**2.3 Answers**



**Answer 1**

$$\int x \ \cos x \ dx$$

$$= \ \boldsymbol{L}(x) \quad \boldsymbol{I}(\cos x) - \int \ \boldsymbol{D}(x) \quad \boldsymbol{I}(\cos x) \ \ dx$$

$$= \quad x \qquad \sin x \ \ - \ \int \ 1 \qquad \sin x \quad dx$$

$$= \ x \ \sin x - \int \sin x \ dx$$

$$= \ x \ \sin x - ( \ - \cos x \,) + c$$

$$= x \sin x + \cos x + c$$

**[ 3 marks ]**

**Answer 2**

$$\int x \ \sin 4x \ dx$$

$$= \ \boldsymbol{L} \qquad \boldsymbol{I} \qquad\qquad \boldsymbol{D} \qquad \boldsymbol{I}$$

$$= x \quad \left(-\frac{\cos 4x}{4}\right) \ - \ \int \ 1 \ \left(-\frac{\cos 4x}{4}\right) \quad dx$$

$$= -\frac{x \cos 4x}{4} + \frac{1}{4} \int \cos 4x \ dx$$

$$= -\frac{x \cos 4x}{4} + \frac{1}{4} \ \frac{\sin 4x}{4} + c$$

$$= -\frac{x \cos 4x}{4} + \frac{\sin 4x}{16} + c$$

**[ 4 marks ]**

**Answer 3**

$$\int x^5 \ \ln x \ dx$$

$$= \int \ln x \ x^5 \ dx$$ Swap the order as $D(\ln x)$ simplifies

$$= \ \boldsymbol{L} \qquad \boldsymbol{I} \qquad\qquad \boldsymbol{D} \qquad \boldsymbol{I}$$

$$= \ln x \quad \frac{x^6}{6} \ - \ \int \ \frac{1}{x} \quad \frac{x^6}{6} \quad dx$$

$$= \frac{x^6 \ln x}{6} - \frac{1}{6} \int x^5 \ dx$$

$$= \frac{x^6 \ln x}{6} - \frac{x^6}{36} + c$$

**[ 3 marks ]**

**Answer 4**

$$\int x\ e^{x}\ dx$$

$$= \boldsymbol{L} \quad \boldsymbol{I} \qquad \boldsymbol{D} \quad \boldsymbol{I}$$

$$= x\ \ e^{x} - \int 1\ \ e^{x}\ dx$$

$$= x\ e^{x} - \int e^{x}\ dx$$

$$= x\ e^{x} - e^{x} + c$$

**[ 3 marks ]**

**Answer 5**

$$\int x\ e^{-5x}\ dx$$

$$= \boldsymbol{L} \qquad \boldsymbol{I} \qquad \boldsymbol{D} \qquad \boldsymbol{I}$$

$$= x\ \ \frac{e^{-5x}}{(-5)} - \int 1\ \frac{e^{-5x}}{(-5)}\ dx$$

$$= -\frac{x\,e^{-5x}}{5} + \frac{1}{5}\int e^{-5x}\ dx$$

$$= -\frac{x\,e^{-5x}}{5} + \frac{1}{5}\ \frac{e^{-5x}}{(-5)} + c$$

$$= -\frac{x\,e^{-5x}}{5} - \frac{e^{-5x}}{25} + c$$

**[ 4 marks ]**

**Answer 6**

$$\int \frac{x}{2\,e^{x}}\ dx = \frac{1}{2}\int x\ e^{-x}\ dx$$

$$= \qquad \boldsymbol{L} \quad \boldsymbol{I} \qquad \boldsymbol{D} \qquad \boldsymbol{I}$$

$$= \frac{1}{2}\left(x\ \frac{e^{-x}}{(-1)} - \int 1\ \frac{e^{-x}}{(-1)}\ dx\right)$$

$$= \frac{1}{2}\left(-x\,e^{-x} + \int e^{-x}\ dx\right)$$

$$= -\frac{x\,e^{-x}}{2} - \frac{1}{2}\,e^{-x} + c$$

**[ 4 marks ]**

**Answer 7**

**( i )**

$$\int (3x+1)^6\,dx = \frac{1}{3}\int 3(3x+1)^6\,dx$$

$$= \frac{1}{3}\;\frac{(3x+1)^7}{7} + c$$

$$= \frac{(3x+1)^7}{21} + c$$

**[ 2 marks ]**

**( ii )**

$$\int x(3x+1)^6\,dx$$

$$= \boldsymbol{L} \qquad \boldsymbol{I} \qquad \boldsymbol{D} \qquad \boldsymbol{I}$$

$$= x\,\frac{(3x+1)^7}{21} - \int 1\,\frac{(3x+1)^7}{21}\,dx$$

$$= \frac{x(3x+1)^7}{21} - \frac{1}{21}\int (3x+1)^7\,dx$$

$$= \frac{x(3x+1)^7}{21} - \frac{1}{21}\,\frac{1}{3}\int 3(3x+1)^7\,dx$$

$$= \frac{x(3x+1)^7}{21} - \frac{1}{21\times 3}\,\frac{(3x+1)^8}{8} + c$$

$$= \frac{x(3x+1)^7}{21} - \frac{(3x+1)^8}{21\times 24} + c$$

**[ 4 marks ]**

**( iii )**

$$= \frac{(3x+1)^7}{21}\left(x - \frac{3x+1}{24}\right) + c$$

$$= \frac{(3x+1)^7}{21}\left(\frac{24x - 3x - 1}{24}\right) + c$$

$$= \frac{(3x+1)^7}{21}\left(\frac{21x - 1}{24}\right) + c$$

$$= \frac{1}{504}\,(3x+1)^7(21x-1) + c$$

**[ 3 marks ]**

**Answer 8**

$$\int x \ \sin 3x \ dx$$

$$= \quad \boldsymbol{L} \qquad \boldsymbol{I} \qquad \boldsymbol{D} \qquad \boldsymbol{I}$$

$$= x \left(-\frac{\cos 3x}{3}\right) - \int 1 \left(-\frac{\cos 3x}{3}\right) dx$$

$$= -\frac{x \cos 3x}{3} + \frac{1}{3} \int \cos 3x \ dx$$

$$= -\frac{x \cos 3x}{3} + \frac{1}{3} \ \frac{\sin 3x}{3} + c$$

$$= -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9} + c$$

$$\therefore \int_0^{\frac{\pi}{3}} x \ \sin 3x \ dx = \left[-\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x\right]_0^{\frac{\pi}{3}}$$

$$= \left[-\frac{1}{3} \ \frac{\pi}{3} \ \cos(\pi) + \frac{1}{9} \sin(\pi)\right] - [\,0\,]$$

$$= \frac{\pi}{9}$$

**[ 6 marks ]**

**Answer 9**

$$\int (2x + 1) \ e^x \ dx$$

$$= \quad \boldsymbol{L} \qquad \boldsymbol{I} \qquad \boldsymbol{D} \quad \boldsymbol{I}$$

$$= (2x + 1) \ e^x - \int 2 \ e^x \ dx$$

$$= (2x + 1) \ e^x - 2 \ e^x + c$$

$$\therefore \int_0^1 (2x + 1) \ e^x \ dx = \left[ (2x + 1) \ e^x - 2 \ e^x \right]_0^1$$

$$= \left[3 e^1 - 2 e^1\right] - \left[e^0 - 2 e^0\right]$$

$$= e - [-1]$$

$$= e + 1$$

**[ 6 marks ]**

**Answer 10**

$$u = 3x - 5 \Rightarrow x = \frac{u}{3} + \frac{5}{3} \Rightarrow \frac{dx}{du} = \frac{1}{3}$$

When $x = 2,\ u = 1$ and when $x = 1,\ u = -2$

$$7\int_1^2 x^2\,(3x - 5)^4\,dx = 7\int_{-2}^{1}\left(\frac{u}{3} + \frac{5}{3}\right)^2 u^4\ \frac{1}{3}\ du$$

$$= \frac{7}{3}\int_{-2}^{1} \frac{1}{3^2}(u + 5)^2\ u^4\ du$$

$$= \frac{7}{27}\int_{-2}^{1} (u + 5)^2\ u^4\ du$$

$$= \frac{7}{27}\int_{-2}^{1} \left(u^2 + 10u + 25\right) u^4\ du$$

$$= \frac{7}{27}\int_{-2}^{1} \left(u^6 + 10\,u^5 + 25\,u^4\right)\ du$$

$$= \frac{7}{27}\left[\frac{u^7}{7} + \frac{10\,u^6}{6} + \frac{25\,u^5}{5}\right]_{-2}^{1}$$

$$= \frac{7}{27}\left\{\left[\frac{1}{7} + \frac{5}{3} + 5\right] - \left[-\frac{128}{7} + \frac{320}{3} - 160\right]\right\}$$

$$= \frac{7}{27}\left\{\left[\frac{143}{21}\right] - \left[-\frac{1504}{21}\right]\right\}$$

$$= \frac{7}{27}\left\{\frac{549}{7}\right\}$$

$$= \frac{61}{3}$$

$$= 20\,\frac{1}{3}$$

**[ 8 marks ]**