## 4.3 Answers



### 4.3.1 Solutions to 4.2 Exercise

**Answer 1**

$$\int x \ \sin 2x \ dx$$

$$= \ \boldsymbol{L} \qquad \boldsymbol{I} \qquad\qquad \boldsymbol{D} \qquad \boldsymbol{I}$$

$$= x \left(-\frac{\cos 2x}{2}\right) - \int 1 \left(-\frac{\cos 2x}{2}\right) dx$$

$$= -\frac{x \cos 2x}{2} + \frac{1}{2}\int \cos 2x \ dx$$

$$= -\frac{x \cos 2x}{2} + \frac{1}{2}\ \frac{\sin 2x}{2} + c$$

$$= -\frac{x \cos 2x}{2} + \frac{\sin 2x}{4} + c$$

$$\therefore \int_0^{\frac{\pi}{2}} x \ \sin 2x \ dx = \left[-\frac{1}{2} x \cos 2x + \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}}$$

$$= \left[-\frac{1}{2}\ \frac{\pi}{2} \ \cos \ (\pi) + \frac{1}{4} \sin \ (\pi)\right] - [\,0\,]$$

$$= \frac{\pi}{4}$$

**[ 6 marks ]**

**Answer 2**

**( a )**

$$\int x \ \sin 3x \ dx$$

$$= \quad \boldsymbol{L} \qquad \boldsymbol{I} \qquad\qquad \boldsymbol{D} \qquad \boldsymbol{I}$$

$$= x \left( -\frac{\cos 3x}{3} \right) - \int 1 \left( -\frac{\cos 3x}{3} \right) dx$$

$$= -\frac{x \cos 3x}{3} + \frac{1}{3} \int \cos 3x \ dx$$

$$= -\frac{x \cos 3x}{3} + \frac{1}{3} \ \frac{\sin 3x}{3} + c$$

$$= -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9} + c$$

**[ 3 marks ]**

**( b )**

$$\int x^2 \cos 3x \ dx$$

$$= \quad \boldsymbol{L} \qquad \boldsymbol{I} \qquad\qquad \boldsymbol{D} \qquad \boldsymbol{I}$$

$$= x^2 \left( \frac{\sin 3x}{3} \right) - \int 2x \left( \frac{\sin 3x}{3} \right) dx$$

$$= \frac{x^2 \sin 3x}{3} - \frac{2}{3} \int x \ \sin 3x \ dx$$

$$= \frac{x^2 \sin 3x}{3} - \frac{2}{3} \left[ -\frac{x \cos 3x}{3} + \frac{\sin 3x}{9} \right] + c$$

$$= \frac{x^2 \sin 3x}{3} + \frac{2x \cos 3x}{9} - \frac{2 \sin 3x}{27} + c$$

**[ 3 marks ]**

**Answer 3**

**( a )**

$$\int x \ \cos 2x \ dx$$

$$= \ \boldsymbol{L} \qquad \boldsymbol{I} \qquad\qquad \boldsymbol{D} \qquad \boldsymbol{I}$$

$$= x \left( \frac{\sin 2x}{2} \right) - \int 1 \left( \frac{\sin 2x}{2} \right) dx$$

$$= \frac{x \sin 2x}{2} - \frac{1}{2} \int \sin 2x \ dx$$

$$= \frac{x \sin 2x}{2} - \frac{1}{2} \left( - \frac{\cos 2x}{2} \right) + c$$

$$= \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + c$$

**[ 4 marks ]**

**( b )**

$$\int x \cos^2 x \ dx = \int x \left( \frac{\cos 2x + 1}{2} \right) dx$$

$$= \frac{1}{2} \int x \cos 2x \ dx + \frac{1}{2} \int x \ dx$$

$$= \frac{1}{2} \left[ \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} \right] + \frac{1}{2} \ \frac{x^2}{2} + c$$

$$= \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + \frac{x^2}{4} + c$$

**[ 3 marks ]**

**Answer 4**

**( i )**

$$\int ln\left(\frac{x}{2}\right) dx = \int ln\,x\,dx - \int ln\,2\ \ dx$$

$$= \int ln\,x \quad \times\ 1\ \ dx - x\,ln\,2$$

$$= \ \ \boldsymbol{L} \quad \boldsymbol{I} \qquad \boldsymbol{D} \quad \boldsymbol{I}$$

$$= ln\,x\ \ x - \int \frac{1}{x}\ \ x\ \ dx - x\,ln\,2$$

$$= x\,ln\,x - \int 1\ \ dx - x\,ln\,2$$

$$= x\,ln\,x - x - x\,ln\,2 + c$$

$$= x\ \ ln\left(\frac{x}{2}\right) - x + c$$

**[ 4 marks ]**

**( ii )**

First the required trigonometric formulae...

$$cos^2 x + sin^2 x = 1$$

$$\text{SUBTRACT} \qquad cos^2 x - sin^2 x = cos\,2x$$

$$2\,sin^2 x = 1 - cos\,2x$$

$$\therefore \quad sin^2 x = \frac{1}{2} - \frac{1}{2}\,cos\,2x$$

Thus;

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} sin^2 x\ \ dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 1 - \ \ cos\,2x \quad dx$$

$$= \frac{1}{2}\left[\ x - \frac{sin\,2x}{2}\ \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left\{\left[\frac{\pi}{2} - \frac{sin\,\pi}{2}\right] - \left[\frac{\pi}{4} - \frac{sin\left(\frac{\pi}{2}\right)}{2}\right]\right\}$$

$$= \frac{1}{2}\left\{\frac{\pi}{2} - 0 - \frac{\pi}{4} + \frac{1}{2}\right\}$$

$$= \frac{\pi}{8} + \frac{1}{4}$$

**[ 3 marks ]**

**Answer 5**

**( a )** $\int x\ e^x\ dx$

$= \boldsymbol{L \quad I \qquad D \quad I}$

$= x\ \ e^x - \int 1\ \ e^x\ dx$

$= x\ e^x - \int e^x\ dx$

$= x\ e^x - e^x + c$

**[ 3 marks ]**

**( b )** $\int x^2\ e^x\ dx$

$= \boldsymbol{L \quad I \qquad D \quad I}$

$= x^2\ \ e^x - \int 2x\ \ e^x\ dx$

$= x^2\ e^x - 2\int x\, e^x\ dx$

$= x^2\ e^x - 2\left( x\ e^x - e^x \right) + c$

**[ 3 marks ]**

**Answer 6**

**( a )** $cos^2 x + sin^2 x = 1$

Divide through by $cos^2 x$

$1 + tan^2 x = sec^2 x$

Thus, $\int tan^2 x\ dx = \int sec^2 x - 1\ dx$

$= tan\, x - x + c$

**[ 2 marks ]**

**( b )** $\int \frac{1}{x^3}\, ln\, x\ \ dx = \int\ ln\, x\ \ x^{-3}\ \ dx$

$= \boldsymbol{L \qquad I \qquad D \quad I}$

$= ln\, x\ \ \frac{x^{-2}}{(-2)} - \int \frac{1}{x}\ \frac{x^{-2}}{(-2)}\ dx$

$= -\frac{ln\, x}{2\, x^2} + \frac{1}{2} \int x^{-3}\, dx$

$= -\frac{ln\, x}{2\, x^2} - \frac{x^{-2}}{4} + c$

$= -\frac{ln\, x}{2\, x^2} - \frac{1}{4\, x^2} + c$

**[ 4 marks ]**

**( c )**

$$u = 1 + e^x \Rightarrow x = ln\ (u - 1) \Rightarrow \frac{du}{dx} = e^x$$

$$\begin{aligned}
\int \frac{e^{3x}}{1 + e^x}\, dx &= \int \frac{e^{2x}\, e^x}{u}\ \frac{1}{e^x}\, du \\
&= \int \frac{e^{2\, ln\, (u - 1)}}{u}\, du \\
&= \int \frac{e^{\, ln\, (u - 1)^2}}{u}\, du \\
&= \int \frac{(u - 1)^2}{u}\, du \\
&= \int \frac{u^2}{u} - \frac{2u}{u} + \frac{1}{u}\, du \\
&= \int u - 2 + \frac{1}{u}\ du \\
&= \frac{u^2}{2} - 2u + ln\, u + c \\
&= \frac{(1 + e^x)^2}{2} - 2(1 + e^x) + ln\ (1 + e^x) + c_1 \\
&= \frac{1}{2} + e^x + \frac{e^{2x}}{2} - 2 - 2\, e^x + ln(1 + e^x) + c_1 \\
&= \frac{e^{2x}}{2} - e^x + ln\ (1 + e^x) + k \qquad \square
\end{aligned}$$

**[ 7 marks ]**