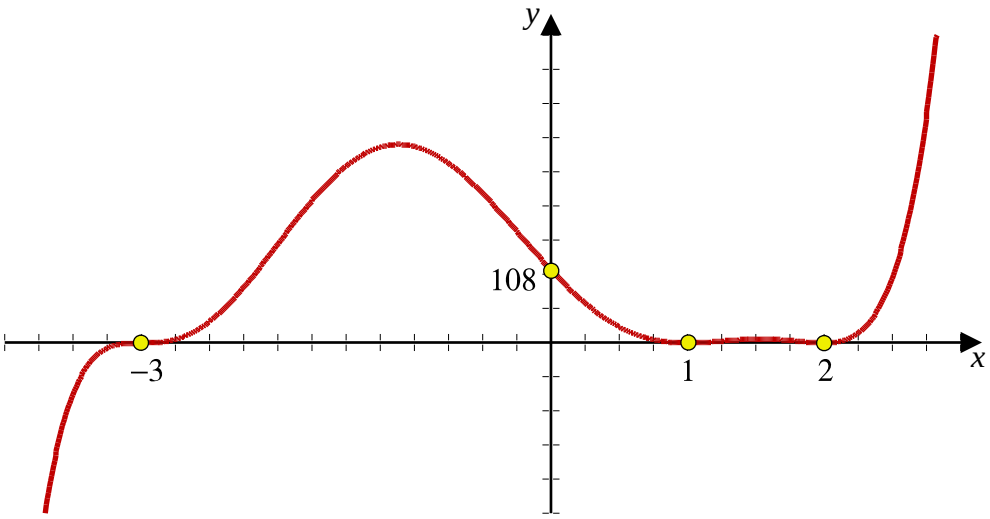
2.4 Answers

A-Level Pure Mathematics : Year 1
Graphwork

2.4.1 Solutions to 2.2 The Together Sketches

(i) $y = (x+3)^3(x-1)^2(x-2)^2$



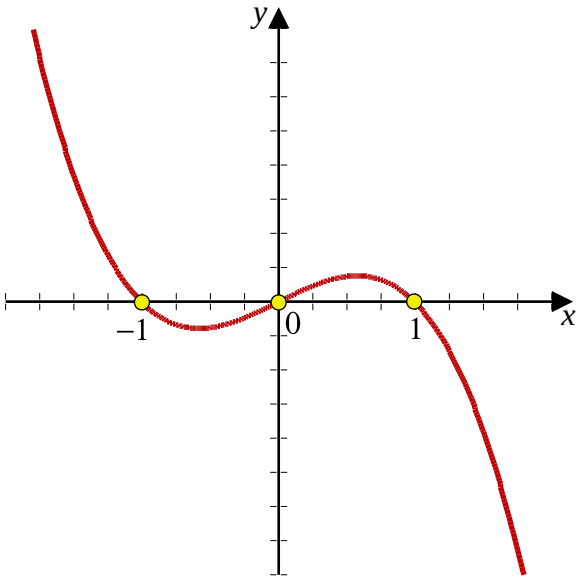
[3 marks]

(ii) $y = x - x^3$

$= x(1-x^2)$

$= x(1-x)(1+x)$

$= -x(x-1)(x+1)$



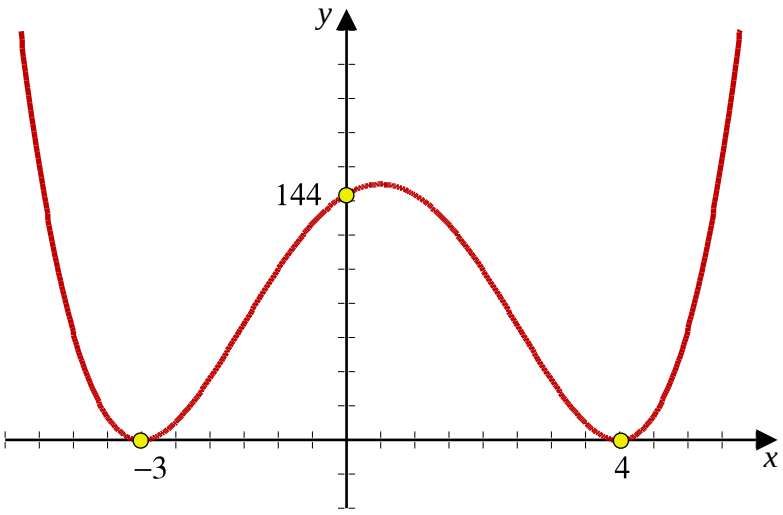
[3 marks]

(iii) $y = (x+3)^2(4-x)^2$

$= (x+3)^2[(4-x)]^2$

$= (x+3)^2[(-1)(x-4)]^2$

$= (x+3)^2(x-4)^2$

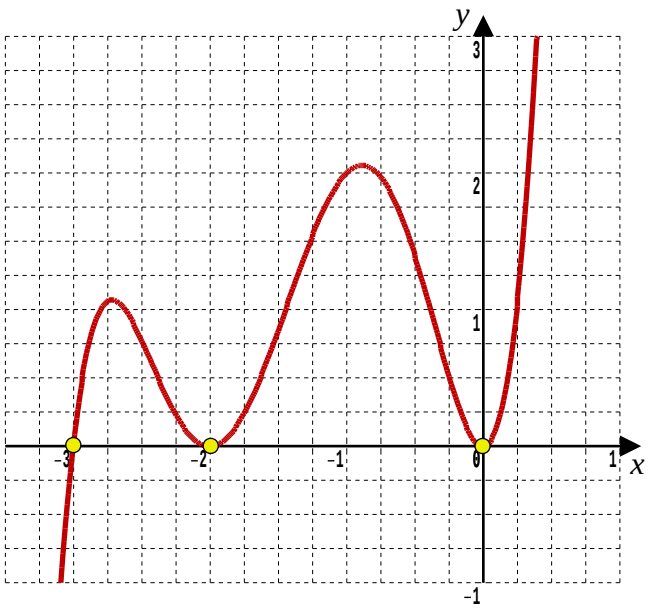


[3 marks]

2.4.2 Solutions to 2.3 Exercise

Answer 1

(i) $y = (x)^2(x+2)^2(x+3)$



[3 marks]

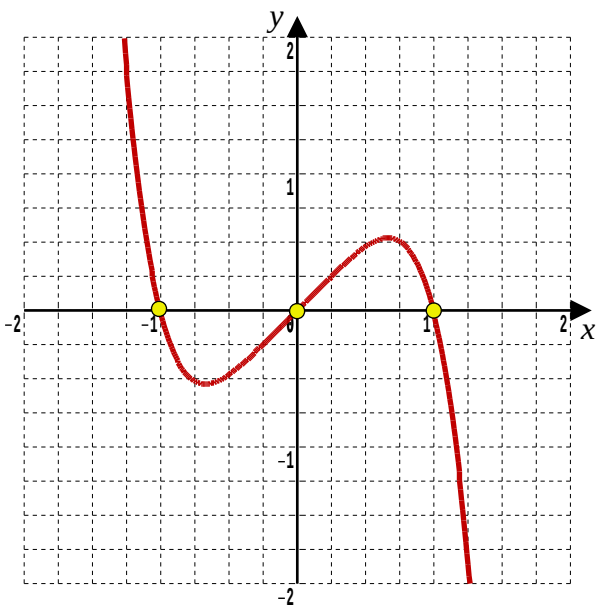
(ii) $y = x - x^5$

$= x(1 - x^4)$

$= x(1 - x^2)(1 + x^2)$

$= x(1 - x)(1 + x)(1 + x^2)$

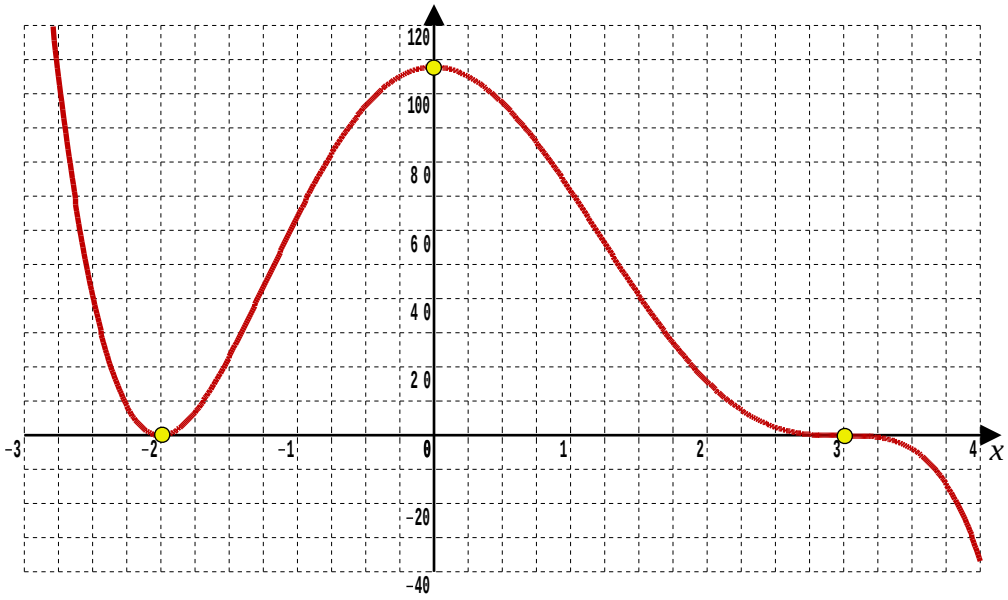
$= (-1)\,x\,(x - 1)(x + 1)(x^2 + 1)$



[3 marks]

(iii) $y = (3 - x)^3(x + 2)^2$

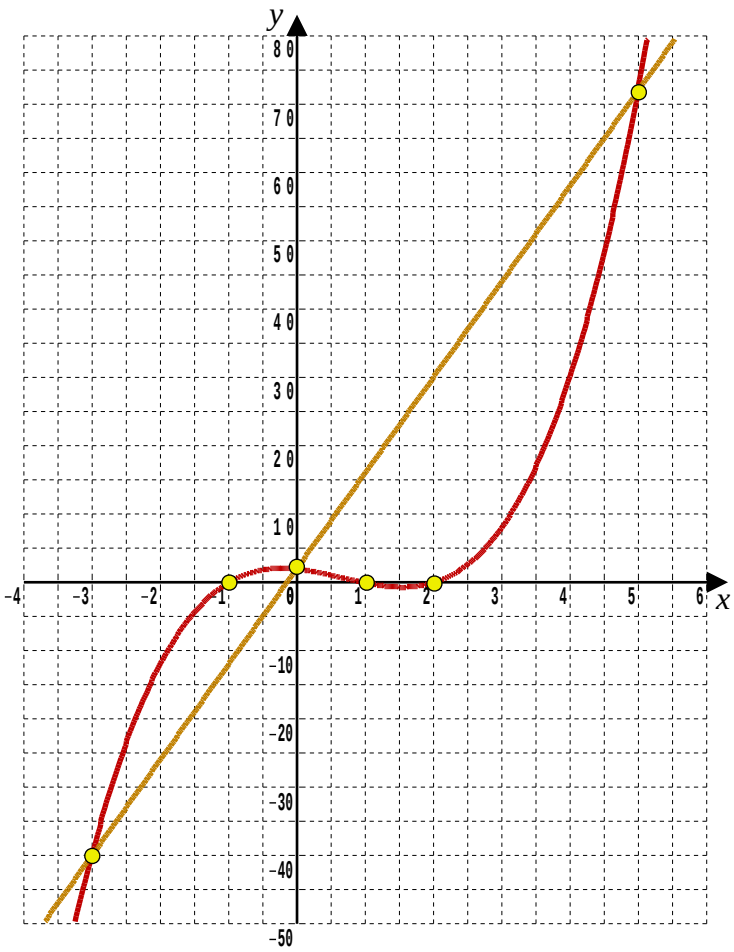
$= (-1)(x - 3)^3(x + 2)^2$



[3 marks]

Answer 2

(i) The curve $y = (x^2 - 1)(x - 2)$ and the line $y = 14x + 2$



[4 marks]

(ii) To find the points of intersection, using algebra,

$$(x^2 - 1)(x - 2) = 14x + 2$$

$$x^3 - 2x^2 - x + 2 - 14x - 2 = 0$$

$$x^3 - 2x^2 - 15x = 0$$

$$x(x^2 - 2x - 15) = 0$$

$$x(x - 5)(x + 3) = 0$$

Either $x = -3$ or $x = 0$ or $x = 5$

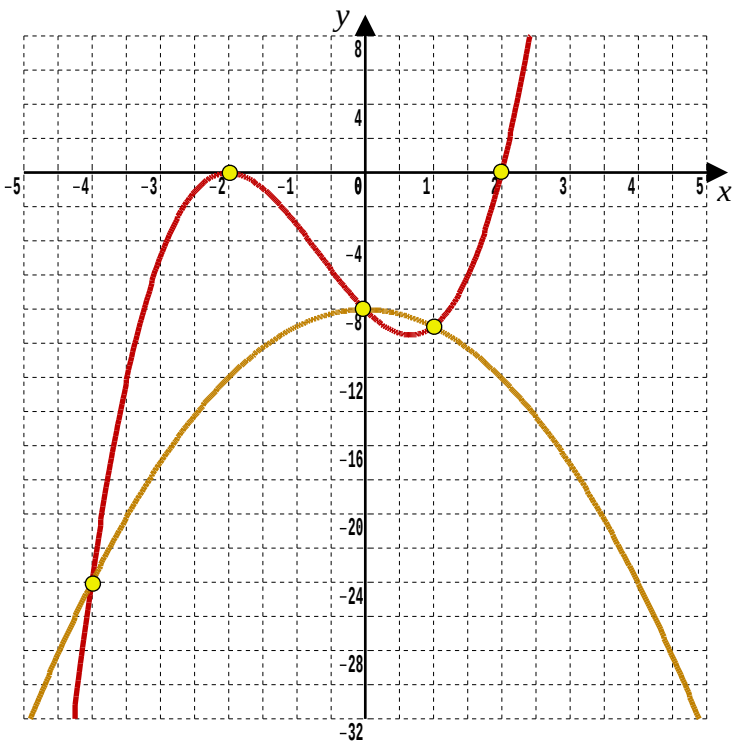
Now use $y = 14x + 2$ to get the intersection points...

$$(-3,\ 40),\ (0,\ 2),\ (5,\ 72)$$

[3 marks]

Answer 3

(i) The cubic curve $y = (x - 2)(x + 2)^2$ and quadratic $y = -x^2 - 8$



[4 marks]

(ii) To find the points of intersection, using algebra,

$$(x - 2)(x + 2)^2 = -x^2 - 8$$

$$(x^2 - 4)(x + 2) = -x^2 - 8$$

$$x^3 + 2x^2 - 4x - 8 = -x^2 - 8$$

$$x^3 + 3x^2 - 4x = 0$$

$$x(x^2 + 3x - 4) = 0$$

$$x(x + 4)(x - 1) = 0$$

Either $x = -4$ or $x = 0$ or $x = 1$

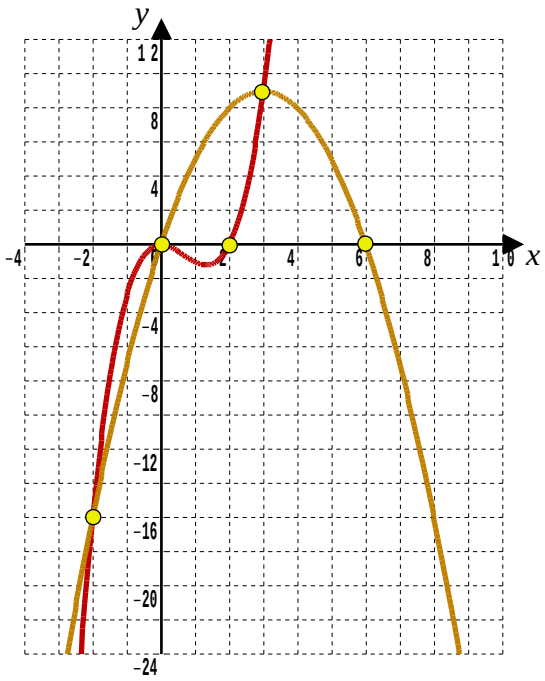
Now use $y = (x - 2)(x + 2)^2$ to get the intersection points...

$$(-4, -24), (0, -8), (1, -9)$$

[3 marks]

Answer 4

(i) The cubic curve $y = x^2(x - 2)$ and quadratic $y = x(6 - x)$



[6 marks]

(b) To find the points of intersection, using algebra,

$$x^2(x - 2) = x(6 - x)$$

$$x^3 - 2x^2 = 6x - x^2$$

$$x^3 - x^2 - 6x = 0$$

$$x(x^2 - x - 6) = 0$$

$$x(x + 2)(x - 3) = 0$$

Either $x = -2$ or $x = 0$ or $x = 3$

Now use $y = x(6 - x)$ to get the intersection points...

$$(-2, -16), \ (0, 0), \ (3, 9)$$

[7 marks]

Answer 5

$$f(x) = x^3 - 2x^2 + px + 36$$

(i) By the factor theorem, $f(3) = 0$

$$(3)^3 - 2(3)^2 + p(3) + 36 = 0$$

$$27 - 18 + 3p + 36 = 0$$

$$3p = -45$$

$$p = -15$$

[3 marks]

(ii)

$$\begin{array}{r|l} x - 3 & x^2 + x - 12 \\ \hline & x^3 - 2x^2 - 15x + 36 \\ & x^3 - 3x^2 \\ \hline & \quad x^2 - 15x \\ & \quad x^2 - 3x \\ \hline & \quad -12x + 36 \\ & \quad -12x + 36 \\ \hline & \quad 0 \end{array} \leftarrow \text{As expected !}$$

$$f(x) = x^3 - 2x^2 - 15x + 36$$

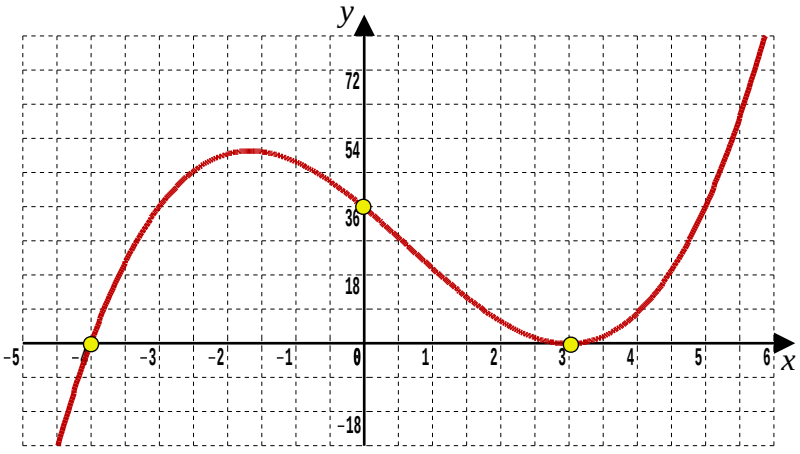
$$= (x - 3)(x^2 + x - 12)$$

$$= (x - 3)(x - 3)(x + 4)$$

$$= (x - 3)^2(x + 4)$$

[4 marks]

(iii)



[3 marks]

Answer 6

(i)

$$
\begin{array}{rl|l}
 & & x^2 \quad + 2x \quad + 4 \\
\hline
x + 2 & x^3 + 0x^2 + 0x + 8 & \\
 & x^3 + 2x^2 & \\
\hline
 & -2x^2 + 0x & \\
 & -2x^2 - 4x & \\
\hline
 & 4x + 8 & \\
 & 4x + 8 & \\
\hline
 & 0 & \leftarrow \text{As expected !}
\end{array}
$$

$$x^3 + 8 = (x + 2)(x^2 + 2x + 4)$$

$$\therefore p = 2 \text{ and } q = 4$$

(ii) For the quadratic factor, $(x^2 + 2x + 4)$

$$\begin{aligned} D &= b^2 - 4ac \\ &= 2^2 - 4 \times 1 \times 4 \\ &= -12 \end{aligned}$$

As the discriminant is zero the quadratic factor contributes no further roots. □

[3 marks]

Answer 7

$$f(x) = x^6 - 3x^5 - 15x^4 + 35x^3 + 90x^2 - 108x - 216$$

(i) $f(1) = 1^6 - 3 \times 1^5 - 15 \times 1^4 + 35 \times 1^3 + 90 \times 1^2 - 108 \times 1 - 216$

$= 1 - 3 - 15 + 35 + 90 - 108 - 216$

$= -216$ $\therefore$ $(x - 1)$ is not a factor of $f(x)$

$f(-1) = 1 + 3 - 15 - 35 + 90 + 108 - 216$

$= -64$ $\therefore$ $(x + 1)$ is not a factor of $f(x)$

[3 marks]

(ii) Using the FACT button on a calculator, $216 = 2^3 \times 3^3$ and as $(x \pm 1)$ can not be factors (from part (i)) it would seem that the factorisation is either $f(x) = (x + 2)^3(x - 3)^3$ or $f(x) = (x - 2)^3(x + 3)^3$

[3 marks]

(iii) Students need only do one of,

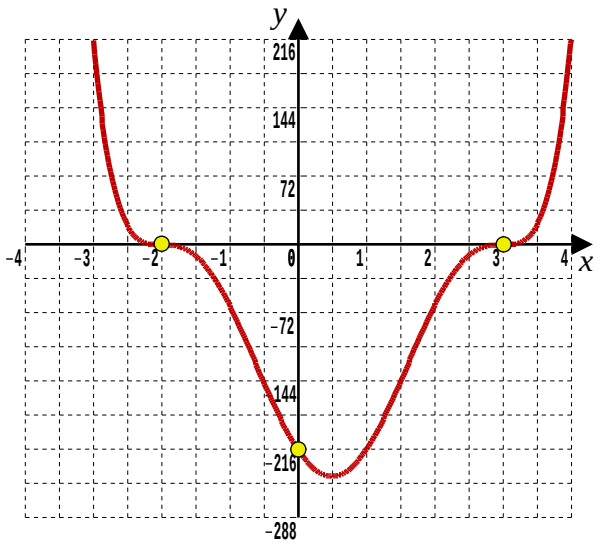
$$f(2) = -64$$
$$f(-2) = 0$$
$$f(3) = 0$$
$$f(-3) = 216$$

as any one of these four results is enough to deduce that,

$$f(x) = (x + 2)^3(x - 3)^3$$

[3 marks]

(iv)



[3 marks]

Answer 8

Points of intersection must satisfy,

$$f(x) = g(x)$$

$$3x^3 + x^2 - x = 2x(x - 1)(x + 1)$$

$$3x^3 + x^2 - x = 2x\left(x^2 - 1\right)$$

$$3x^3 + x^2 - x - 2x^3 + 2x = 0$$

$$x^3 + x^2 + x = 0$$

$$x\left(x^2 + x + 1\right) = 0$$

Either $x = 0$ or $x^2 + x + 1 = 0$

However, $x^2 + x + 1 = 0$ has a discriminant of -3

As this is negative there are no further roots.

The only point of intersection is thus $(0,\ 0)$

[6 marks]