3.5 Answers

A-Level Pure Mathematics : Year 1
Graphwork

Answer 1

(a) Let $y = 0$ in which case,

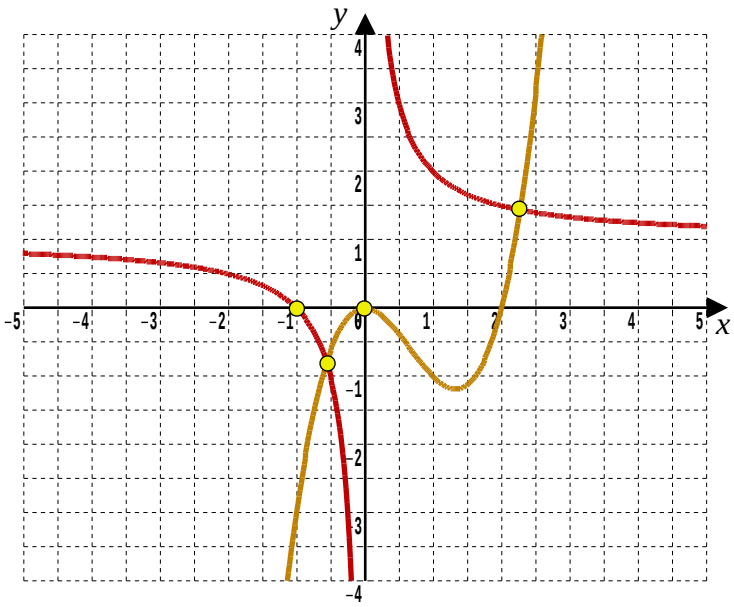
$$\frac{1}{x} + 1 = 0$$

$$1 + x = 0$$

$$x = -1$$

[1 mark]

(b)



Curve D has intercepts with the x-axis at (0, 0) or (2, 0)

[3 marks]

(c) The equation,

$$x^2(x - 2) = \frac{1}{x} + 1$$

will have two solutions as it will find where the two curves, graphed above intersect.

[1 mark]

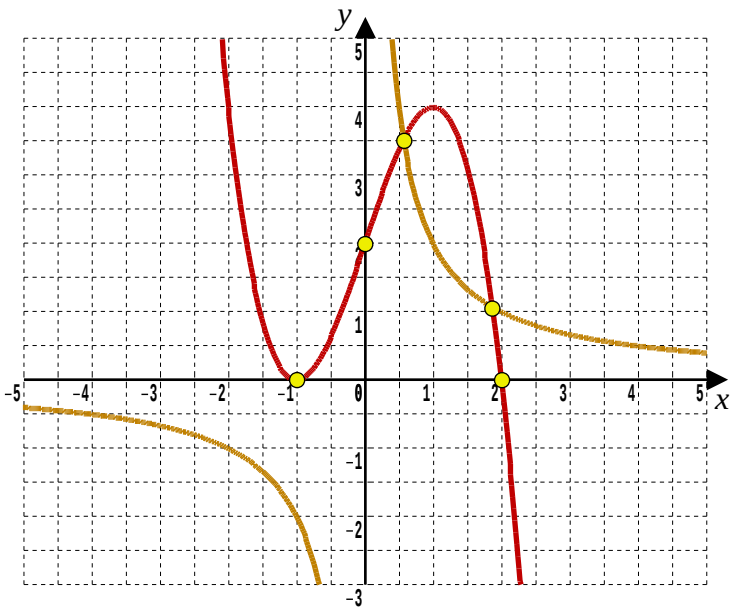
Answer 2

$$y = (x + 1)^2(2 - x)$$

(a) When $x = 1$, $y = 4$ $\quad \therefore a = 4$

[1 mark]

(b)



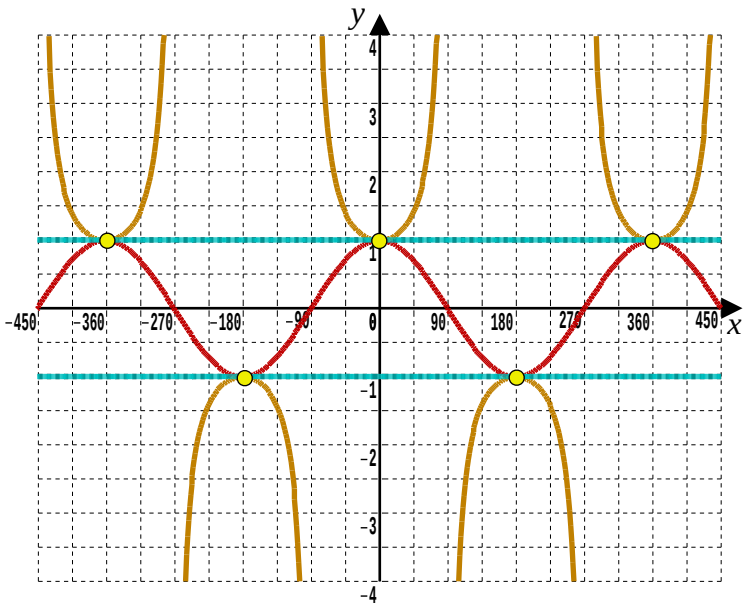
[5 marks]

(c) Two solutions as the two curves intercept in two places.

[1 mark]

Answer 3

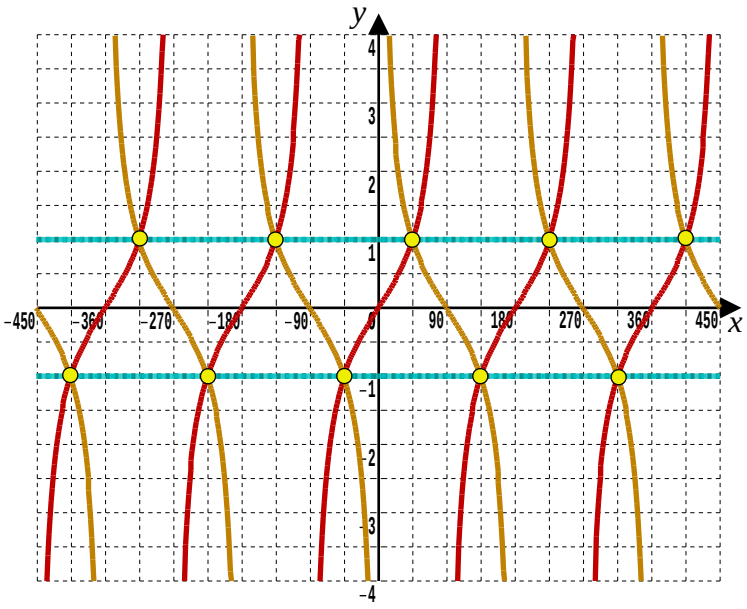
(i)



Red : $f(x) = cos(x)$ Gold : $\frac{1}{f(x)} = \frac{1}{cos(x)}$

[3 marks]

(ii)



Red : $f(x) = tan(x)$ Gold : $\frac{1}{f(x)} = \frac{1}{tan(x)}$

[3 marks]

Answer 4

(i) The minimum point is $(1, -16)$

[1 mark]

(ii) $(x - 1)^2 - 16 = x^2 - 2x - 15$

This will cross the y-axis at $(0, -15)$

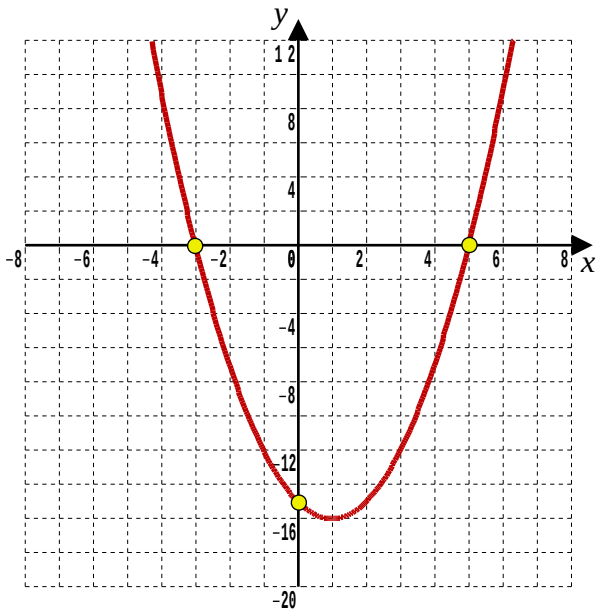
[1 mark]

(iii) $(x + 3)(x - 5)$

This will cross the x-axis at $(-3, 0)$ and $(5, 0)$

[1 mark]

(iv)



[2 marks]

Answer 5

$$y = (x - 4)^2 + 1$$

The curve of this equation would cross the x-axis when $y = 0$ and this would require that the following equation be satisfied,

$$(x - 4)^2 + 1 = 0$$

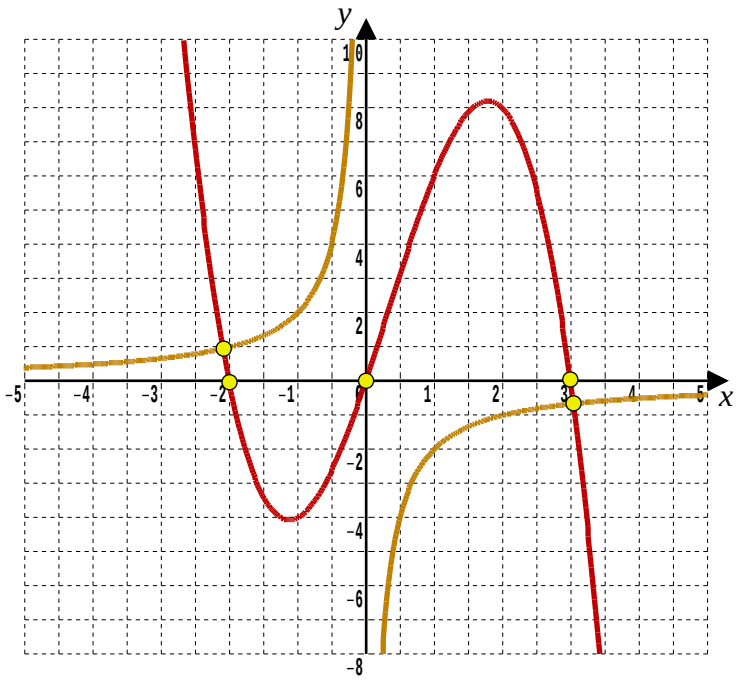
This clearly has no solutions for the least value of $(x - 4)^2$ is zero but, even then, 1 is added on.

$\therefore$ The curve of this equation does not cross the x-axis. $\square$

[2 marks]

Answer 6

(a)



Red: $y = x(x + 2)(3 - x)$ Gold: $y = -\frac{2}{x}$

[6 marks]

(b) There will be two solutions to the equation

$$x(x + 2)(3 - x) = -\frac{2}{x}$$

because the left hand side is the red graph and the right hand side is the gold graph and, from the sketch, the two graphs intersect in two places.

[2 marks]

Answer 7

(a) $x^2 + 2x + 3 = \left[x^2 + 2x \right] + 3$

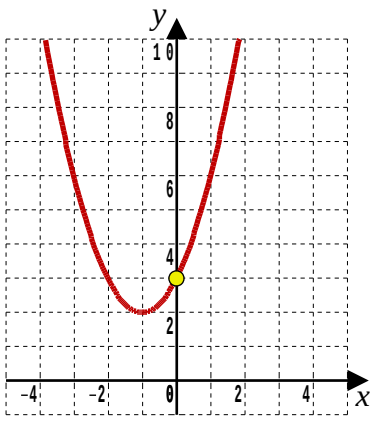
$= \left[(x + 1)^2 - 1 \right] + 3$

$= (x + 1)^2 + 2$

$\therefore\ a = 1$ and $b = 2$

[2 marks]

(b)



[3 marks]

(c) $D = 2^2 - 4 \times 1 \times 3$

$= -8$

As this is negative, the equation has no roots.

Graphically, this corresponds to there being no x-axis interceptions.

(d) $D = k^2 - 4 \times 1 \times 3$

$= k^2 - 12$

$\therefore$ For no real roots, $k^2 - 12 < 0$

$k^2 < 12$

$-2\sqrt{3} < k < 2\sqrt{3}$

[4 marks]

Answer 8

(a)

$$x^2 - 6x + 18 = \left[x^2 - 6x \right] + 18$$

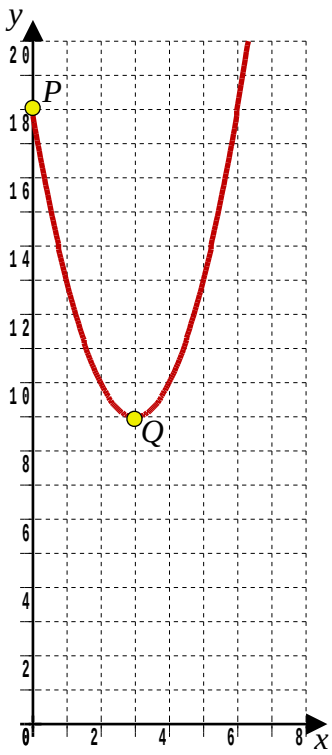
$$= \left[(x - 3)^2 - 9 \right] + 18$$

$$= (x - 3)^2 + 9$$

$$\therefore \quad a = 3 \text{ and } b = 9$$

[3 marks]

(b)



[3 marks]

(c)

$$(x - 3)^2 + 9 = 41$$

$$(x - 3)^2 = 32$$

$$x - 3 = \pm 4\sqrt{2}$$

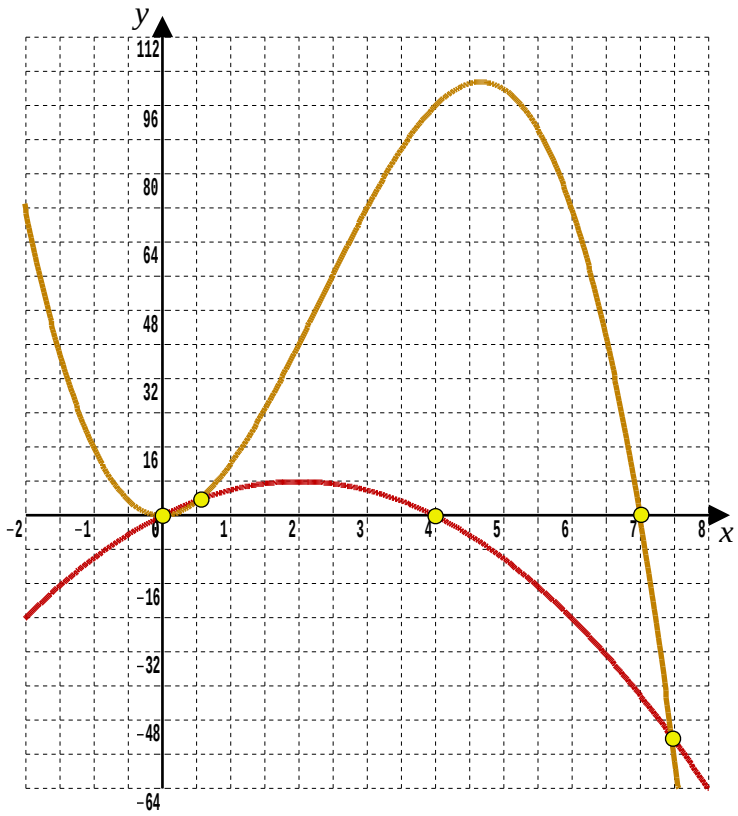
$$x = 3 \pm 4\sqrt{2}$$

$$\therefore \quad p = 3 \text{ and } r = 4 \text{ (reject } r = -4 \text{ as told } x \geqslant 0)$$

[5 marks]

Answer 9

(a)



[5 marks]

(b) Points of intersection will be found from,

$$x(4 - x) = x^2(7 - x)$$

$$4x - x^2 = 7x^2 - x^3$$

$$x^3 - 8x^2 + 4x = 0$$

$$x\left(x^2 - 8x + 4\right) = 0$$

[3 marks]

(c) The solution, $x = 0$ is rejected as the coordinates of A are both positive.

$$\therefore\ x^2 - 8x + 4 = 0$$
$$\left[x^2 - 8x \right] + 4 = 0$$
$$\left[(x-4)^2 - 16 \right] + 4 = 0$$
$$(x-4)^2 = 12$$
$$x = 4 - 2\sqrt{3} \quad \text{(must be lesser of two)}$$
$$y = \left(4 - 2\sqrt{3}\right)\left(4 - 4 + 2\sqrt{3}\right)$$
$$= \left(4 - 2\sqrt{3}\right)\left(2\sqrt{3}\right)$$
$$= 8\sqrt{3} - 12$$
$$= -12 + 8\sqrt{3}$$

$\therefore$ Intersection sought is $\left(4 - 2\sqrt{3}, -12 + 8\sqrt{3}\right)$

$\therefore p = 4,\ q = -2,\ r = -12$ and $s = 8$

[7 marks]