Additional Mathematics Assessment
Paper 2
Wednesday 26th May 2021
11.30 - 12.50 (80 minutes)

Answers

Answer 1

(a) $ab = 6\sqrt{2} \times 2\sqrt{6}$

$= 6\sqrt{2} \times 2\sqrt{2}\sqrt{3}$

$= 24\sqrt{3}$

[2 marks]

(b) $\dfrac{a}{b} = \dfrac{6\sqrt{2}}{2\sqrt{6}}$

$= \dfrac{6\sqrt{2}}{2\sqrt{2}\sqrt{3}}$

$= \dfrac{3}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \sqrt{3}$

[2 marks]

Answer 2

Possible factors include $(x \pm 1)$, $(x \pm 2)$

$$\text{Let } f(x) = x^3 - 3x^2 + 3 \times 2 - 2$$

$$\text{in which case, } f(2) = 2^3 - 3 \times 2^2 + 3 \times 2 - 2$$

$$= 8 - 12 + 6 - 2$$

$$= 0$$

$\therefore$ $(x - 2)$ is a factor of $f(x)$ by the factor theorem

$$\begin{array}{r|rrrr}
 & x^2 & -\ x & +\ 1 & \\
\hline
x - 2 & x^3 - 3x^2 & + 3x & - 2 & \\
 & x^3 - 2x^2 & & & \\
\hline
 & -\ x^2 & + 3x & & \\
 & -\ x^2 & + 2x & & \\
\hline
 & & x & - 2 & \\
 & & x & - 2 & \\
\hline
 & & & 0 & \leftarrow \text{As expected !}
\end{array}$$

$$\text{Thus, } x^3 - 3x^2 + 3x - 2 = (x - 2)(x^2 - x + 1)$$

[3 marks]

Answer 3

$$y = 3 - x^3$$

$$\frac{dy}{dx} = -3x^2$$

$$\text{When } x = 1, \ \frac{dy}{dx} = -3$$

The tangent passes through $(1, 2)$ with gradient -3

$$y = mx + c$$

$$2 = -3 \times 1 + c \Rightarrow c = 5$$

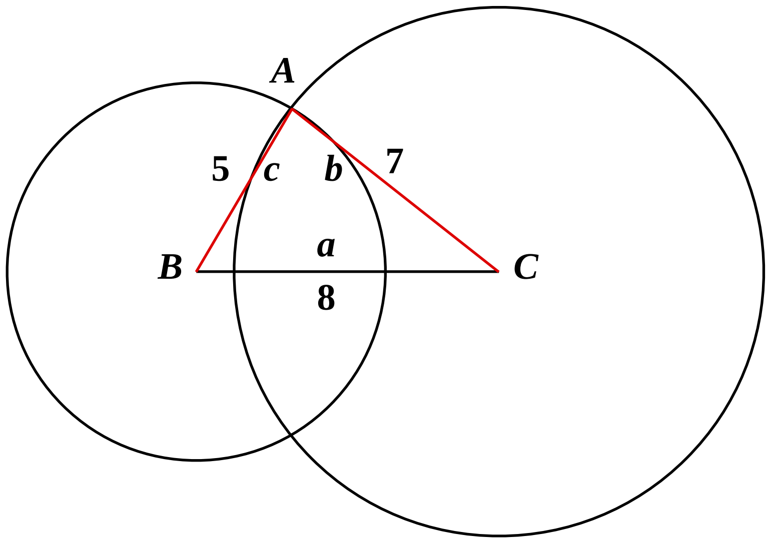
$\therefore$ The tangent is $y = -3x + 5$

This will cut the y-axis at $(0, 5)$

[5 marks]

Answer 4

A quick ruler and compass construction may help pick the correct angle rather than working them all out...



$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{8^2 + 5^2 - 7^2}{2 \times 8 \times 5}$$

$$= \frac{64 + 25 - 49}{80}$$

$$= \frac{1}{2}$$

$\therefore B = 60°$ (This is angle ABC)

[4 marks]

Answer 5

(a)

$$\sin 2x = 0.4$$
$$2x = 23.58,\ 156.42,\ 383.58,\ 516.42$$
$$x = 11.8°,\ 78.2°,\ 191.8°,\ 258.2°$$

[3 marks]

(b)

$$\sin^2 x = 2\cos x - 1$$
$$1 - \cos^2 x = 2\cos x - 1$$
$$\cos^2 x + 2\cos x - 2 = 0$$
$$\cos x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-2)}}{2 \times 1}$$
$$= \frac{-2 \pm \sqrt{12}}{2}$$
$$= -1 \pm \sqrt{3}$$

For $\cos x = -1 + \sqrt{3}$, $x = 42.9°,\ 317.1°$

(For $\cos x = -1 - \sqrt{3}$, there are no solutions)

[5 marks]

Answer 6

$$y = x^3 - 6x^2 + 9x + 7$$
$$\frac{dy}{dx} = 3x^2 - 12x + 9$$

For turning points, $\frac{dy}{dx} = 0$

$$3x^2 - 12x + 9 = 0$$

Divide through by 3

$$x^2 - 4x + 3 = 0$$
$$(x - 1)(x - 3) = 0$$

Either $x = 1$ or $x = 3$

which gives as turning points, $(1,\ 11)$, $(3,\ 7)$

It could be argued that from the shape and orientation of the cubic curve, it is (3, 7) that is the minimum. Or it could be shown that the second derivative at that point is positive. That is,

$\frac{d^2y}{dx^2} = 6x - 12$ which, when $x = 3$, is positive $\Rightarrow (3,\ 7)$ is the minimum

[6 marks]

Answer 7

(a)

$$\left| AB \right| = \sqrt{(\Delta y)^2 + (\Delta x)^2}$$
$$= \sqrt{(8 - 2)^2 + (5 - 1)^2}$$
$$= \sqrt{6^2 + 4^2}$$
$$= \sqrt{36 + 16}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13}$$

[2 marks]

(b) **(i)**

$$Midpoint = M\left(\frac{1 + 5}{2}, \ \frac{2 + 8}{2} \right)$$
$$= M(3, \ 5)$$

[1 mark]

(ii)

$$m_{AB} = \frac{\Delta y}{\Delta x}$$
$$= \frac{8 - 2}{5 - 1}$$
$$= \frac{6}{4}$$
$$= \frac{3}{2}$$
$$\therefore m_{PERPENDICULAR} = -\frac{2}{3}$$

The perpendicular bisector will have a straight line equation of the form, $y = mx + c$

$$5 = -\frac{2}{3} \times 3 + c \Rightarrow c = 7$$
$$\therefore y = -\frac{2}{3}x + 7$$

[4 marks]

Answer 8

(a)

$$\begin{aligned} Area &= length \times breadth \\ &= (40 - 2x) \times x \\ &= 40x - 2x^2 \\ &= (-2)\left(x^2 - 20x\right) \\ &= (-2)\left((x - 10)^2 - 100\right) \\ &= (-2)(x - 10)^2 + 200 \end{aligned}$$

That is $a = -2$, $b = -10$ and $c = 200$

[4 marks]

(b) The smallest that the $(x - 10)^2$ can be is zero, corresponding to $x = 10$

This will then give the maximum area as it takes nothing away from the 200

$$\therefore\ A_{MAX} = 200\ \text{m}^2$$

[2 marks]

(c)

$$\begin{aligned} Area &= 40x - 2x^2 \\ 150 &= 40x - 2x^2 \end{aligned}$$

Divide through by 2

$$\begin{aligned} 75 &= 20x - x^2 \\ x^2 - 20x + 75 &= 0 \\ (x - 5)(x - 15) &= 0 \end{aligned}$$

Either $x = 5\ \text{m}$ or $x = 15\ \text{m}$

[3 marks]

Answer 9

(a) $(x - 10)^2 + y^2 = 3^2$

[1 mark]

(b) By considering the radius of each circle, $lsf = \dfrac{9}{3} = 3$

Let the length of OP be x so it is required to show that x is 15

$$\begin{aligned} \therefore\ \frac{x}{x - 10} &= 3 \\ x &= 3(x - 10) \\ x &= 3x - 30 \\ 2x &= 30 \\ x &= 15 \quad \square \end{aligned}$$

[4 marks]

Answer 10

Let $f(x) = x^3 - 3x + k$ and note that, as $x = 2$ is a root, $f(2) = 0$

$$\therefore\ 2^3 - 3 \times 2 + k = 0$$
$$8 - 6 + k = 0$$
$$k = -2$$

$$\begin{array}{r|l} & x^2 + 2x + 1 \\ \hline x - 2 & x^3 + 0x^2 - 3x - 2 \\ & \underline{x^3 - 2x^2} \\ & \quad 2x^2 - 3x \\ & \quad \underline{2x^2 - 4x} \\ & \qquad x - 2 \\ & \qquad \underline{x - 2} \\ & \qquad\quad 0 \leftarrow \text{As expected !} \end{array}$$

Thus, $x^3 - 3x - 2 = (x - 2)(x^2 + 2x + 1)$

$$= (x - 2)(x + 1)^2$$

$\therefore$ The only other root is $x = -1$

[5 marks]

Answer 11

(a) $x^2 + y^2 = 5$ and $y = 2x + k$ intersect when,

$$x^2 + (2x + k)^2 = 5$$
$$x^2 + 4x^2 + 4kx + k^2 = 5$$
$$5x^2 + 4kx + (k^2 - 5) = 0 \qquad \square$$

[2 marks]

(b) To be a tangent to the circle, the discriminant of the part (a) equation must to equal to zero to give a repeated root.
(i.e. a single point of contact)

$$D = 0$$
$$b^2 - 4ac = 0$$
$$(4k)^2 - 4 \times 5 \times (k^2 - 5) = 0$$
$$16k^2 - 20k^2 + 100 = 0$$
$$4k^2 = 100 \Rightarrow k = \pm 5$$

[2 marks]