Additional Mathematics Assessment
Paper 1
Thursday 6th May 2021
8.30 am - 9.50 am (80 minutes)

Answers

Answer 1

(a)

$$y = x^3 + 2x - 7$$
$$\frac{dy}{dx} = 3x^2 + 2$$

[2 marks]

(b)

At a turning point, $\dfrac{dy}{dx} = 0$

$\therefore$ We require, $3x^2 + 2 = 0$

$$x^2 = -\frac{2}{3}$$

But the square root of a negative number has no real solutions

$\therefore$ The curve has no turning points.

[2 marks]

Answer 2

$$\frac{2}{3+\sqrt{2}} = \frac{2}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$$
$$= \frac{6-2\sqrt{2}}{9-2}$$
$$= \frac{6-\sqrt{8}}{7}$$

i.e. $a = 6$, $b = 8$ and $c = 7$

[3 marks]

Answer 3

(a)

$$\frac{x}{x+2} - \frac{6}{x-1} = \frac{x}{x+2} \times \frac{x-1}{x-1} - \frac{6}{x-1} \times \frac{x+2}{x+2}$$
$$= \frac{x^2 - x - 6x - 12}{(x+2)(x-1)}$$
$$= \frac{x^2 - 7x - 12}{(x+2)(x-1)}$$

[3 marks]

(b)

$$\frac{x}{x+2} - \frac{6}{x-1} = 4$$

$$\frac{x^2 - 7x - 12}{(x+2)(x-1)} = 4 \qquad \text{using the part (a) result}$$

$$x^2 - 7x - 12 = 4\left((x+2)(x-1) \right)$$

$$x^2 - 7x - 12 = 4\left(x^2 + x - 2\right)$$

$$x^2 - 7x - 12 = 4x^2 + 4x - 8$$

$$3x^2 + 11x + 4 = 0$$

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \times 3 \times 4}}{2 \times 3}$$

$$= \frac{-11 \pm \sqrt{73}}{6}$$

[4 marks]

Answer 4

(a)

$$2x^2 + 8x - 12 = 2\left[x^2 + 4x\right] - 12$$

$$= 2\left[(x+2)^2 - 4\right] - 12$$

$$= 2(x+2)^2 - 8 - 12$$

$$= 2(x+2)^2 - 20$$

[4 marks]

(b) The minimum value occurs when $(x+2)^2 = 0$

$\therefore$ The minimum value is -20

[1 mark]

Answer 5

$$sin(x + 30) = 0.2$$

$$x + 30 = arcsin\,0.2$$

$$= 11.54°,\ 180° - 11.54°,\ 360° + 11.54°$$

$$= 11.54°,\ 168.46°,\ 371.53°$$

$$x = 138.5°,\ 341.5° \qquad \text{for} \quad 0° \leqslant x \leqslant 360°$$

[4 marks]

Answer 6

$$\text{Let } f(x) = x^3 - 5x^2 + x + 15$$

(a) If $(x - 3)$ is a factor then $f(3)$ will equal zero, by the factor theorem.

$$\begin{aligned} f(3) &= 3^3 - 5 \times 3^2 + 3 + 15 \\ &= 27 - 45 + 3 + 15 \\ &= 0 \end{aligned}$$

$\therefore$ $(x - 3)$ is indeed a factor $\square$

[1 mark]

(b)

$$\begin{array}{r|l} & \quad x^2 - 2x - 5 \\ \cline{2-2} x - 3 & x^3 - 5x^2 + x + 15 \\ & \underline{x^3 - 3x^2} \\ & \quad -2x^2 + x \\ & \quad \underline{-2x^2 + 6x} \\ & \qquad -5x + 15 \\ & \qquad \underline{-5x + 15} \\ & \qquad\qquad 0 \leftarrow \text{As expected !} \end{array}$$

$$x^3 - 5x^2 + x + 15 = 0$$

$$(x - 3)\left(x^2 - 2x - 5\right) = 0$$

Either $x - 3 = 0$ in which case $x = 3$

Or $x^2 - 2x - 5 = 0$ in which case...

$$\begin{aligned} x &= \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-5)}}{2 \times 1} \\ &= \frac{2 \pm \sqrt{24}}{2} \\ &= \frac{2 \pm 2\sqrt{6}}{2} \\ &= 1 \pm \sqrt{6} \end{aligned}$$

[4 marks]

Answer 7

(a) The length of a side of square *ABFE* will be $\sqrt{1600} = 40$ m

This will be the length of the hypotenuse of the cross sectional triangle.

The adjacent of that triangle will be

$$40 \times \cos 20 = 37.59$$

Giving the area of rectangle *ABCD* as $37.59 \times 40 = 1504$ m^2

[3 marks]

(b) Using right angled triangle *EAX*...

$$|XE| = \sqrt{40^2 + 12^2}$$
$$= \sqrt{1744} \text{ m}$$

Using right angled triangle *FBX*...

$$|XF| = \sqrt{40^2 + (40 - 12)^2}$$
$$= \sqrt{2384} \text{ m}$$

Use the cosine rule on ΔEXF...

$$\cos EXF = \frac{1744 + 2384 - 1600}{2\sqrt{1744}\sqrt{2384}}$$
$$= 0.6199$$
$$\therefore EXF = 51.7°$$

[5 marks]

Answer 8

(a) **(i)**

$$s = 48t - t^3$$
$$v = \frac{ds}{dt}$$
$$= 48 - 3t^2$$
$$a = \frac{dv}{dt}$$
$$= -6t$$

So when $t = 1$, $a = -6$ cm s^{-2}

[5 marks]

(ii)

$$v = 0$$
$$48 - 3t^2 = 0$$
$$t^2 = \frac{48}{3}$$
$$t = 4 \text{ seconds}$$

[2 marks]

(iii)

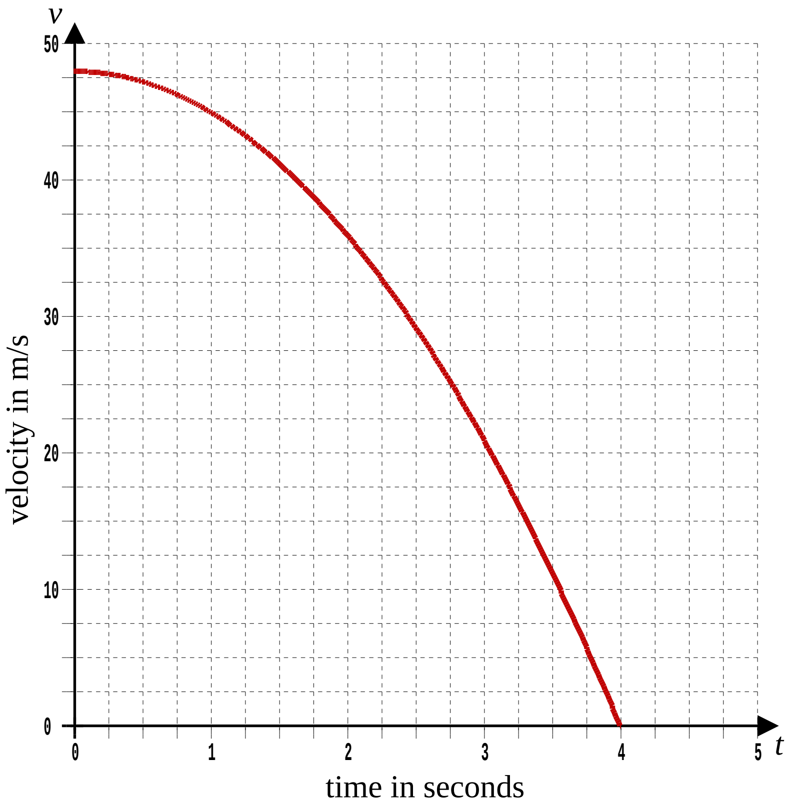
$$s = 48t - t^3$$

$$\text{When } t = 4,\ s = 48 \times 4 - 4^3$$

$$= 128 \text{ cm}$$

[2 marks]

(b)



[1 mark]

Answer 9

(i)

$$V_{CYLINDER} = \pi r^2 h$$

$$16\pi = \pi r^2 h$$

$$h = \frac{16\pi}{\pi r^2}$$

$$h = \frac{16}{r^2}$$

[1 mark]

(ii)

$$Surface\ Area = 2 \times circle + Rectangle$$

$$= 2\pi r^2 + 2\pi rh$$

$$= 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$$

$$= 2\pi r^2 + \frac{32\pi}{r}$$

[3 marks]

(iii)

$$S = 2\pi r^2 + \frac{32\pi}{r}$$

$$= 2\pi r^2 + 32\pi r^{-1}$$

$$\frac{dS}{dr} = 4\pi r - 32\pi r^{-2}$$

$$= 4\pi r - \frac{32\pi}{r^2}$$

$$\therefore \frac{ds}{dr} = 0 \text{ when } 4\pi r - \frac{32\pi}{r^2} = 0$$

$$4\pi r = \frac{32\pi}{r^2}$$

$$r^3 = \frac{32\pi}{4\pi}$$

$$= 8$$

$$r = 2 \text{ cm}$$

[5 marks]

(iv)

$$\frac{dS}{dr} = 4\pi r - 32\pi r^{-2}$$

$$\frac{d^2S}{dr^2} = 4\pi + 64\pi r^{-3}$$

$$\text{When } r = 2, \quad \frac{d^2S}{dr^2} = 4\pi + \frac{64\pi}{2^3}$$

$$= 12\pi$$

[4 marks]

(v)

$$S = 2\pi r^2 + \frac{32\pi}{r}$$

Minimum S is when $r = 2$ cm $\Rightarrow S = 24\pi$

[1 mark]