

6.3 Answers

A-Level Pure Mathematics : Year 2

Integration III

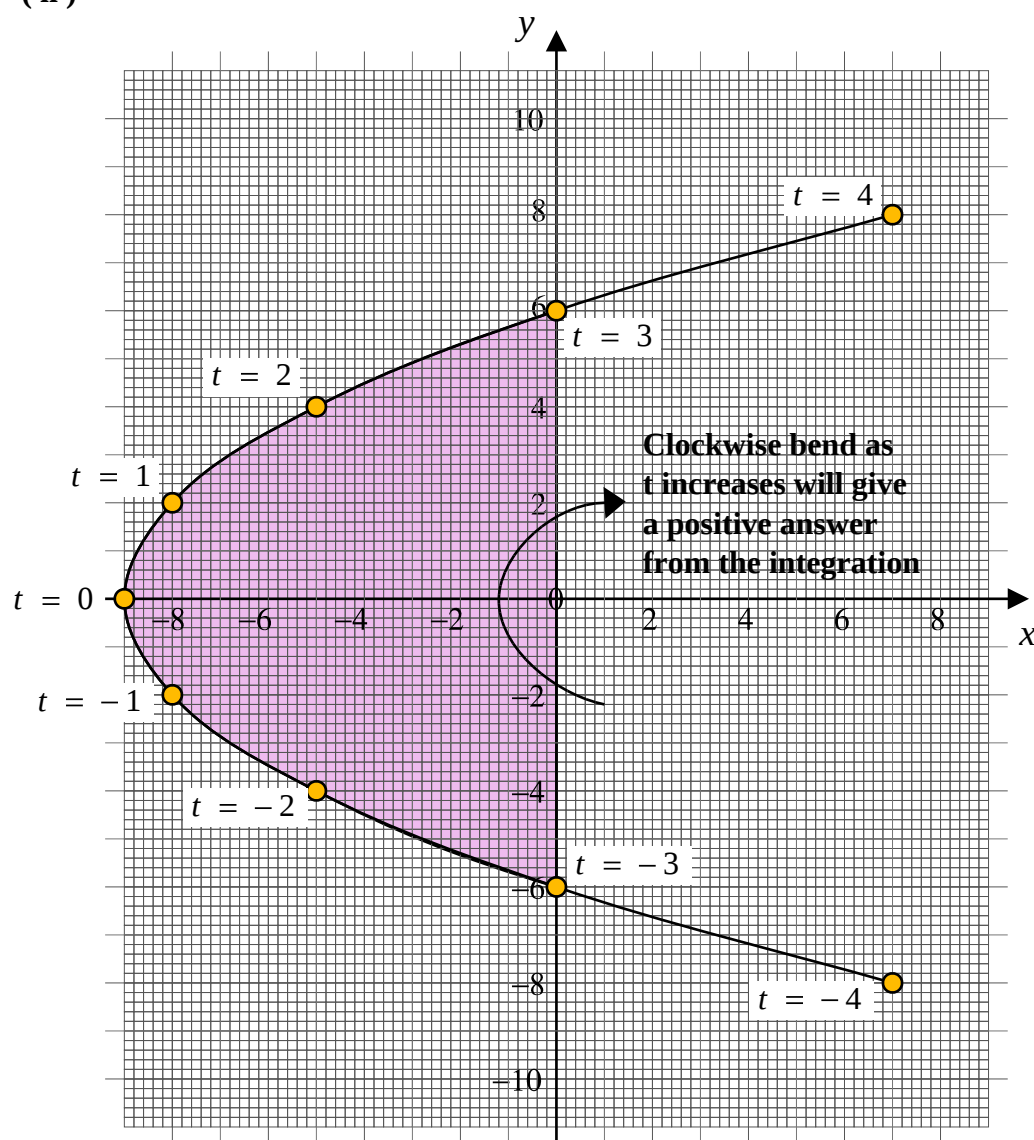
6.3.1 Solution to Introduction Question

(i)

t	-4	-3	-2	-1	0	1	2	3	4
x	7	0	-5	-8	-9	-8	-5	0	7
y	-8	-6	-4	-2	0	2	4	6	8

[3 marks]

(ii)



[3 marks]

(iii) Area shaded in as shown above.

[1 mark]

(iv)

$$x = t^2 - 9 \Rightarrow \frac{dx}{dt} = 2t$$

$$\begin{aligned} \text{Area} &= \int y \, dx \\ &= \int_{t=-3}^{t=3} 2t \cdot 2t \, dt \\ &= 4 \int_{-3}^3 t^2 \, dt \\ &= 4 \left[\frac{t^3}{3} \right]_{-3}^3 \\ &= 4 \{ [9] - [-9] \} \\ &= 72 \end{aligned}$$

[4 marks]

(v)

$$\begin{aligned} y = 2t &\Rightarrow y^2 = 4t^2 \Rightarrow t^2 = \frac{y^2}{4} \\ x = t^2 - 9 &\Rightarrow x = \frac{y^2}{4} - 9 \Rightarrow y^2 = 4(x + 9) \\ y &= \pm 2(x + 9)^{\frac{1}{2}} \end{aligned}$$

[2 marks]

(vi)

$$\begin{aligned} \int y \, dx &= 2 \times \int_{x=-9}^{x=0} 2(x + 9)^{\frac{1}{2}} \, dx \\ &= 4 \int_{-9}^0 (x + 9)^{\frac{1}{2}} \, dx \\ &= 4 \left[\frac{2(x + 9)^{\frac{3}{2}}}{3} \right]_{-9}^0 \\ &= \frac{8}{3} \left[(x + 9)^{\frac{3}{2}} \right]_{-9}^0 \\ &= \frac{8}{3} \{ 9^{\frac{3}{2}} - 0 \} \\ &= 72 \quad \text{as before} \end{aligned}$$

[3 marks]

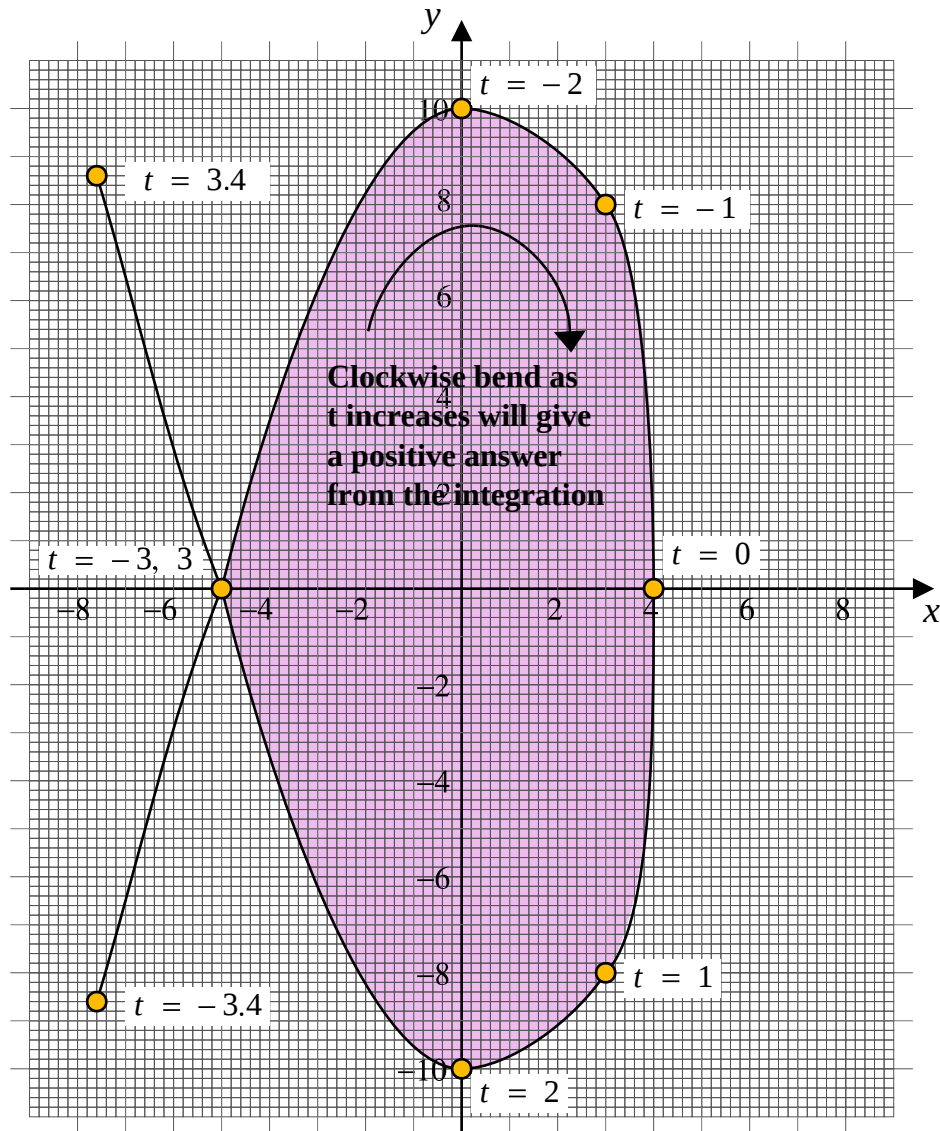
6.3.2 Solution to Exercise 6.2

(i)

t	-3.4	-3	-2	-1	0	1	2	3	3.4
x	-7.6	-5	0	3	4	3	0	-5	-7.6
y	-8.7	0	10	8	0	-8	-10	0	8.7

[3 marks]

(ii)



[3 marks]

(iii) Area shaded in as shown above.

[1 mark]

$$(iv) \quad x = 4 - t^2 \Rightarrow \frac{dx}{dt} = -2t$$

$$\begin{aligned} Area &= \int y dx \\ &= \int_{t=-3}^{t=3} t(t^2 - 9)(-2t) dt \\ &= -2 \int_{-3}^3 t^4 - 9t^2 dt \\ &= -2 \left[\frac{t^5}{5} - 3t^3 \right]_{-3}^3 \\ &= -2 \{ [48.6 - 81] - [-48.6 - (-81)] \} \\ &= 129.6 \end{aligned}$$

[4 marks]

$$\begin{aligned} (v) \quad x &= 4 - t^2 \Rightarrow t^2 = 4 - x \\ y &= t(t^2 - 9) \Rightarrow y = \pm(4 - x)^{\frac{1}{2}}(-1)(x + 5) \\ \therefore y &= \pm(4 - x)^{\frac{1}{2}}(x + 5) \end{aligned}$$

[4 marks]

$$(vi) \quad u = 4 - x \Rightarrow x = 4 - u \Rightarrow \frac{dx}{du} = -1$$

$$\text{When } x = -5, u = 9$$

$$\text{When } x = 4, u = 0$$

$$\begin{aligned} Area &= 2 \int_{-5}^4 (4 - x)^{\frac{1}{2}}(x + 5) dx \\ &= 2 \int_9^0 u^{\frac{1}{2}}(9 - u)(-1) du \\ &= 2 \int_9^0 u^{\frac{3}{2}} - 9u^{\frac{1}{2}} du \\ &= 2 \left[\frac{2u^{\frac{5}{2}}}{5} - 6u^{\frac{3}{2}} \right]_9^0 \\ &= 2 \times \{ 0 - (-64.8) \} \\ &= 129.6 \quad \text{as before} \end{aligned}$$

[5 marks]