[4 marks]

Answer 3

(i)

$$\begin{aligned} V_{HOLED} &= (2x)^3 - x \times x \times 2x \\ &= 8x^3 - 2x^3 \\ &= 6x^3 \\ &= 6 \times 3.5^3 \\ &= 257.25 \text{ cm}^3 \end{aligned}$$

[3 marks]

(ii)

$$\begin{aligned} \textit{Surface Area} &= 6 \times (2x)^2 - 2x^2 + 4 \times x \times 2x \\ &= 24x^2 - 2x^2 + 8x^2 \\ &= 30x^2 \\ &= 30 \times 3.5^2 \\ &= 367.5 \text{ cm}^2 \end{aligned}$$

[3 marks]

Answer 4

(i) $\sqrt{50}$

$= 5\sqrt{2}$

(ii) $\sqrt{6} \times \sqrt{48}$

$= \sqrt{3}\sqrt{2} \times \sqrt{16}\sqrt{3}$

$= 12\sqrt{2}$

(iii) $\frac{14}{\sqrt{2}}$

$= \frac{14}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= 7\sqrt{2}$

[1, 1, 1 marks]

Answer 5

(a) **(i)** $\angle ACB = 29°$

[1 mark]

(ii) An angle at the circumference is half the angle at the centre
(when both subtends the same arc)
"subtend" means "stand on"

[1 mark]

(b) **(i)** $\angle BCD = 119°$

[1 mark]

(ii) An angle that subtends a diameter is 90° ($\angle ACD$)

$\angle BCD = \angle ACB + \angle ACD$

$= 29 + 90$

[1 mark]

Answer 6

$$3451 \times 0.88^3 = 2352 \text{ Knuts}$$

[3 marks]

Answer 7

$$|AC| = \sqrt{4^2 + 5^2}$$
$$= \sqrt{41}$$
$$\angle VAC = \arctan\left(\frac{8}{\frac{1}{2}\sqrt{41}}\right)$$
$$= 68.2°$$

[4 marks]

Answer 8

$$x = 4 \times 5^{28} \qquad y = 8 \times 5^{25}$$
$$= 2^2 \times 5^{28} \qquad = 2^3 \times 5^{25}$$

(i) $hcf\{x, y\} = 2^2 \times 5^{25}$

[2 marks]

(ii)
$$lcm\{x, y\} = \frac{xy}{hcf\{x, y\}}$$
$$= \frac{2^2 \times 5^{28} \times 2^3 \times 5^{25}}{2^2 \times 5^{25}}$$
$$= 2^3 \times 5^{28}$$

[2 marks]

Answer 9

(i)
$$p(red,\ red) = \frac{1}{7}$$
$$\frac{n}{15} \times \frac{n-1}{14} = \frac{1}{7}$$
$$n(n-1) = 15 \times 2$$
$$n^2 - n - 30 = 0 \qquad \square$$

[4 marks]

(ii)
$$(n+5)(n-6) = 0$$
$$n = -5 \quad \text{or} \quad n = 6$$

[2 marks]

(iii) As a probability can not be negative, only $n = 6$ is appropriate.

[1 mark]

(iv) 6 beads of the original 15 where red

[1 mark]

IGCSE Mathematics

Grade Grabber 9

50 Mark Paper

Set Notation Recap:

Symbol	***Interpretations***		
$\cap$	intersection	and	the part of
$\cup$	union	or	merge
A'	not A		
$\subset$	subset	is contained in	
$\not\subset$	not a subset	is not contained in	
$\in$	element	is a member of	
$\notin$	not an element	is not a member of	
$\varnothing$	the empty set	{ }	
$n(A)$	the number of elements in set A		
ε	the universal set		

Question 1

Given that ε = { the positive integers }

P = { prime numbers }

E = { even numbers }

(a) List the following sets, either in words or by listing their elements;

(i) E'

[1 mark]

(ii) $P \cap E$

[1 mark]

(iii) $n(P \cap E)$

[1 mark]

(iv) $E \cup E'$

[1 mark]

(v) $P \cap P'$

[1 mark]

(b) Is $28577463968538 \in E$?

[1 mark]

(c) Is $9 \in P$?

[1 mark]

(d) Is $E' \subset P$?

[1 mark]

(e) Is $27 \notin P$?

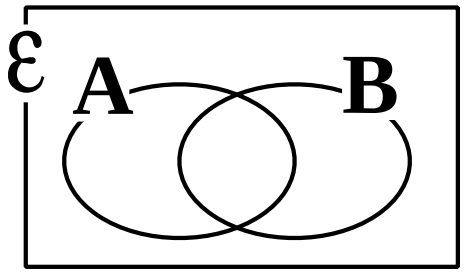
[1 mark]

(f) Is it true that the set {factors of 15 } $\not\subset E$?

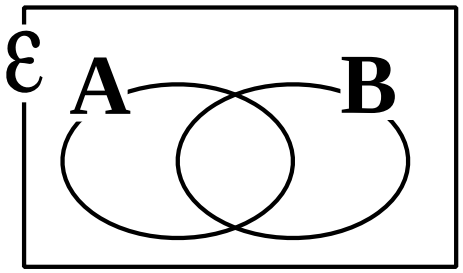
[1 mark]

Question 2

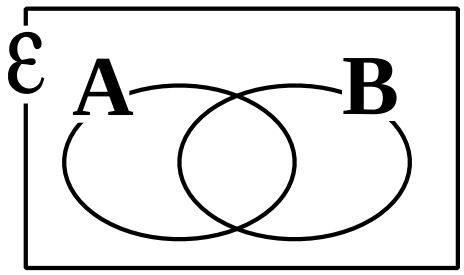
On each diagram, shade in the region specified,



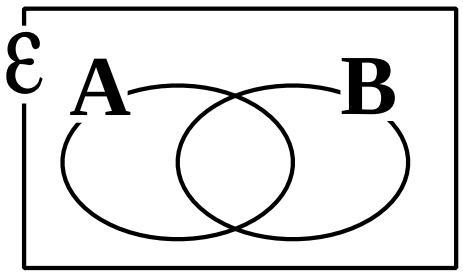
(i) A'



(ii) $A \cup B'$



(iii) $A' \cup B$



(iv) $(A \cup B') \cap (A' \cup B)$

[4 marks]

Question 3

Expand the brackets and simplify:

(i) $(2x - 3)(7x - 5)$

[2 marks]

(ii) $(3 + \sqrt{2})(3 - \sqrt{2})$

[2 marks]

(iii) $(2x^2)^3$

[2 marks]

Question 4

Find $\frac{dy}{dx}$ given that $y = 5x^3 + \frac{1}{x^3} + 2x + \frac{1}{4}$

[4 marks]

Question 5

A straight line passes through the points (2, 6) and (12, 11)

(i) What is the gradient of the line ?

[2 marks]

(ii) Write down the equation of the straight line in the form $y = mx + c$ where m and c are constants to be found.

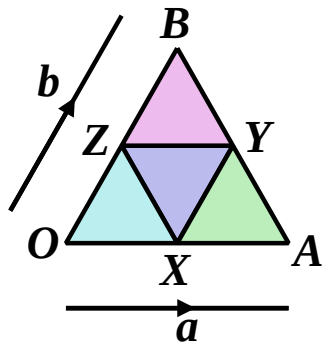
[2 marks]

Question 6

The diagram below shows four congruent equilateral triangles arranged edge-to-edge to form a large equilateral triangle.

Notice that; $\overrightarrow{OA} = \boldsymbol{a}$

and; $\overrightarrow{OB} = \boldsymbol{b}$



Describe, in terms of the vectors ***a*** and ***b***, the following;

(i) $\overrightarrow{OX}$ **(ii)** $\overrightarrow{AX}$ **(iii)** $\overrightarrow{ZB}$

(iv) $\overrightarrow{YZ}$ **(v)** $\overrightarrow{ZO}$ **(vi)** $\overrightarrow{BA}$

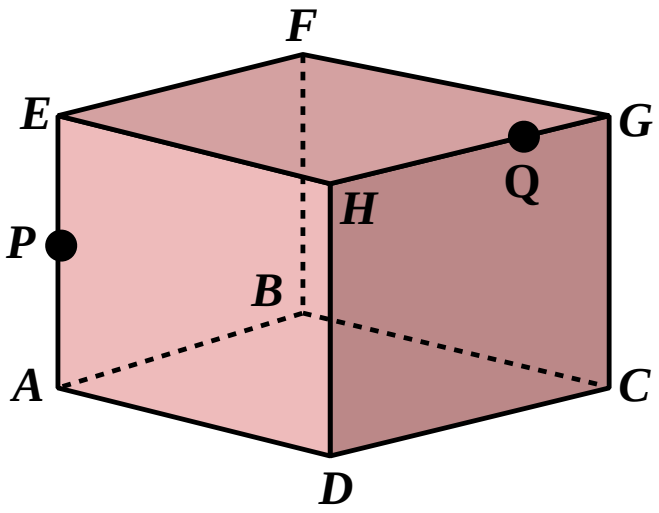
(vii) $\overrightarrow{ZX}$ **(viii)** $\overrightarrow{OY}$ **(ix)** $\overrightarrow{AZ}$

[9 marks]

Question 7

ABCDEFGH is a cube of side 4 cm.

AP = 2 cm and *HQ* = 3 cm.



Calculate;

(i) the lengths of *EQ* and *PD*

[2 marks]

(ii) the length of *PQ*

[1 mark]

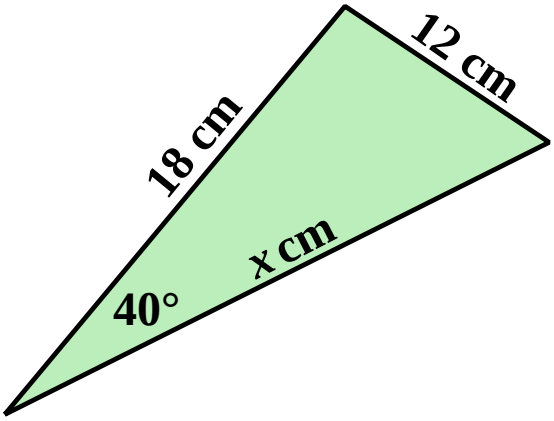
(iii) the angle between *PQ* and the plane *EFGH*

[2 marks]

(iv) the angle *QPD*

[3 marks]

Question 8



(i) Find the length of the side marked x in the diagram above.

[3 marks]

(ii) Find the area of the triangle.

[2 marks]

Answers to Grade Grabber 9

Answer 1

(a) **(i)** $E' = \{ 1, 3, 5, 7, 9, 11, ... \}$
$= \{$ the odd numbers $\}$

[1 mark]

(ii) $P \cap E = \{ 2 \}$

[1 mark]

(iii) $n(P \cap E) = 1$

[1 mark]

(iv) $E \cup E' = \varepsilon$
$= \{$ the positive integers $\}$

[1 mark]

(v) $P \cap P' = \varnothing$

[1 mark]

(b) Is $28577463968538 \in E$? Yes

[1 mark]

(c) Is $9 \in P$? No

[1 mark]

(d) Is $E' \subset P$? No

[1 mark]

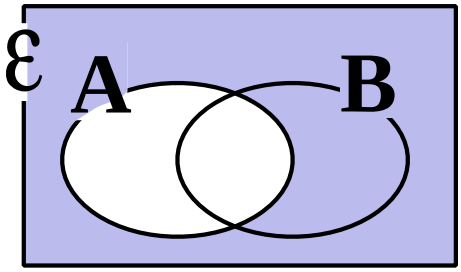
(e) Is $27 \notin P$? Yes

[1 mark]

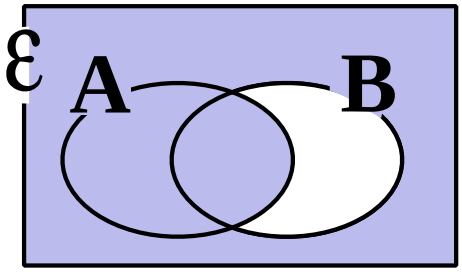
(f) Is it true that the set {factors of 15 } $\not\subset E$? Yes

[1 mark]

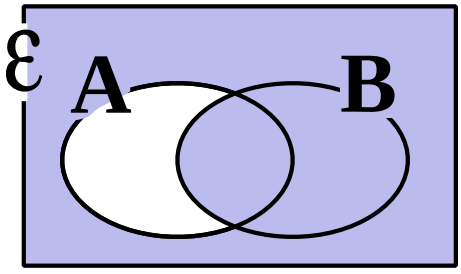
Answer 2



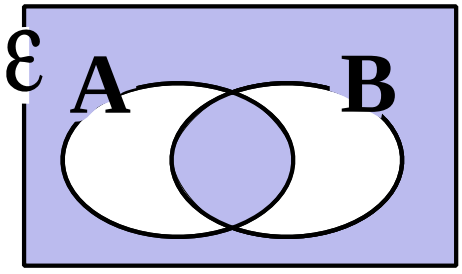
(i) A'



(ii) $A \cup B'$



(iii) $A' \cup B$



(iv) $(A \cup B') \cap (A' \cup B)$

[4 marks]

Answer 3

(i) $(2x - 3)(7x - 5) = 14x^2 - 31x + 15$

[2 marks]

(ii) $(3 + \sqrt{2})(3 - \sqrt{2}) = 7$ using a difference of two squares

[2 marks]

(iii) $(2x^2)^3 = 8x^6$

[2 marks]

Question 4

$$y = 5x^3 + \frac{1}{x^3} + 2x + \frac{1}{4}$$

$$= 5x^3 + x^{-3} + 2x + \frac{1}{4}$$

$$\frac{dy}{dx} = 15x^2 - 3x^{-4} + 2$$

$$= 15x^2 - \frac{3}{x^4} + 2$$

[4 marks]

Answer 5

(i)

$$m = \frac{\Delta Y}{\Delta X}$$

$$= \frac{11 - 6}{12 - 2}$$

$$= \frac{5}{10}$$

$$= \frac{1}{2}$$

[2 marks]

(ii)

$$y = mx + c$$

$$6 = \frac{1}{2} \times 2 + c$$

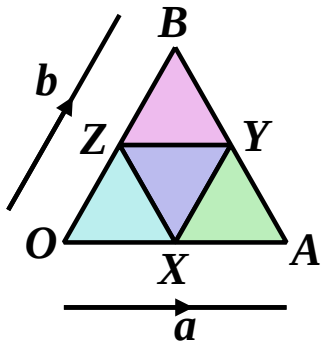
$$6 = 1 + c$$

$$c = 5$$

$$\therefore y = \frac{1}{2}x + 5$$

[2 marks]

Answer 6



(i) $\overrightarrow{OX} = \frac{1}{2}\boldsymbol{a}$ **(ii)** $\overrightarrow{AX} = -\frac{1}{2}\boldsymbol{a}$ **(iii)** $\overrightarrow{ZB} = \frac{1}{2}\boldsymbol{b}$

(iv) $\overrightarrow{YZ} = -\frac{1}{2}\boldsymbol{a}$ **(v)** $\overrightarrow{ZO} = -\frac{1}{2}\boldsymbol{b}$ **(vi)** $\overrightarrow{BA} = \boldsymbol{a} - \boldsymbol{b}$

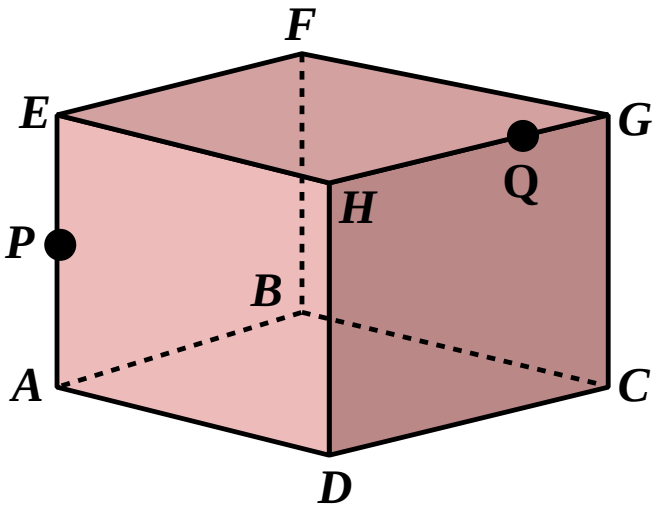
(vii) $\overrightarrow{ZX} = \frac{1}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$

(viii) $\overrightarrow{OY} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

(ix) $\overrightarrow{AZ} = -\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

[9 marks]

Answer 7



(i) $|EQ| = \sqrt{4^2 + 3^2}$ $|PD| = \sqrt{2^2 + 4^2}$

$= 5$ cm $= 2\sqrt{5}$ cm (About 4.472 cm)

[2 marks]

(ii) $|PQ| = \sqrt{2^2 + 5^2}$

$= \sqrt{29}$ cm (About 5.385 cm)

[1 mark]

(iii) the angle between PQ and the plane $EFGH$ is $\angle PQE$

$$\angle PQE = \arctan\left(\frac{2}{\sqrt{29}}\right)$$

$$= 20.4°$$

[2 marks]

(iv) For angle QPD first use Pythagoras' Theorem, then the Cosine Rule backwards on triangle QPD

$$|QD| = \sqrt{4^2 + 3^2}$$

$$= 5 \text{ cm}$$

$$\cos P = \frac{q^2 + d^2 - p^2}{2qd}$$

$$= \frac{(2\sqrt{5})^2 + (\sqrt{29})^2 - 5^2}{2 \times 2\sqrt{5} \times \sqrt{29}}$$

$$= \frac{20 + 29 - 25}{4\sqrt{145}}$$

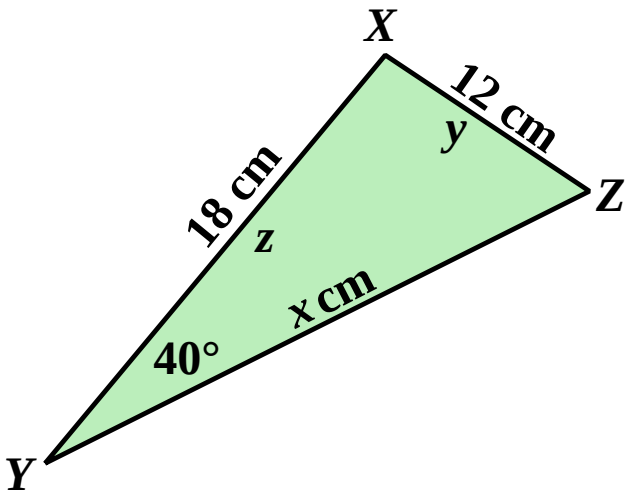
$$= 0.4983$$

$$P = 60.1°$$

[3 marks]

Answer 8

Label the triangle *XYZ*...



(i) By the inverted Sine Rule,

$$\frac{\sin Z}{18} = \frac{\sin 40}{12}$$

$$\sin Z = \frac{18 \sin 40}{12}$$

$$= 0.96418$$

$$Z = 74.62°$$

As the angles in a triangle sum to 180°,

$$\therefore X = 180 - 40 - 74.62$$

$$= 65.38°$$

Now to use the Sine Rule,

$$\frac{x}{\sin 65.38} = \frac{12}{\sin 40}$$

$$x = \frac{12 \sin 65.38}{\sin 40}$$

$$= 16.97 \text{ cm}$$

[3 marks]

(ii) Taking care to use the included angle,

$$Area \triangle = \frac{1}{2} x z \sin Y$$

$$= \frac{1}{2} \times 16.97 \times 18 \times \sin 40°$$

$$= 98.2 \text{ cm}^2$$

[2 marks]

IGCSE Mathematics

Grade Grabber 10

40 Mark Paper

Question 1

(i) Find the Highest Common Factor of 63 and 105

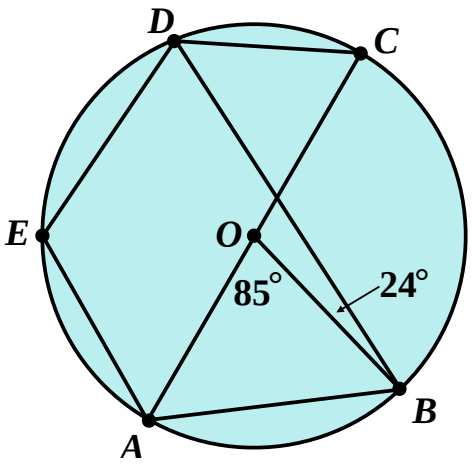
[1 mark]

(ii) Find the Lowest Common Multiple of 63 and 105

[1 mark]

Question 2

In the diagram, *A, B, C, D* and *E* are points on the circumference of a circle centre *O*.
The line AOC is a diameter of the circle.
Angle *AOB* is equal to 85° and angle *DBO* is equal to 24°



Find: **(i)** angle *OAB*

[2 marks]

(ii) angle *BDC*

[2 marks]

(iii) angle *OCD*

[2 marks]

(iv) angle *AED*

[2 marks]

Question 3

Solve these equations;

(i) $8x + 13 = 41$

[1 mark]

(ii) $4(5x - 7) = 2(7x + 4)$

[2 marks]

(iii) $x^2 - 11x + 24 = 0$

[2 marks]

Question 4

The perimeter of a triangle is 108 cm.
The lengths of its sides are in the ratio 2 : 3 : 4

Work out the size of the largest angle.

[4 marks]

Question 5

A coin is biased such that the probability of it landing heads is three times the probability of it landing tails.
The coin is tossed twice.

What is the probability that

(i) It lands "Heads" both times ?

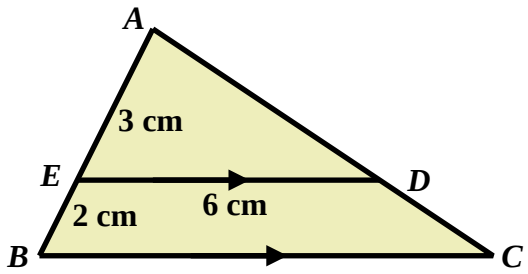
[2 marks]

(ii) It lands "Heads" exactly once ?

[3 marks]

Question 6

In the figure below, *ED* is parallel to *BC*.



(a) Calculate:

(i) the linear scale factor of the enlargement mapping triangle *AED* onto triangle *ABC*.

[1 mark]

(ii) the length of *BC*

[1 mark]

(b) If the area of triangle *ABC* is 20 cm^2, what is the area of triangle *AED* ?

[3 marks]

Question 7

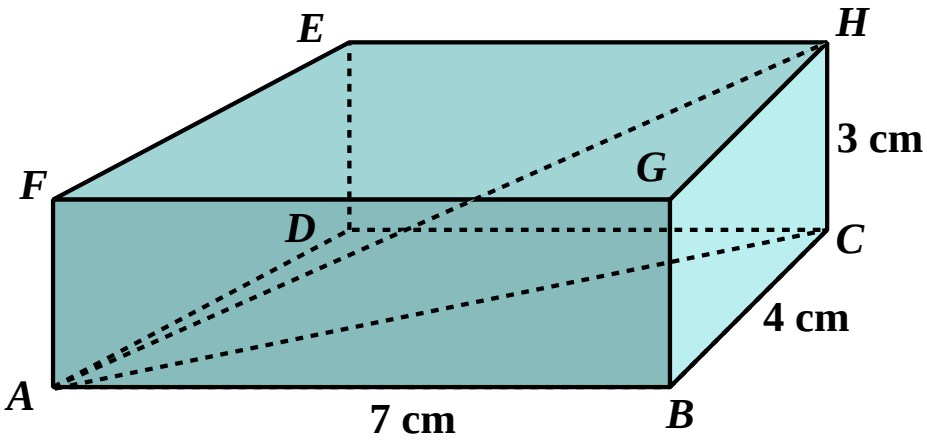
Solve the equation :

$$\frac{5}{t-3} - \frac{2}{t+2} = 0$$

[3 marks]

Question 8

A cuboid *ABCDEFGH* measures 7 cm by 4 cm by 3 cm, as shown below.



Calculate, in degrees, the angle *HAC*.
Give your answer accurate to 1 decimal place.

[4 marks]

Question 9

The curve *C* has equation $y = 4x^3 - 3x$

Find the range of values of x for which the gradient of *C* is negative.

[4 marks]

Answers to Grade Grabber 10

Answer 1

Using the "FACT" button on a calculator,

$$63 = 3 \times 3 \times 7$$

$$105 = 3 \times 5 \times 7$$

(i) $hcf\{65,\ 105\} = 21$

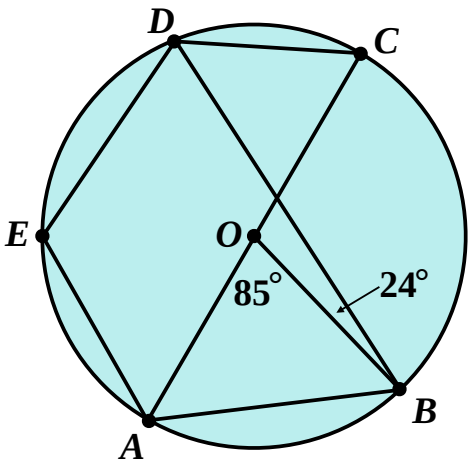
[1 mark]

(ii) $lcm\{65,\ 105\} = \dfrac{63 \times 105}{hcf\{65,\ 105\}}$

$= 315$

[1 mark]

Answer 2



(i) $\angle OAB = \dfrac{180 - 85}{2}$ as $\triangle OAB$ is isosceles

$= 47.5°$

[2 marks]

(ii) $\angle BDC = \angle CAB$ both angles subtend arc CB

$= \angle OAB$

$= 47.5°$

[2 marks]

(iii) $\angle OCD = \angle ACD$

$= \angle ABD$ both angles subtend arc AD

$= 24 + 47.5 = 71.5°$

[2 marks]

(iv) $\angle AED = 180 - \angle ACD$ opposite angles in a cyclic quadrilateral

$= 180 - 71.5 = 108.5°$

[2 marks]

Answer 3

(i) $8x + 13 = 41$

$8x = 28$

$x = 3.5$

[1 mark]

(ii) $4(5x - 7) = 2(7x + 4)$

$20x - 28 = 14x + 8$

$6x = 36$

$x = 6$

[2 marks]

(iii) $x^2 - 11x + 24 = 0$

$(x - 3)(x - 8) = 0$

Either $x = 3$ or $x = 8$

[2 marks]

Answer 4

$$\frac{108}{9} = 12$$

$\therefore$ The triangle has sides of length 24 cm 36 cm and 48 cm

The largest angle, call it x, is opposite the longest side;

$$\cos x = \frac{24^2 + 36^2 - 48^2}{2 \times 24 \times 36}$$

$$= -\frac{1}{4}$$

$$x = 104.5°$$

[4 marks]

Answer 5

$$p(heads) = \frac{3}{4} \qquad p(tails) = \frac{1}{4}$$

(i) $p(heads,\ heads) = \frac{3}{4} \times \frac{3}{4}$

$$= \frac{9}{16}$$

(ii) $p(heads,\ tails) + p(tails,\ heads) = \frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4}$

$$= \frac{6}{16} \qquad \left(\frac{3}{8}\right)$$

[2, 2 marks]

Answer 6

It may help to draw $\triangle AED$ separately to $\triangle ABC$

(a) **(i)** $lsf = \frac{5}{3}$

[1 mark]

(ii) $|BC| = 6 \times \frac{5}{3}$

$= 10 \text{ cm}$

[1 mark]

(b) $Area\ \triangle AED = \left(\frac{3}{5}\right)^2 \times 20$

$= 7.2 \text{ cm}^2$

[3 marks]

Answer 7

$$\frac{5}{t-3} - \frac{2}{t+2} = 0$$

multiply through by $(t-3)(t+2)$

$$5(t+2) - 2(t-3) = 0$$

$$5t + 10 - 2t + 6 = 0$$

$$3t + 16 = 0$$

$$t = -\frac{16}{3}$$

[3 marks]

Answer 8

$|AC| = \sqrt{7^2 + 4^2} = \sqrt{65}$ (About 8.06 cm)

$\angle HAC = arctan\left(\frac{3}{\sqrt{65}}\right) = 20.4°$

[4 marks]

Answer 9

$$y = 4x^3 - 3x \Rightarrow \frac{dy}{dx} = 12x^2 - 3$$

$$\therefore \frac{dy}{dx} = 0 \text{ when } x = \pm\frac{1}{2}$$

The gradient is negative for $-\frac{1}{2} < x < \frac{1}{2}$

[4 marks]

IGCSE Mathematics

Grade Grabber 11

40 Mark Paper

Question 1

Photograph by Martin Hansen

The time taken, T seconds, to boil one litre of water (at room temperature) is inversely proportional to the power, P watts, of the kettle.

When a kettle rated with $P = 2$ kilowatts, the time take, $T = 72$ seconds

Find the time it will take to boil one litre of water (at room temperature) when a kettle rated with $P = 0.8$ kilowatts is used.

[4 marks]

Question 2

By using the theorem of Pythagoras, or otherwise, find the distance between the points (3, 1) and (23, 16)

[3 marks]

Question 3

Writing answers in standard form and correct to three significant figures, calculate;

(i) $4.683 \times 10^6 + 8.908 \times 10^5$

[1 mark]

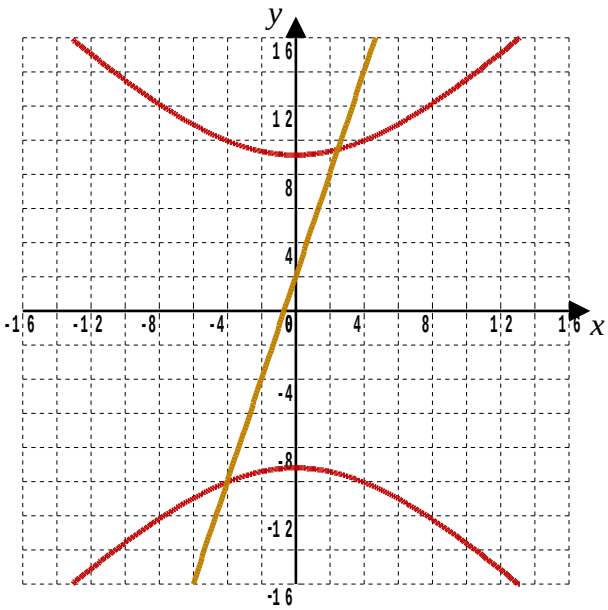
(ii) $5.2281 \times 10^{-4} \times 3.735 \times 10^{-2}$

[1 mark]

(iii) π^{20}

[1 mark]

Question 4



The graph shows a hyperbola (in red) and a straight line (in gold)

The hyperbola has equation $y^2 - x^2 = 84$ and the line has equation $y = 3x + 2$

Use algebra to determine the two points at where the hyperbola and line intersect.

[5 marks]

Question 5

Expand the brackets and simplify; $\left(3 - \sqrt{6}\right)\left(\sqrt{2} + \sqrt{3}\right)$

Show clear algebraic working.

[3 marks]

Question 6

(i) Show that $\dfrac{x^3 - 4}{x^2} = x - 4x^{-2}$

[1 mark]

(ii) Hence, or otherwise, use calculus to find the turning point on the curve;

$$y = \frac{x^3 - 4}{x^2}$$

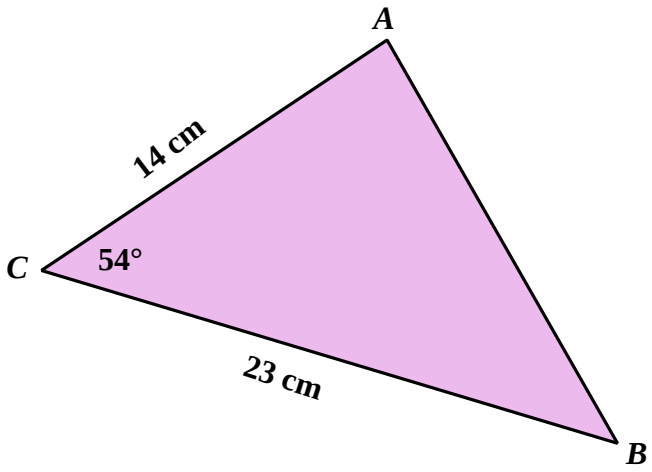
[4 marks]

(iii) State, with a reason, if the turning point is a minimum or a maximum

[2 marks]

Question 7

Calculate the length of *AB*



[3 marks]

Question 8

Solve the following equation giving exact answers,

$$\frac{x}{x-2} + \frac{4}{x+3} = 7$$

[5 marks]

Question 9

(i) Write the recurring decimal 0.272 727 272 727…
as a fraction in the form

$$\frac{p}{q}$$

where p and q are integers with no factor in common.

[3 marks]

(ii) Write the recurring decimal 0.*aba bab aba bab aba* …
as a fraction in the form

$$\frac{p}{q}$$

where p is expressed in terms of a and b
and q is an integer.

[4 marks]

Answers to Grade Grabber 11

Answer 1

$$T = \frac{k}{P}$$

$$72 = \frac{k}{2000}$$

$$k = 72 \times 2$$

$$= 144$$

$$\therefore\ T = \frac{144}{P}$$

When the power is 0.8 kilowatts,

$$T = \frac{144}{0.8}$$

$$= 180 \text{ seconds} \qquad (3 \text{ minutes})$$

[4 marks]

Answer 2

$$Distance = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$= \sqrt{(23 - 3)^2 + (16 - 1)^2}$$

$$= \sqrt{20^2 + 15^2}$$

$$= 25$$

[3 marks]

Answer 3

(i) $4.683 \times 10^6 + 8.908 \times 10^5$

$= 5.57 \times 10^6$

[1 mark]

(ii) $5.2281 \times 10^{-4} \times 3.735 \times 10^{-2}$

$= 1.95 \times 10^{-5}$

[1 mark]

(iii) π^{20}

$= 8.77 \times 10^9$

[1 mark]

Answer 4

$$y^2 - x^2 = 84$$
$$(3x + 2)^2 - x^2 = 84$$
$$9x^2 + 12x + 4 - x^2 = 84$$
$$8x^2 + 12x - 80 = 0$$
$$2x^2 + 3x - 20 = 0 \qquad \text{by dividing through by 4}$$
$$(2x - 5)(x + 4) = 0$$
$$\text{Either } x = \frac{5}{2} \text{ or } x = -4$$

$\therefore$ The points of intersection are $\left(\frac{5}{2}, \frac{19}{2}\right), (-4, -10)$

[5 marks]

Answer 5

$$(3 - \sqrt{6})(\sqrt{2} + \sqrt{3}) = 3\sqrt{2} + 3\sqrt{3} - \sqrt{12} - \sqrt{18}$$
$$= 3\sqrt{2} + 3\sqrt{3} - 2\sqrt{3} - 3\sqrt{2}$$
$$= \sqrt{3}$$

[3 marks]

Answer 6

(i)
$$\text{LHS} = \frac{x^3 - 4}{x^2}$$
$$= \frac{x^3}{x^2} - \frac{4}{x^2}$$
$$= x - 4x^{-2}$$
$$= \text{RHS} \qquad \square$$

[1 mark]

(ii)
$$\frac{dy}{dx} = 1 + 8x^{-3}$$

For a turning point, $\frac{dy}{dx} = 0 \quad \Rightarrow 1 + \frac{8}{x^3} = 0$

$$\therefore \; x^3 + 8 = 0$$

$x = -2 \Rightarrow$ Turning point is $(-2, -3)$

[4 marks]

(iii)
$$\frac{d^2y}{dx^2} = -24x^{-4}$$

This is negative for all values of x, zero excepted, and so the turning point found will be a maximum (corresponding to clockwise, negative, bend)

[4 marks]

Answer 7

By the cosine Rule,

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ &= 23^2 + 14^2 - 2 \times 23 \times 14 \times \cos 54 \\ &= 346.466 \\ c &= 18.6 \text{ cm} \end{aligned}$$

[3 marks]

Answer 8

$$\frac{x}{x-2} + \frac{4}{x+3} = 7$$

Multiply through by $(x-2)(x+3)$

$$\begin{aligned} x(x+3) + 4(x-2) &= 7(x-2)(x+3) \\ x^2 + 3x + 4x - 8 &= 7\left(x^2 + x - 6\right) \\ x^2 + 7x - 8 &= 7x^2 + 7x - 42 \\ 6x^2 - 34 &= 0 \\ x^2 &= \frac{34}{6} \\ x &= \pm\frac{\sqrt{51}}{3} \end{aligned}$$

[5 marks]

Answer 9

(i) Let $x = 0.272\,727\,272\,727\ldots$ in which case,

$$\begin{aligned} 100x &= 27.272\,727\,272\,727\ldots \\ x &= 0.272\,727\,272\,727\ldots \end{aligned}$$

$$\text{Subtract} \quad 99x = 27 \qquad \Rightarrow x = \frac{3}{11}$$

[3 marks]

(ii) Let $x = 0.aba\,bab\,aba\,bab\ldots$

$$\begin{aligned} 100x &= ab.aba\,bab\,aba\,bab\ldots \\ x &= 0.aba\,bab\,aba\,bab\ldots \end{aligned}$$

$$\text{Subtract} \quad 99x = ab \Rightarrow \qquad \Rightarrow x = \frac{10a + b}{99}$$

[3 marks]

IGCSE Mathematics

Grade Grabber 12

40 Mark Paper

Question 1

Expand and simplify; $(x + 4)(x - 3)(x + 3)$

Classic Blunder : Not spotting the difference of two squares.

[3 marks]

Question 2

Factorise fully; $24e^4g + 32e^3g^5$

Classic Blunder : Not knowing what the word “factorise” means.

[2 marks]

Question 3

Tracy journeyed by train from Shrewsbury to London.
The train travelled a distance of 264 km.
The time taken was 2 hours 45 minutes.
Work out the average speed of the rain in kilometres per hour.

Classic Blunder : Thinking 2 hours 45 minutes is 2.45 hours.

[3 marks]

Question 4

Show that $2 - (x - 2) \div \left(\frac{x^2 - 4}{2x + 3} \right)$ can be written as $\frac{a}{x + b}$

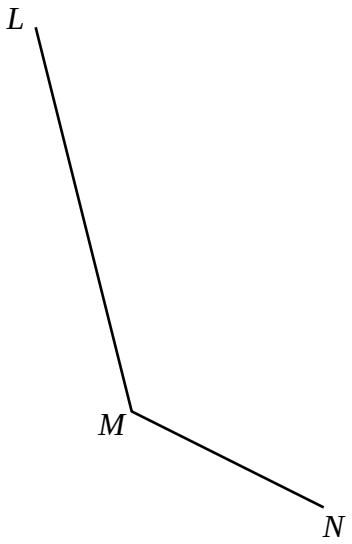
where a and b are integers.

Classic Blunder : Not sorting out the division using KOF as the first step
It's all about BODMAS !

[5 marks]

Question 5

Use ruler and compass to construct the bisector of obtuse angle *LMN*
You must show all your construction lines



Classic Blunder : Not having a ruler and compass.

[3 marks]

Question 6

Solve; $x^2 - 3x - 18 = 0$

Classic Blunder : Not immediately factorising the quadratic,

[3 marks]

Question 7

Morgan, Lucy and Fadi share some money in the ratio 2 : 5 : 3
Lucy gets £26 more than Fadi.
Work out the total amount of money that was shared out between the three people.

Classic Blunder : Not reading the question carefully.

[4 marks]

Question 8

A rectangular lawn has a length of $5x$ metres and a width of $2x$ metres.
The lawn has a path of width 1 metre on three of its sides.

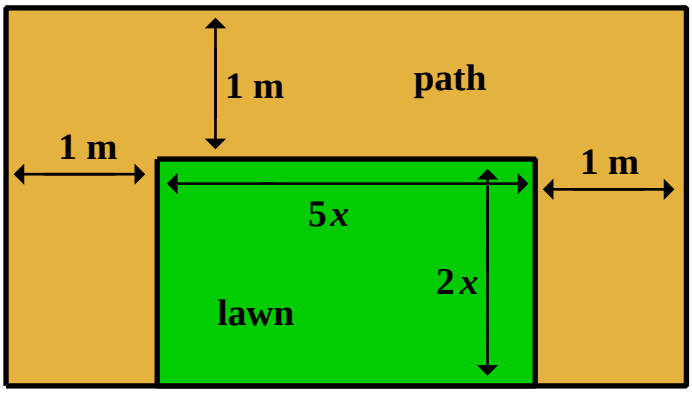


Diagram NOT accurately drawn

The total area of the lawn and the path is 60 m^2

(i) Show that $10x^2 + 9x - 58 = 0$

Classic Blunder : Not realising the total area of 60 must equal $(5x + 2) (2x + 1)$

[3 marks]

(ii) Calculate the area of the lawn.
Show clear algebraic working.

HINT : You could use FACT 58 on your calculator to help guess the brackets or simply use the formula to solve quadratic equations

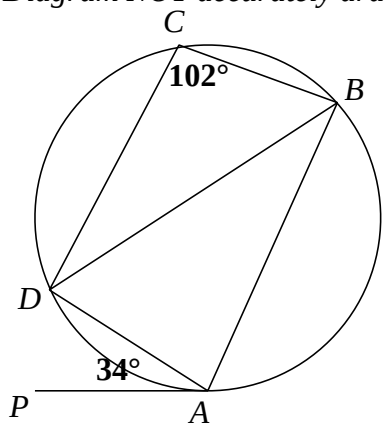
i.e. $ax^2 + bx + c = 0$ has solutions $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Classic Blunder : Not doing part **(b)** because you couldn't do part **(a)**

[4 marks]

Question 9

Diagram NOT accurately drawn



A, B, C and *D* are points on a circle
PA is a tangent to the circle, angle *PAD* = 34° and angle *BCD* = 102°
Calculate the size of angle *ADB*
Give a reason for each stage of your working

Classic Blunder : Not using the Alternate Segment Theorem.

[5 marks]

Question 10

The curve with equation $y = 28x^2 + \frac{7}{x}$ has one stationary point.

Find the coordnates of this stationary point.
Show your working clearly.

Classic Blunder : Not knowing that a stationary point is a point with gradient zero.

[5 marks]

Answers to Grade Grabber 12

Answer 1

$$\begin{aligned}(x+4)(x-3)(x+3) &= (x+4)(x^2-9)\\ &= x^3+4x^2-9x-36\end{aligned}$$

[3 marks]

Answer 2

$$24e^4g+32e^3g^5 = 8e^3g(3e+4g^4)$$

[2 marks]

Answer 3

$$\begin{aligned}\textit{Average Speed} &= \frac{\textit{Total Distance}}{\textit{Total Time}}\\ &= \frac{264}{2.75}\\ &= 96 \text{ km h}^{-1}\end{aligned}$$

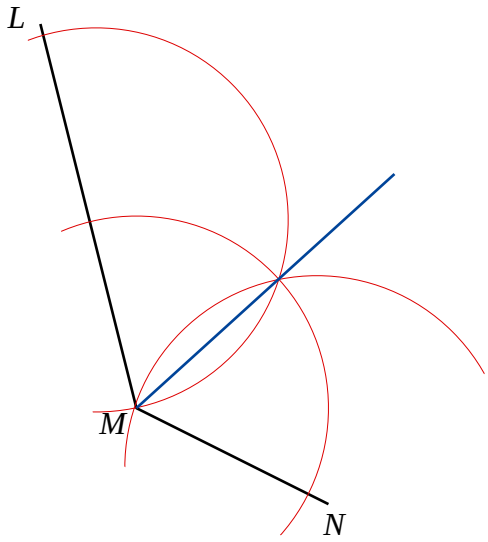
[3 marks]

Answer 4

$$\begin{aligned}\text{LHS} &= 2-(x-2)\div\left(\frac{x^2-4}{2x+3}\right)\\ &= 2-(x-2)\times\frac{2x+3}{x^2-4}\\ &= 2-\frac{(x-2)(2x+3)}{(x-2)(x+2)}\\ &= 2-\frac{(2x+3)}{(x+2)}\\ &= \frac{2(x+2)-(2x+3)}{(x+2)}\\ &= \frac{2x+4-2x-3}{x+2}\\ &= \frac{1}{x+2}\\ &= \text{RHS as it's of the form } \frac{a}{x+b} \text{ with } a=1,\ b=2\end{aligned}$$

[5 marks]

Answer 5



[3 marks]

Answer 6

$$x^2 - 3x - 18 = 0$$

$$(x + 3)(x - 6) = 0$$

Either $x = -3$ or $x = 6$

[3 marks]

Answer 7

2 parts must be £26

$2 \times £13 \ : \ 5 \times £13 \ : \ 3 \times £13$

Total is, $10 \times £13 = £130$

[4 marks]

Answer 8

(i)

$$(5x + 2)(2x + 1) = 60$$

$$10x^2 + 9x + 2 = 60$$

$$10x^2 + 9x - 58 = 0 \qquad \square$$

[3 marks]

$$(10x + 29)(x - 2) = 0$$

As a length cannot be negative, $x = 2$ is the only valid solution

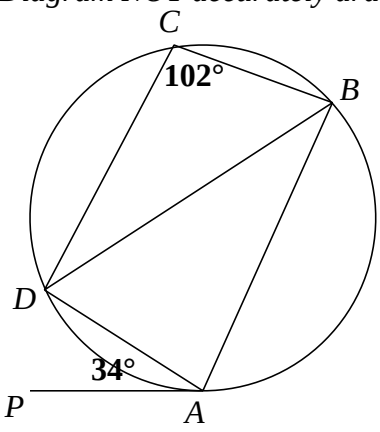
$$Area\ lawn = 10 \times 4$$

$$= 40\ m^2$$

[4 marks]

Answer 9

Diagram NOT accurately drawn



$\angle DAB = 78°$ as opposite sides in cyclic quadrilateral $ABCD$ sum to 180°

$\angle ABD = 34°$ alternate angle theorem

$\angle ADB = 68°$ angles in a triangle sum to 180°

[5 marks]

Answer 10

$$y = 28x^2 + \frac{7}{x}$$

$$= 28x^2 + 7x^{-1}$$

$$\frac{dy}{dx} = 56x - 7x^{-2}$$

For a stationary point, $\frac{dy}{dx} = 0$

$$56x - 7x^{-2} = 0$$

Multiply through by x^2

$$56x^3 - 7 = 0$$

$$x^3 = \frac{7}{56}$$

$$= \frac{1}{8}$$

$$x = \frac{1}{2} \qquad \text{Point is } \left(\frac{1}{2},\ 21\right)$$

[5 marks]